Nonlinear dispersive waves in acoustics

Vassos Achilleos

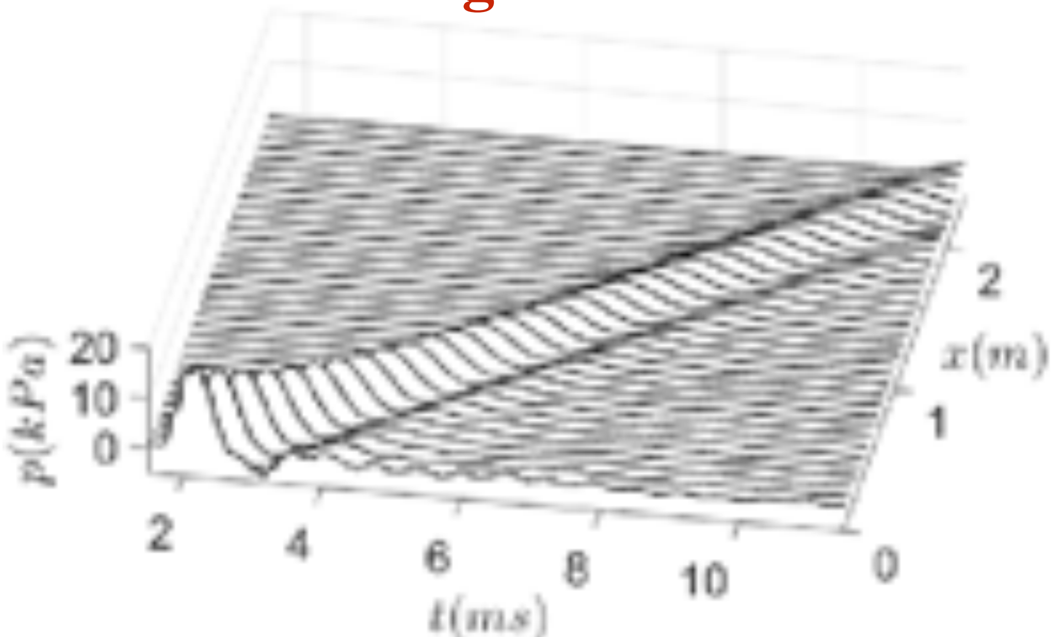
O. Richoux, G. Theocharis, C. Desjouy, V. Tournat

- Laboratoire d'Acoustique de l'Université du Mans (LAUM), UMR 6613, Institut d'Acoustique - Graduate School (IA-GS), CNRS, Le Mans Université, France

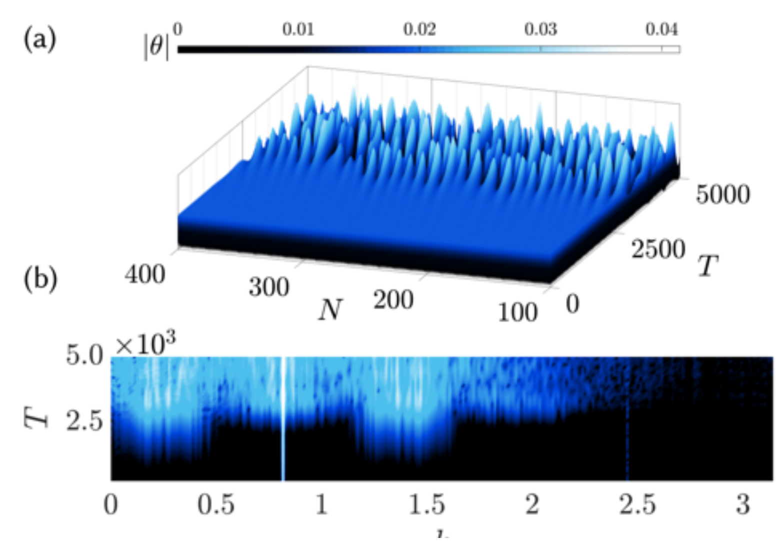
D. Frantzeskakis

- Section of Condensed Matter Physics, National and Kapodistrian University of Athens, University Campus, GR-157 84 Athens, Greece)

I. I. Sougleridis



A. Demiquel



Part 1: Solitons in airborne acoustics

Localised nonlinear waves in dispersive media

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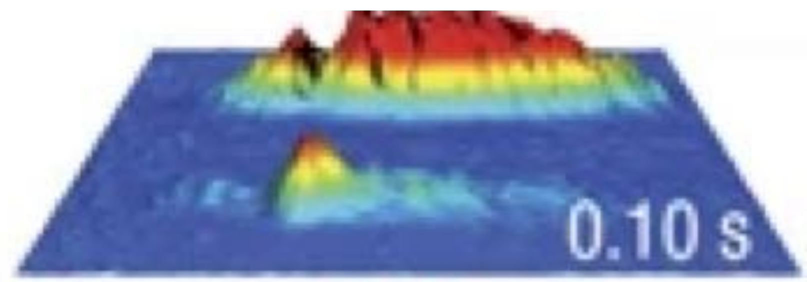
Localised nonlinear waves in dispersive media

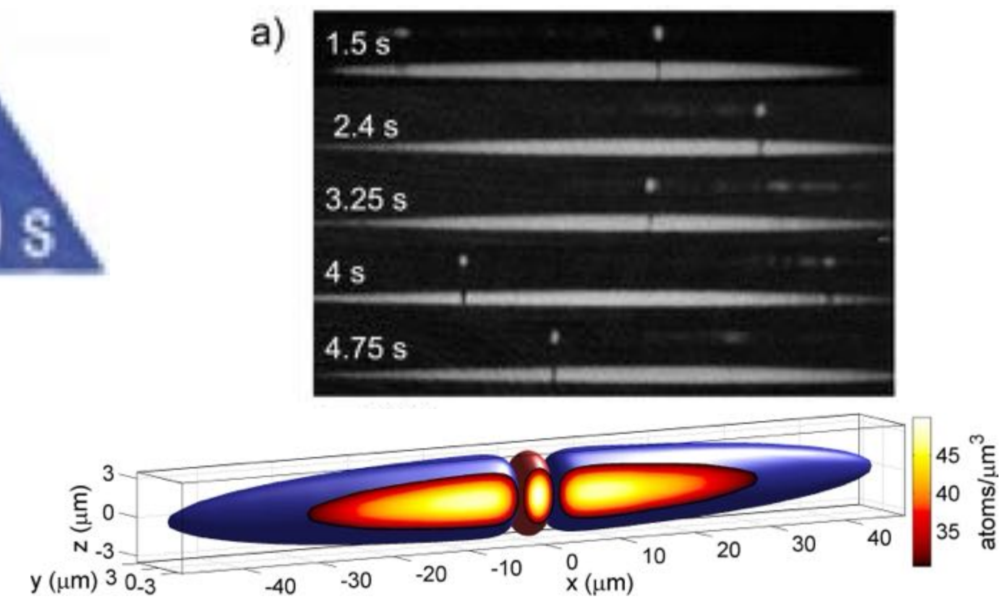
Free water waves

Superfluids : Bose-Einstein Condenstates

Nature Physics **4**, *496–501 (2008)*

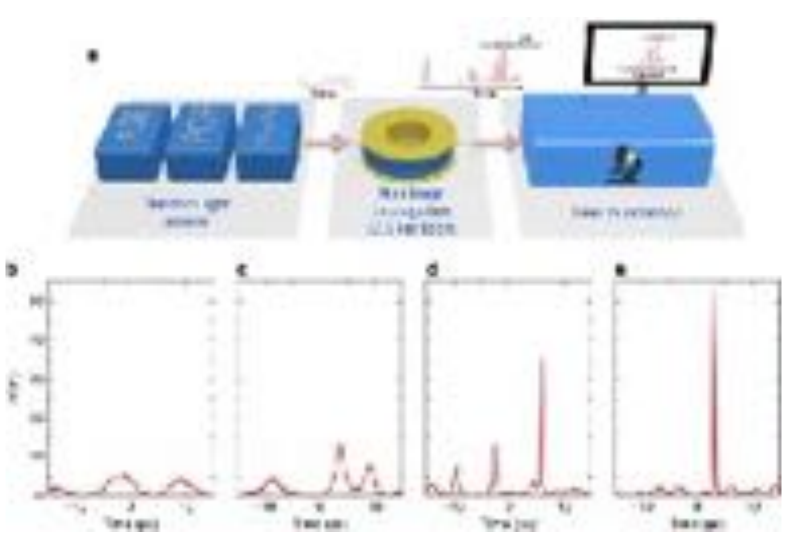


Phys. Lett. A 375 (2011) 642–646



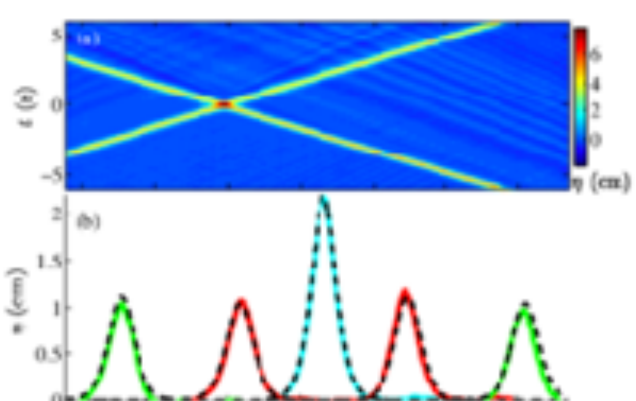
Optical fibers

Nature Communications
7, Article number: 13136 (2016)



Water tanks

Phys. Rev. Lett. **122**, 214502 (2019)



Localised nonlinear waves in dispersive media

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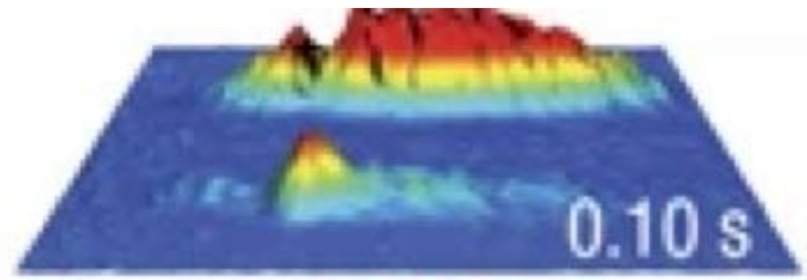

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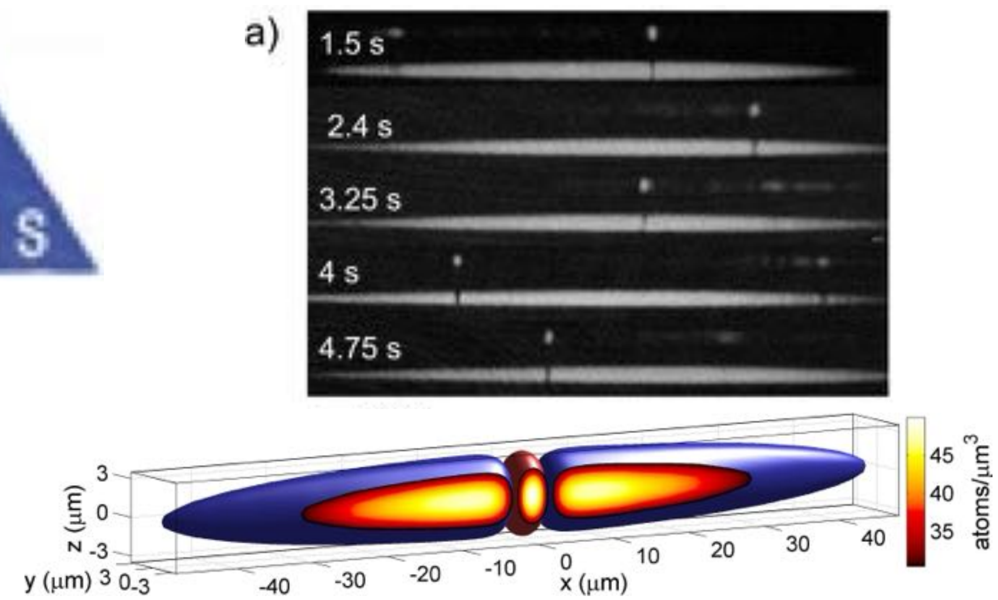

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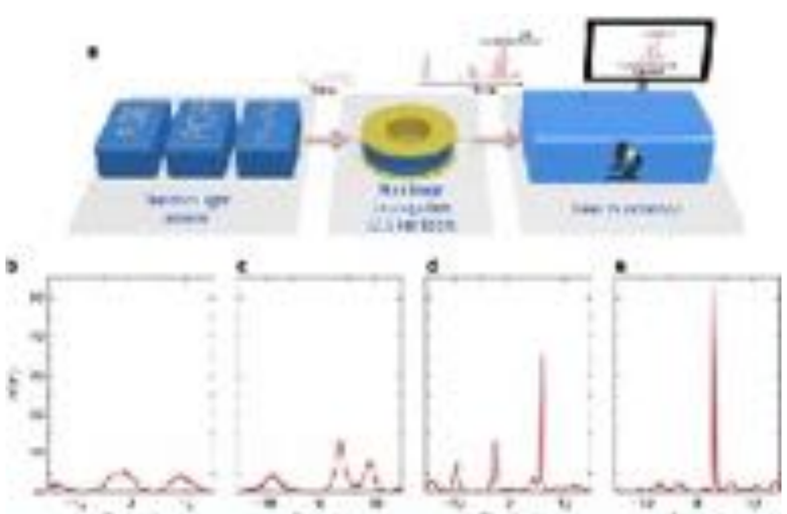

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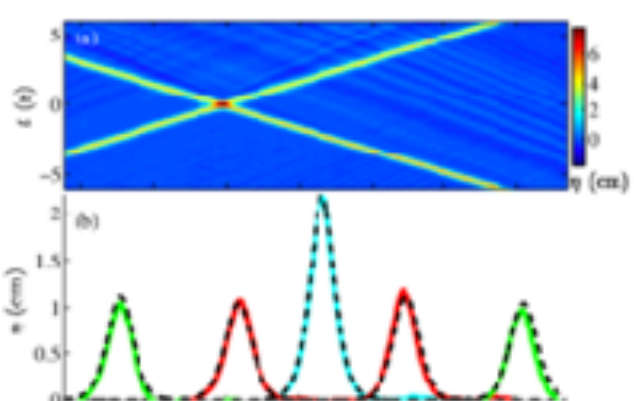
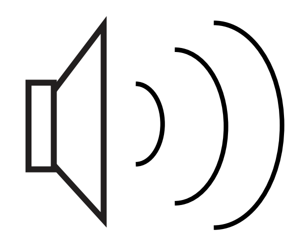
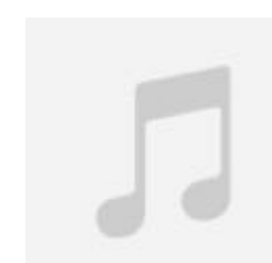


Why not in audible sound?





Part 1: Solitons in airborne acoustics

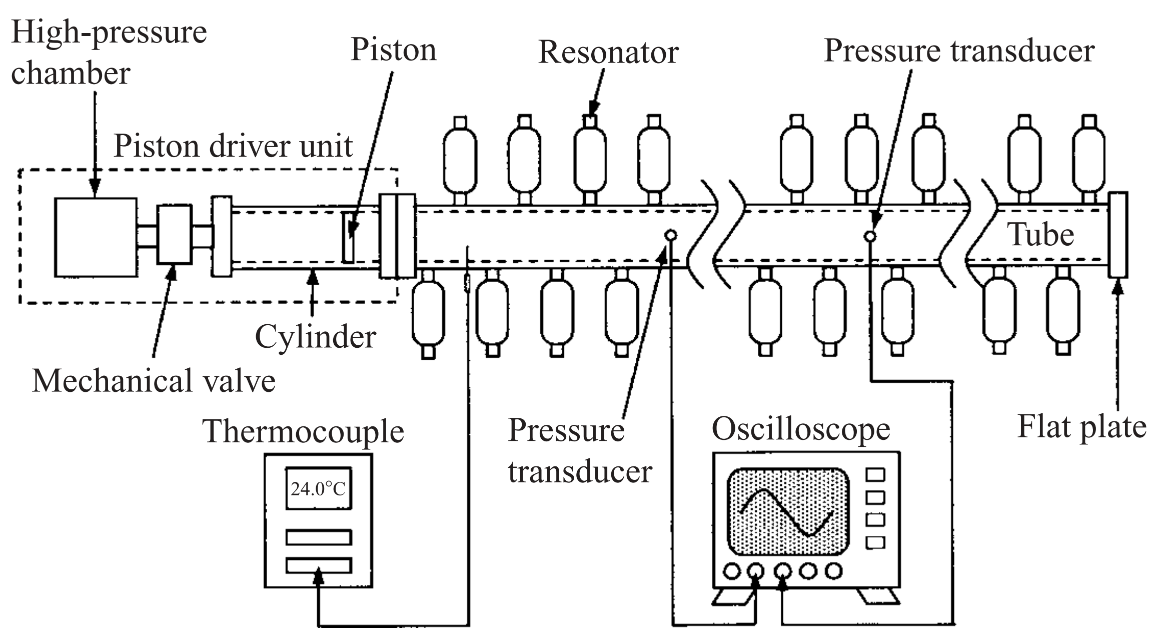
J. Fluid Mech. (2004), *vol.* 504, *pp.* 271–299. © 2004 Cambridge University Press 271
DOI: 10.1017/S0022112004008109 Printed in the United Kingdom

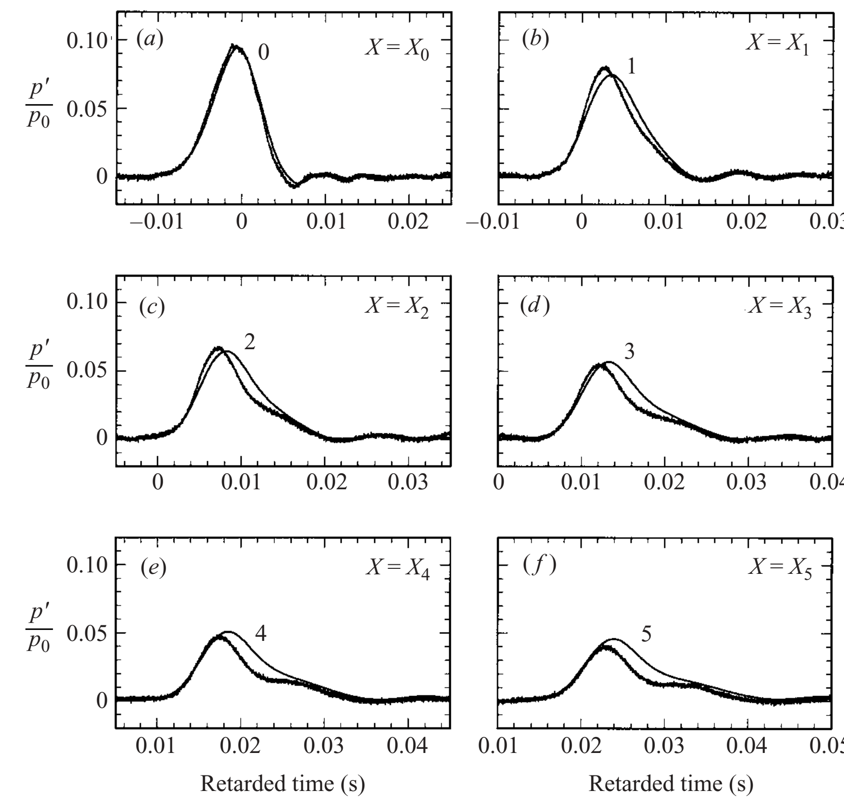
Verification of acoustic solitary waves

By N. SUGIMOTO, M. MASUDA, K. YAMASHITA AND H. HORIMOTO

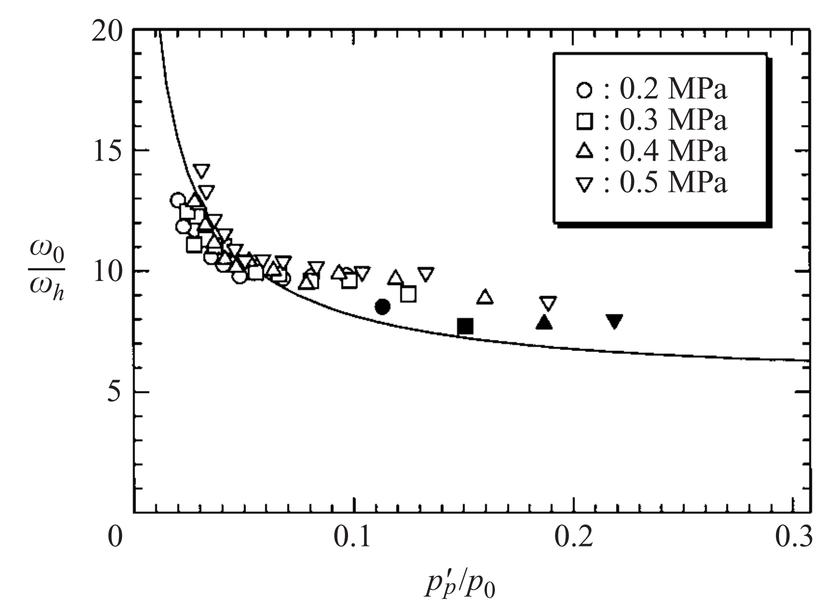
Division of Nonlinear Mechanics, Department of Mechanical Science, Graduate School of Engineering Science, University of Osaka, Toyonaka, Osaka 560-8531, Japan

(Received 22 July 2003 and in revised form 18 November 2003)





width of the soliton



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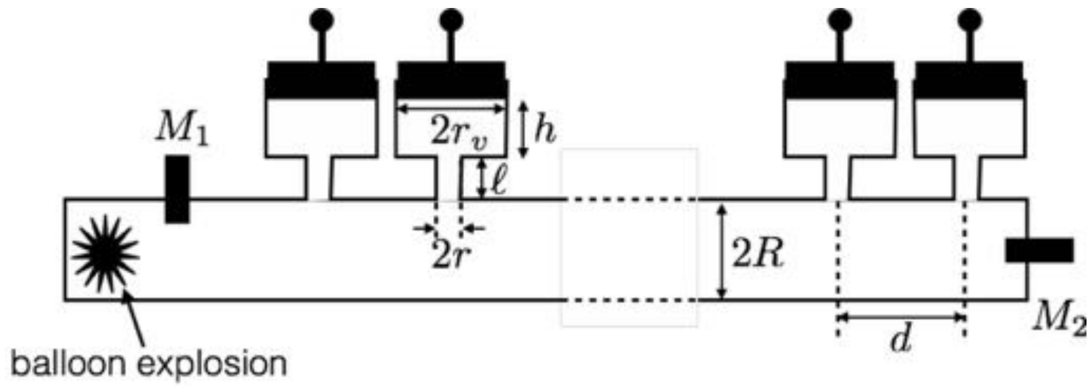
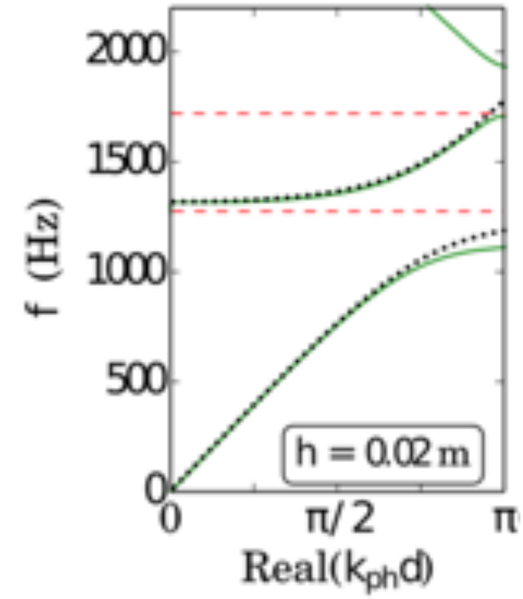
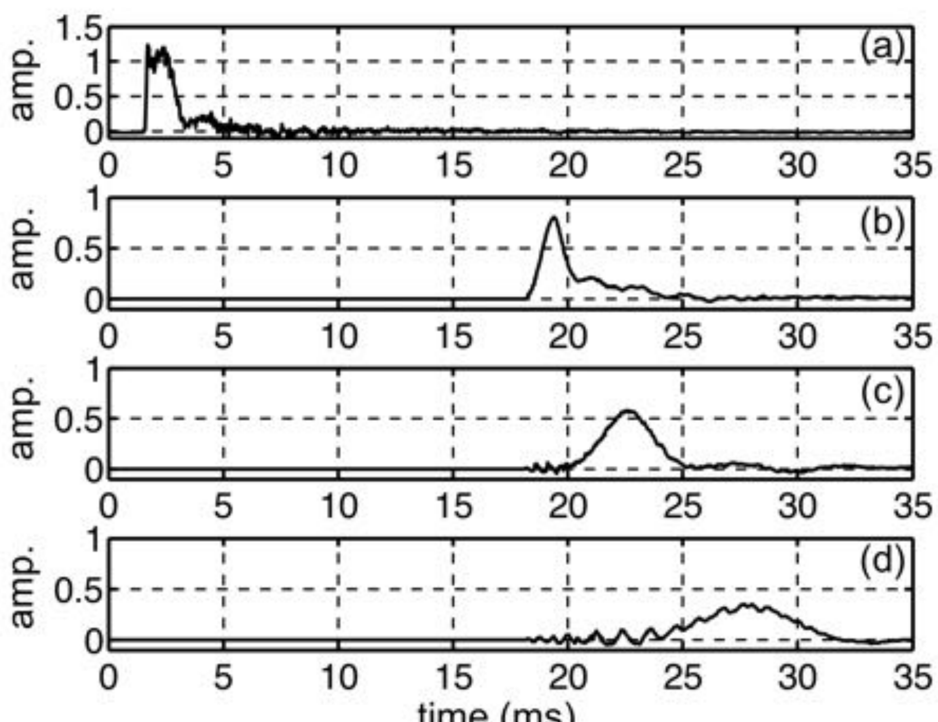




propagation velocity depends on particle velocity

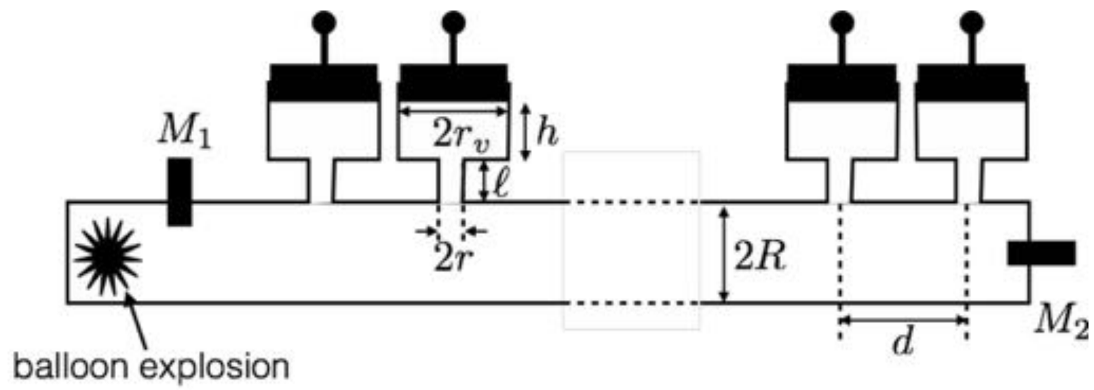
$$\frac{dx}{dt} = c_0 + \beta u \qquad \beta = 1 + \frac{B}{2A}$$

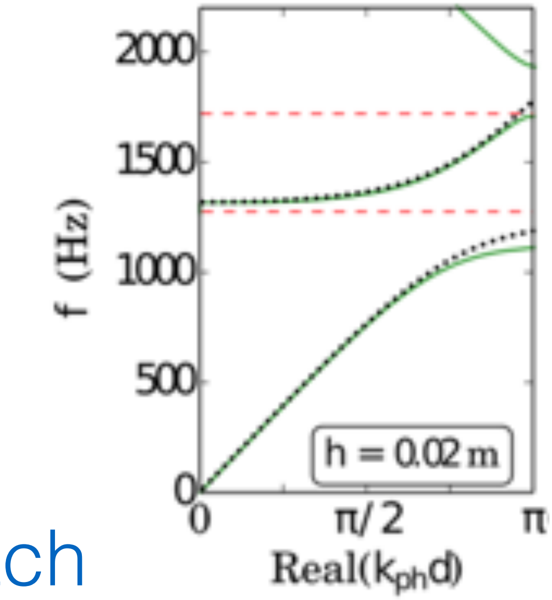


experiments

VA. O. Richoux, G. Theocharis. D J. Frantzeskakis
Phys. Rev. E **91**, 023204 (2015)

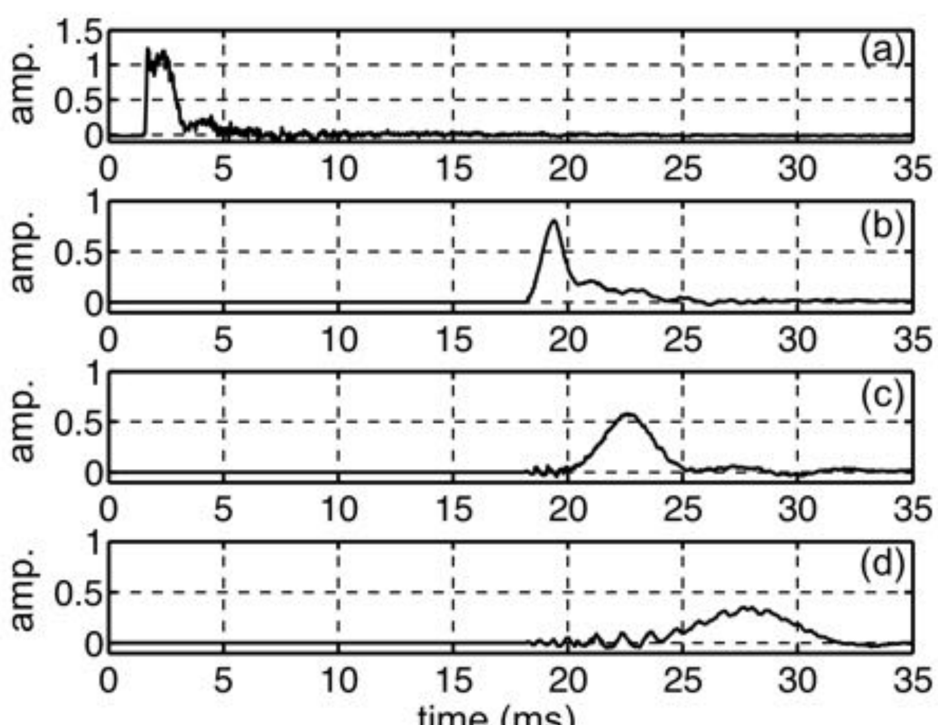
Part 1: Solitons in airborne acoustics





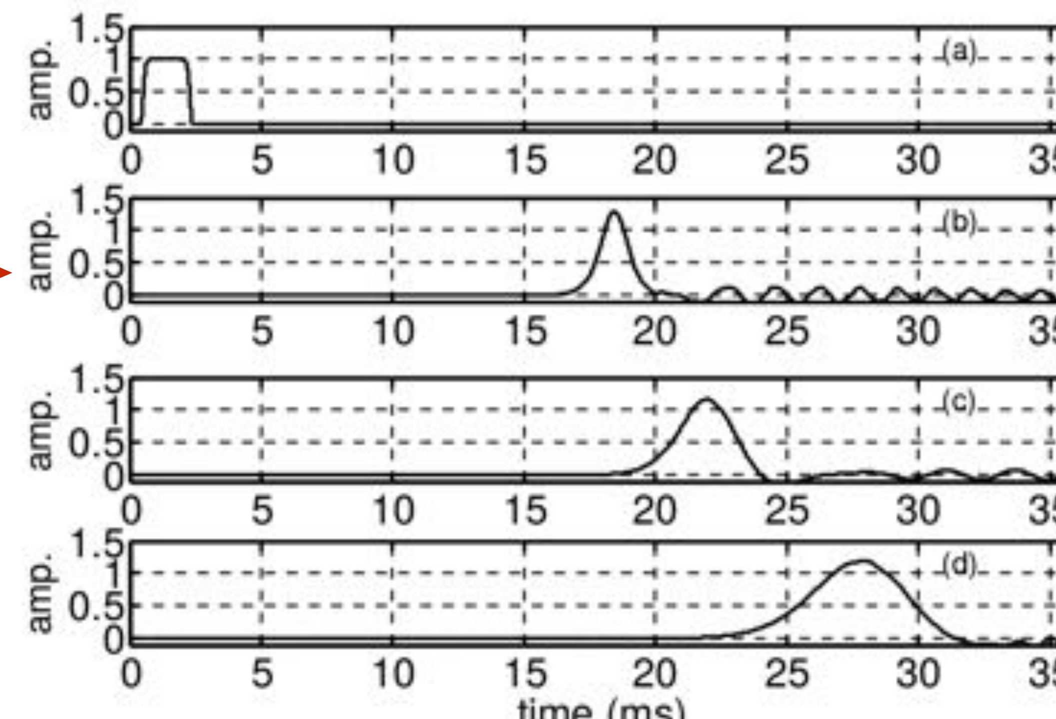
Electroacoustic analogue - Transmission line approach

$$\frac{d^2 p_n}{dt^2} - \frac{c_0^2}{\kappa d^2}\left(1+\frac{1}{\omega_0^2}\frac{d^2}{dt^2}\right)\hat{\delta}^2 p_n + \frac{1}{\kappa}\frac{d^2}{dt^2}\left(1+\frac{1}{\omega_0^2}\frac{d^2}{dt^2}\right)\left[p_n\left(1-2\frac{\beta_0}{\varrho_0 c_0^2}p_n\right)\right] = 0$$



analogue

experiments



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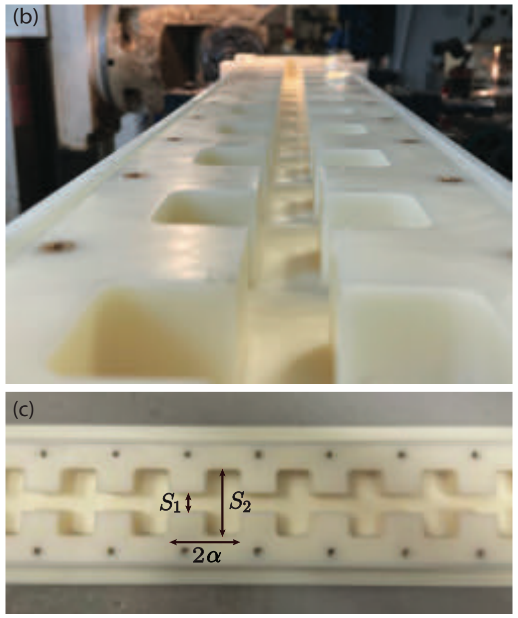

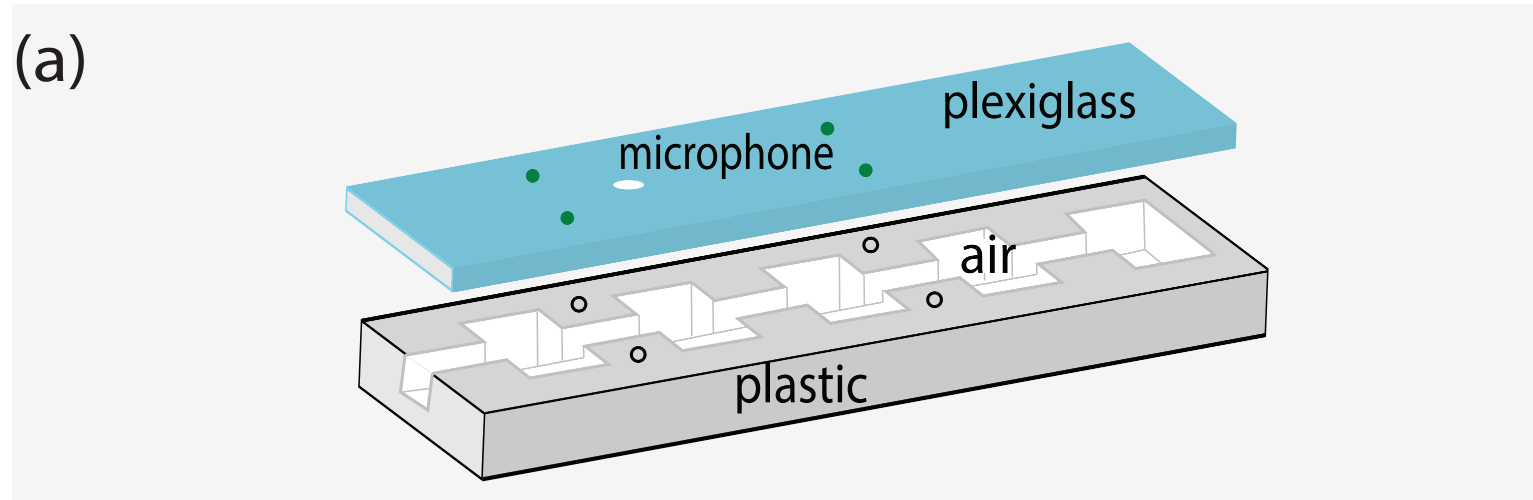

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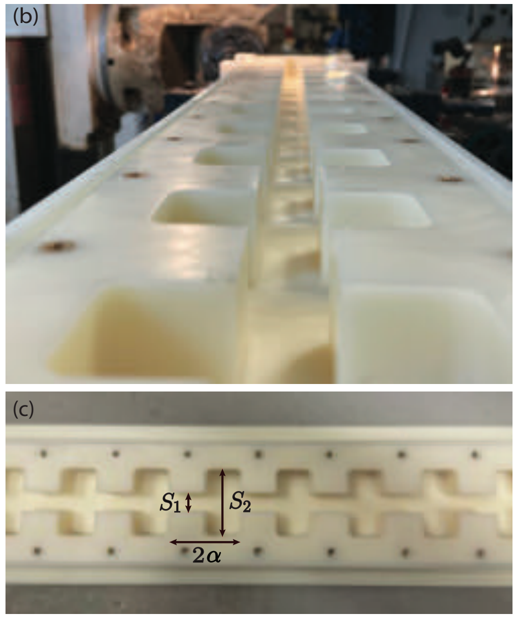
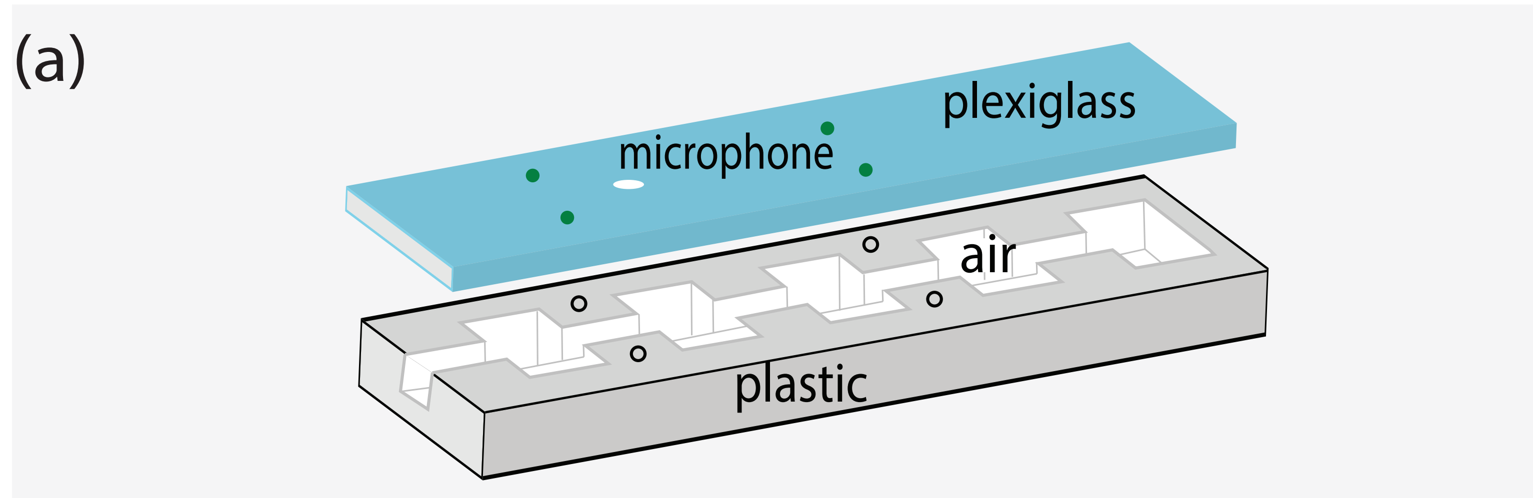
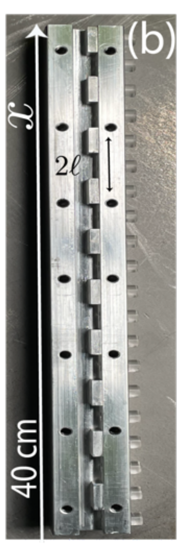
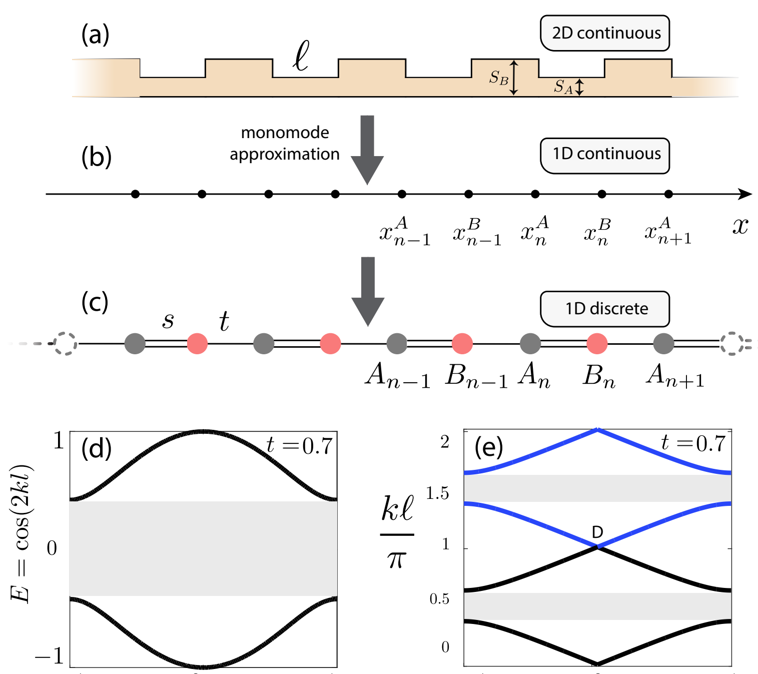
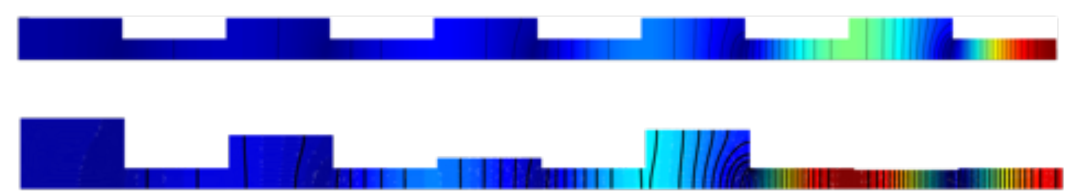


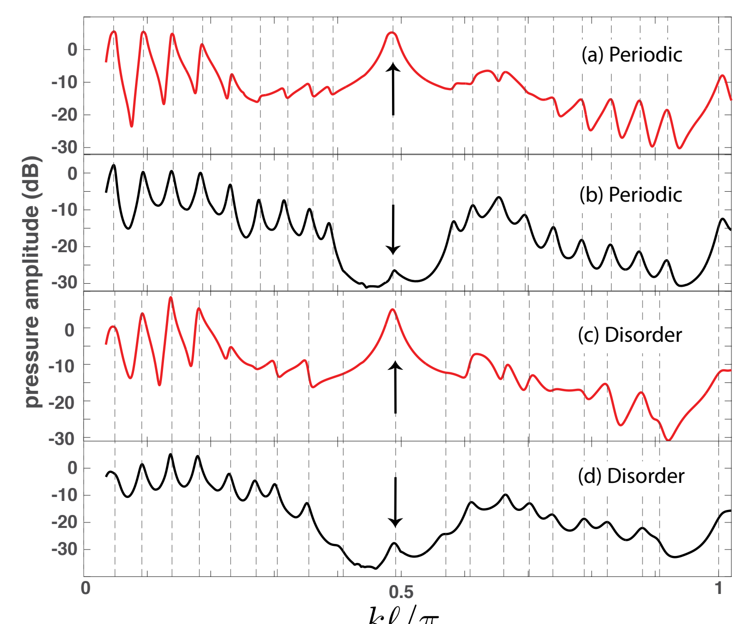


Inspired by the topological SSH model in acoustics



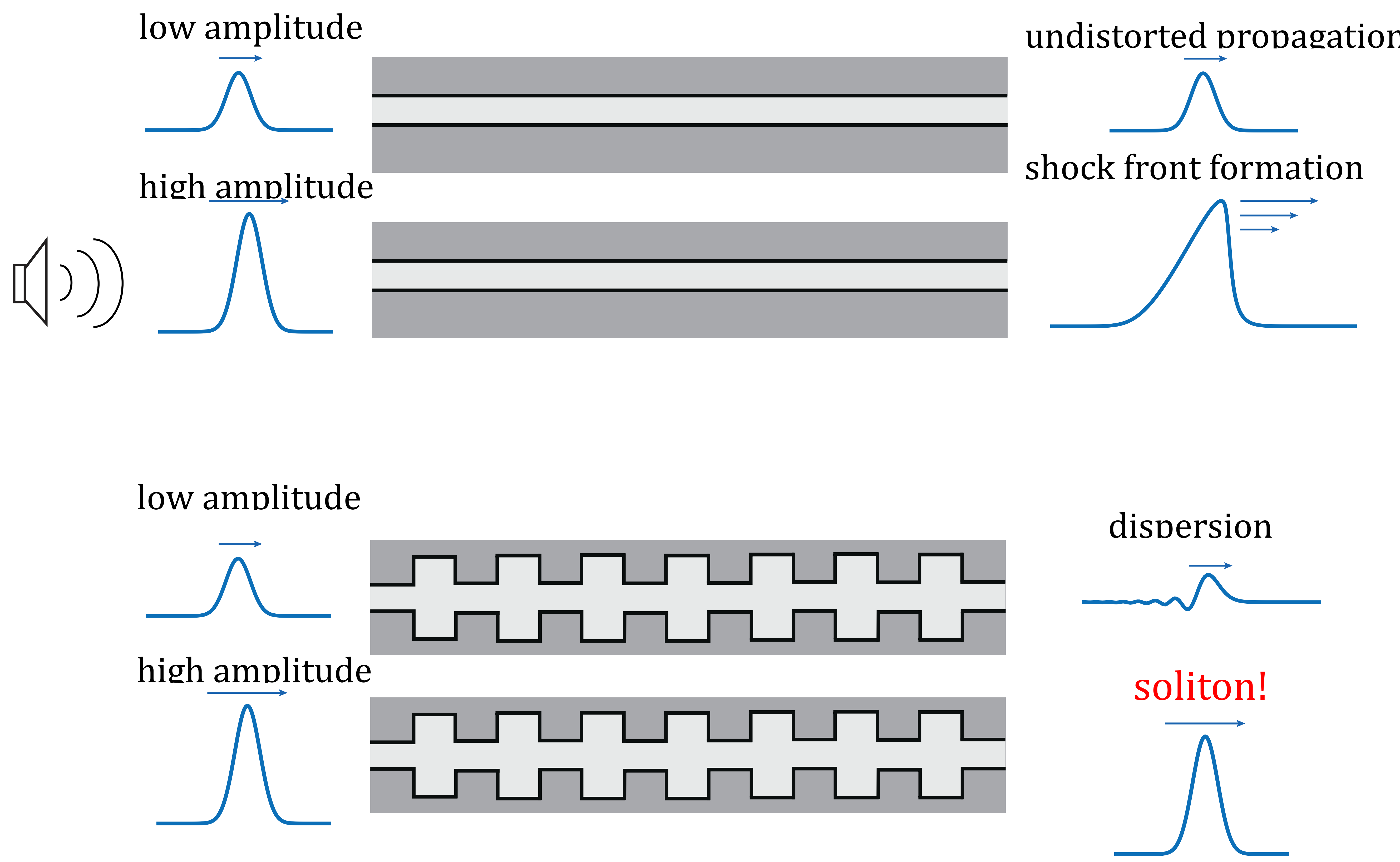






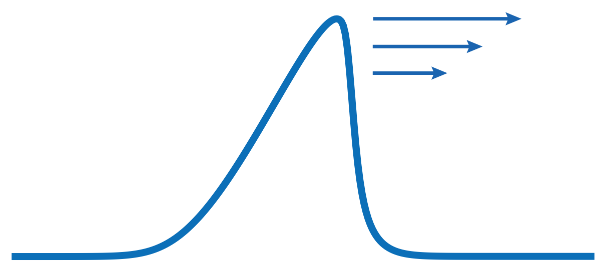
$$H\mathbf{X} = E\mathbf{X}$$

Part 1: Solitons in airborne acoustics

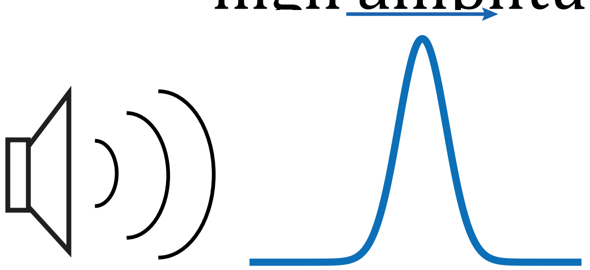


Part 1: Solitons in airborne acoustics

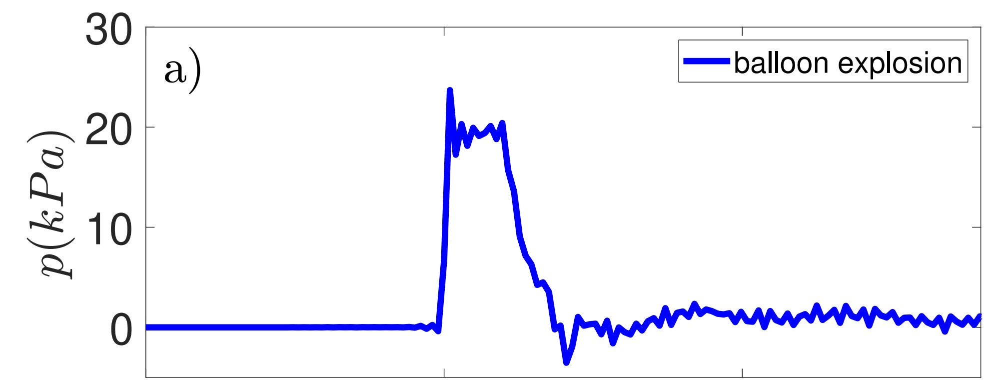
shock front formation

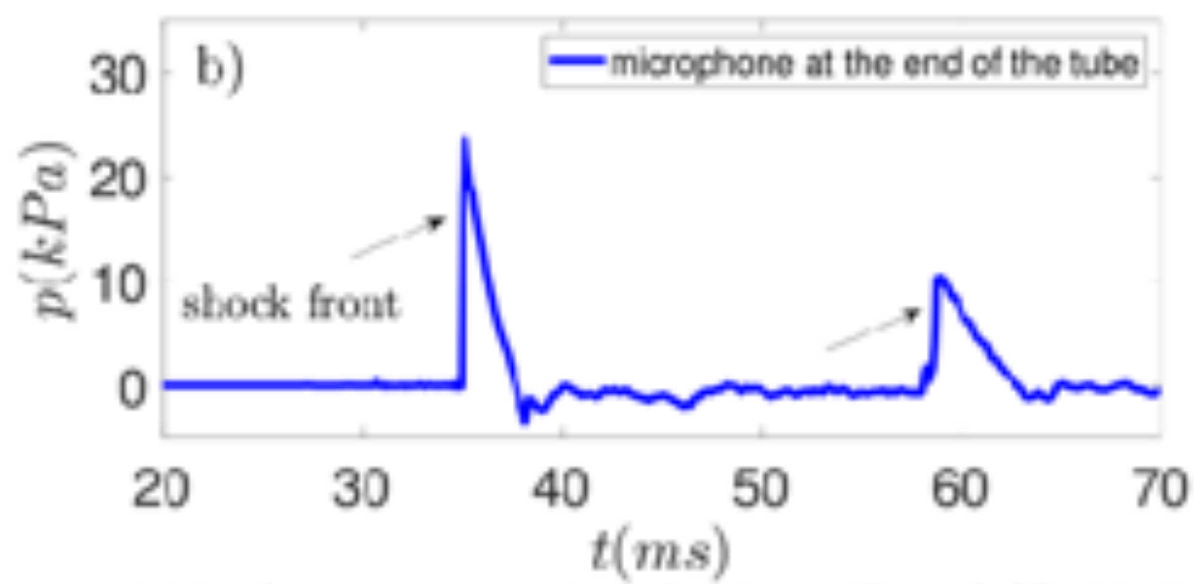


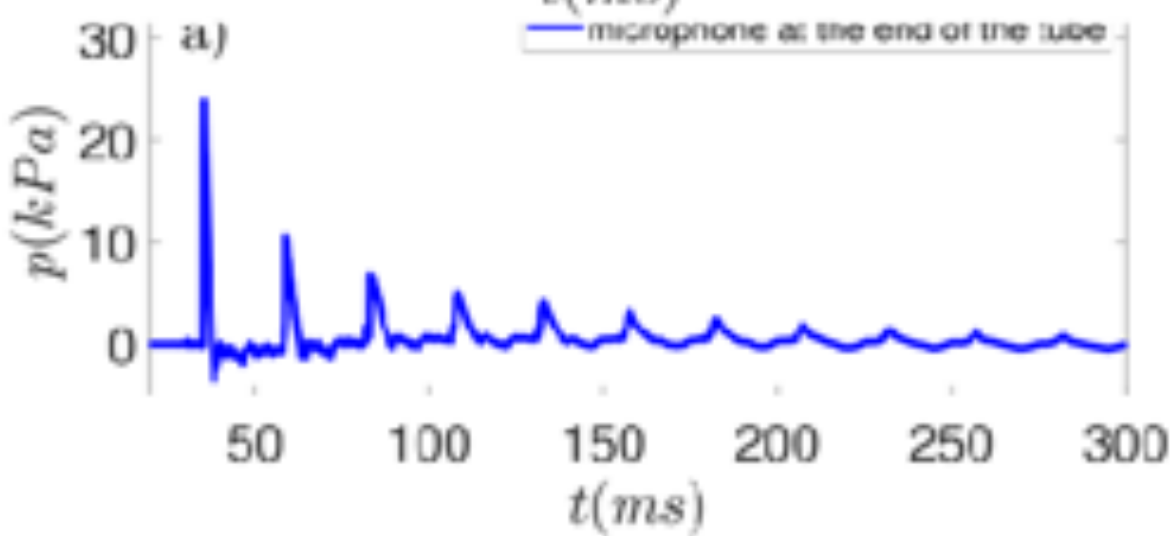
high amplitude



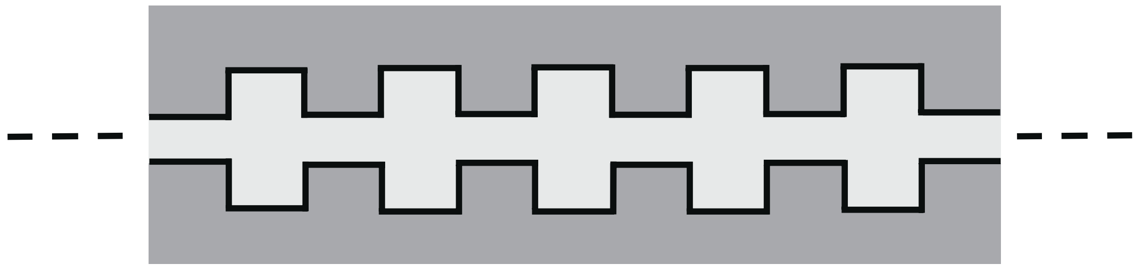
Experimental measurments





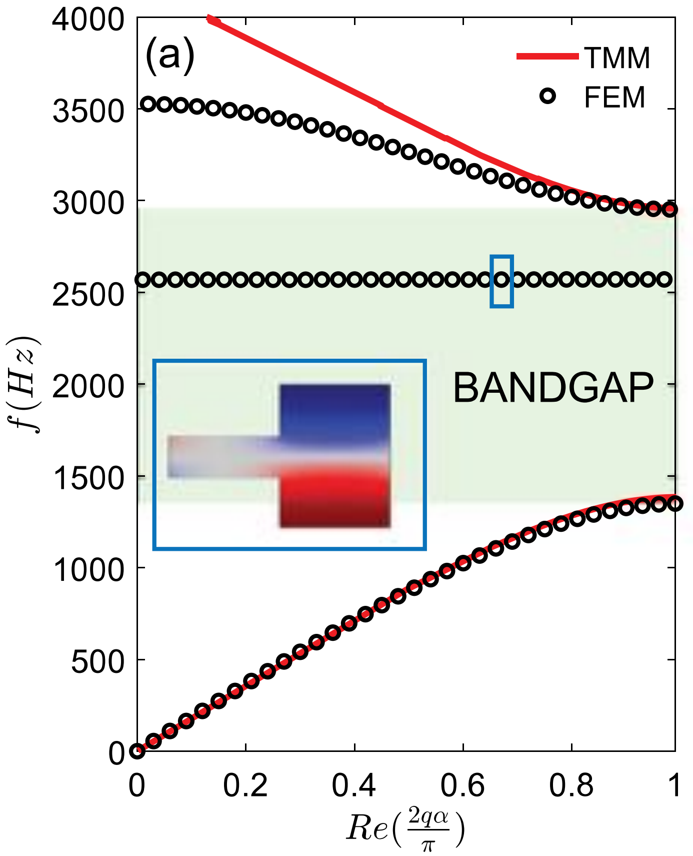


Part 1: Solitons in airborne acoustics



1-D approximation - Transfer matrix method

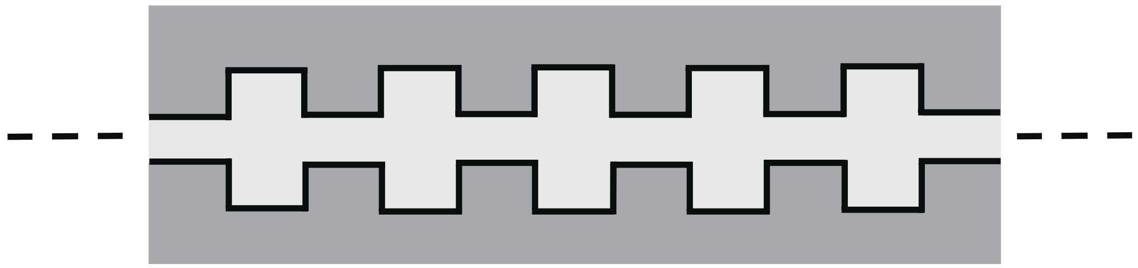
$$\cos(2q\alpha) = \cos^2(k\tilde{\alpha}) - \frac{S_1^2 + S_2^2}{2S_1S_2}\sin^2(k\tilde{\alpha}) \qquad \omega = ck$$



FEM simulations

1D-TMM

Part 1: Solitons in airborne acoustics



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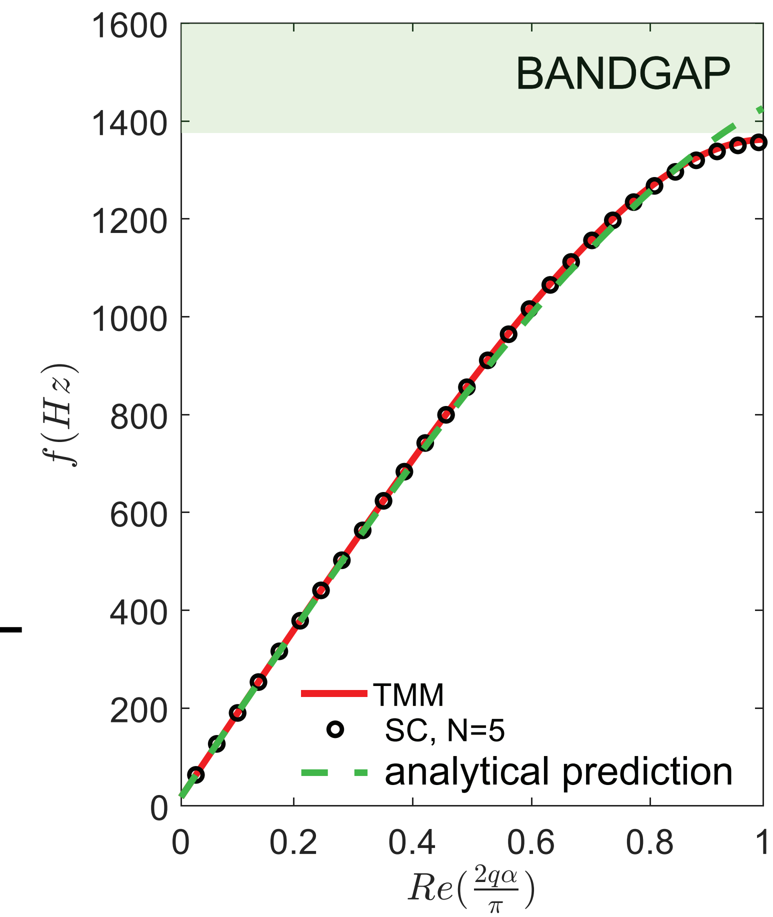


Effective medium

$$p_{tt} - b(p^2)_{tt} = \tilde{c}^2 p_{xx} + \beta_m p_{xxtt} + \beta_x p_{xxxx}$$

To find the coefficients we compare the two equations in the long wavelength limit

$$\omega\tilde{\alpha} = \tilde{c}_0\left(q\alpha - \frac{\beta_m + \beta_x}{2}(q\alpha)^3 + \frac{(\beta_m + \beta_x)(3\beta_m - \beta_x)}{8}(q\alpha)^5 + O\left((q\alpha)^7\right)\right)$$

$$\omega\tilde{\alpha} = \tilde{c}_0\left(q\alpha - \frac{\delta}{2}(q\alpha)^3 + 3\delta(Q - \delta)(q\alpha)^5 + O\left((q\alpha)^7\right)\right)$$

Effective nonlinear PDE and the soliton solution

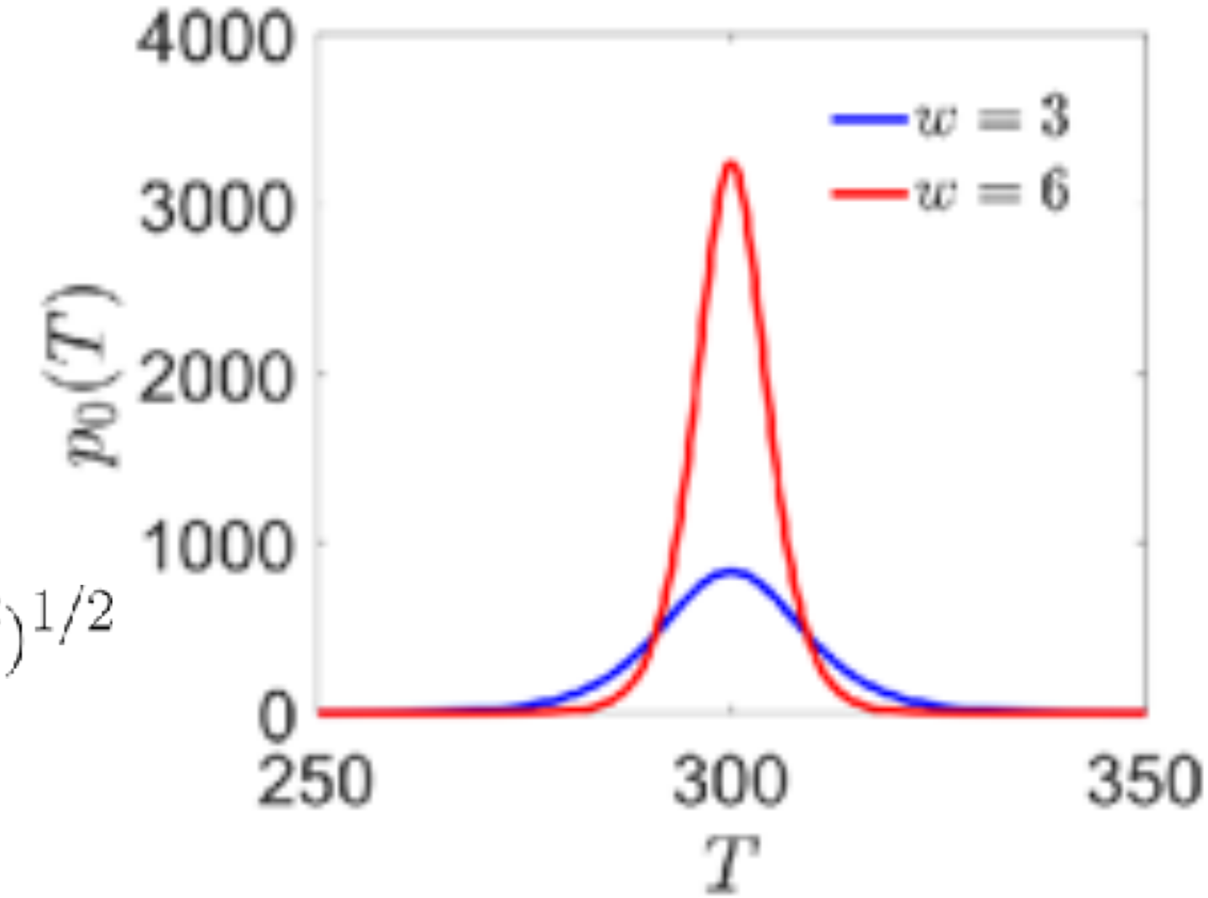
- - - effective dispersive nonlinear medium - - -

$$p_{tt} - b(p^2)_{tt} = \tilde{c}^2 p_{xx} + \beta_m p_{xxtt} + \beta_x p_{xxxx}$$

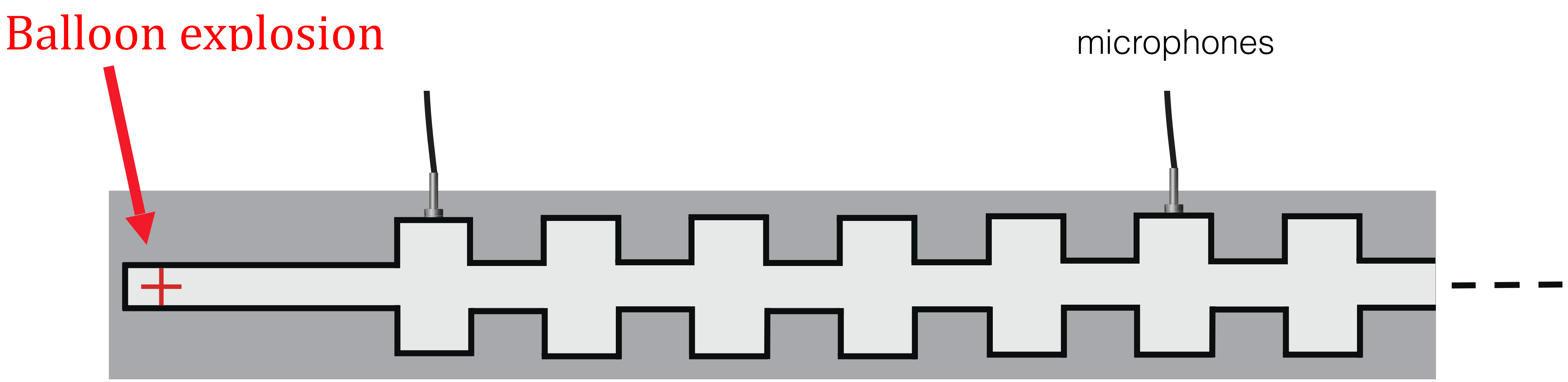
Soliton solutions (homoclinic orbits)

$$p(x,t) = A\,\mathrm{sech}^2\left(w\left(x - \nu t\right)\right)$$

$$A = \frac{3\left(\nu^2 - \tilde{c}^2\right)}{2b\nu^2}, \quad w = ((\nu^2 - \tilde{c}^2)/4(\beta_x + \beta_m \nu^2)^{1/2}$$

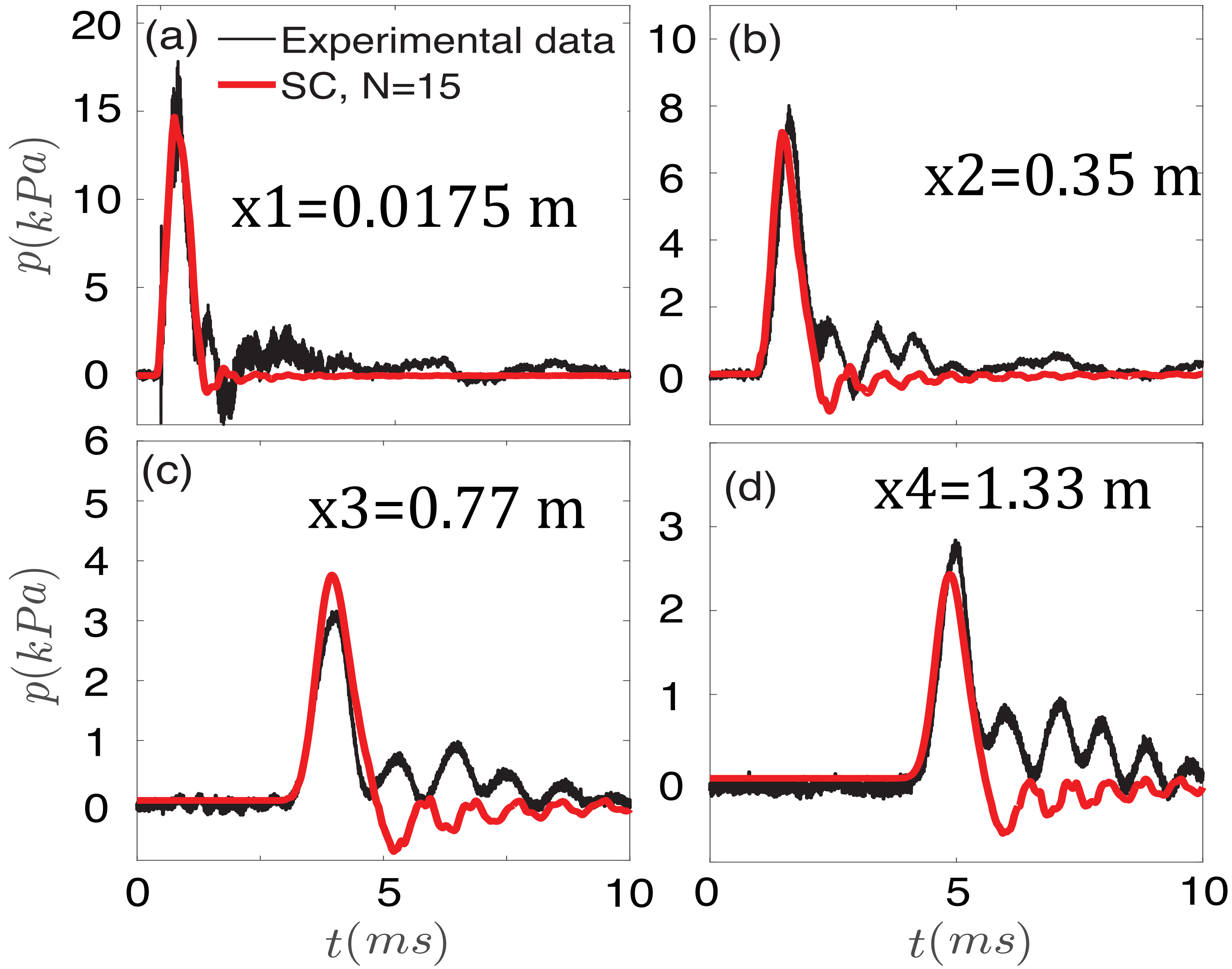


Experiments



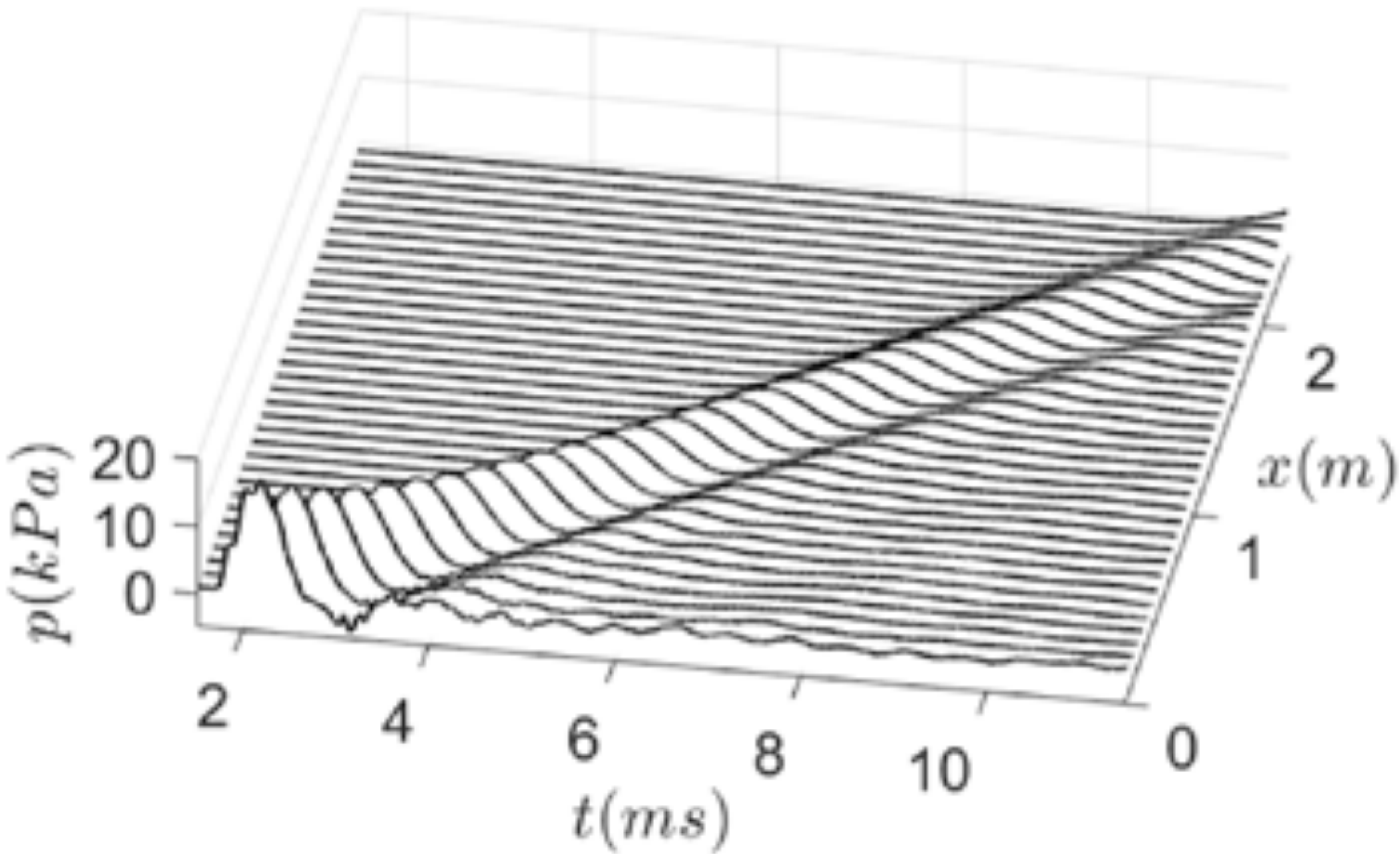
Experiments

One of many realisations



Experiments and effect of losses

2D numerical simulations - soliton decay



Experiments effect of losses

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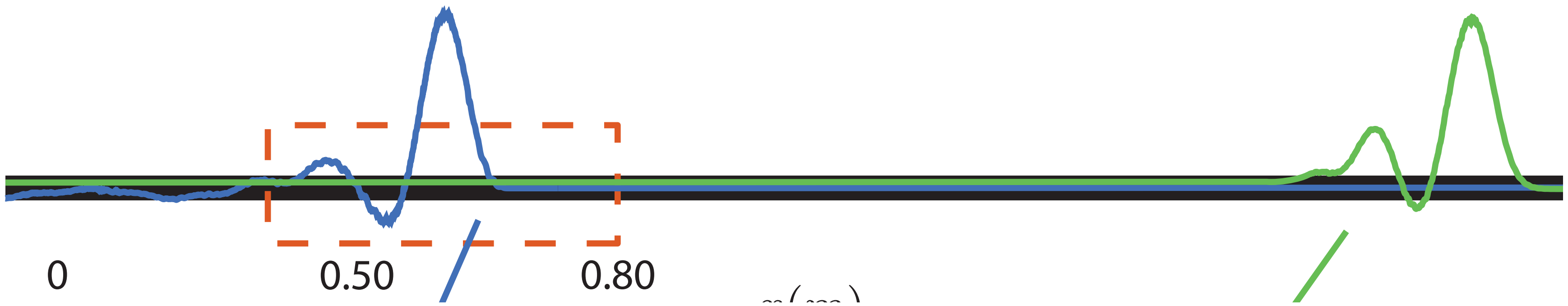

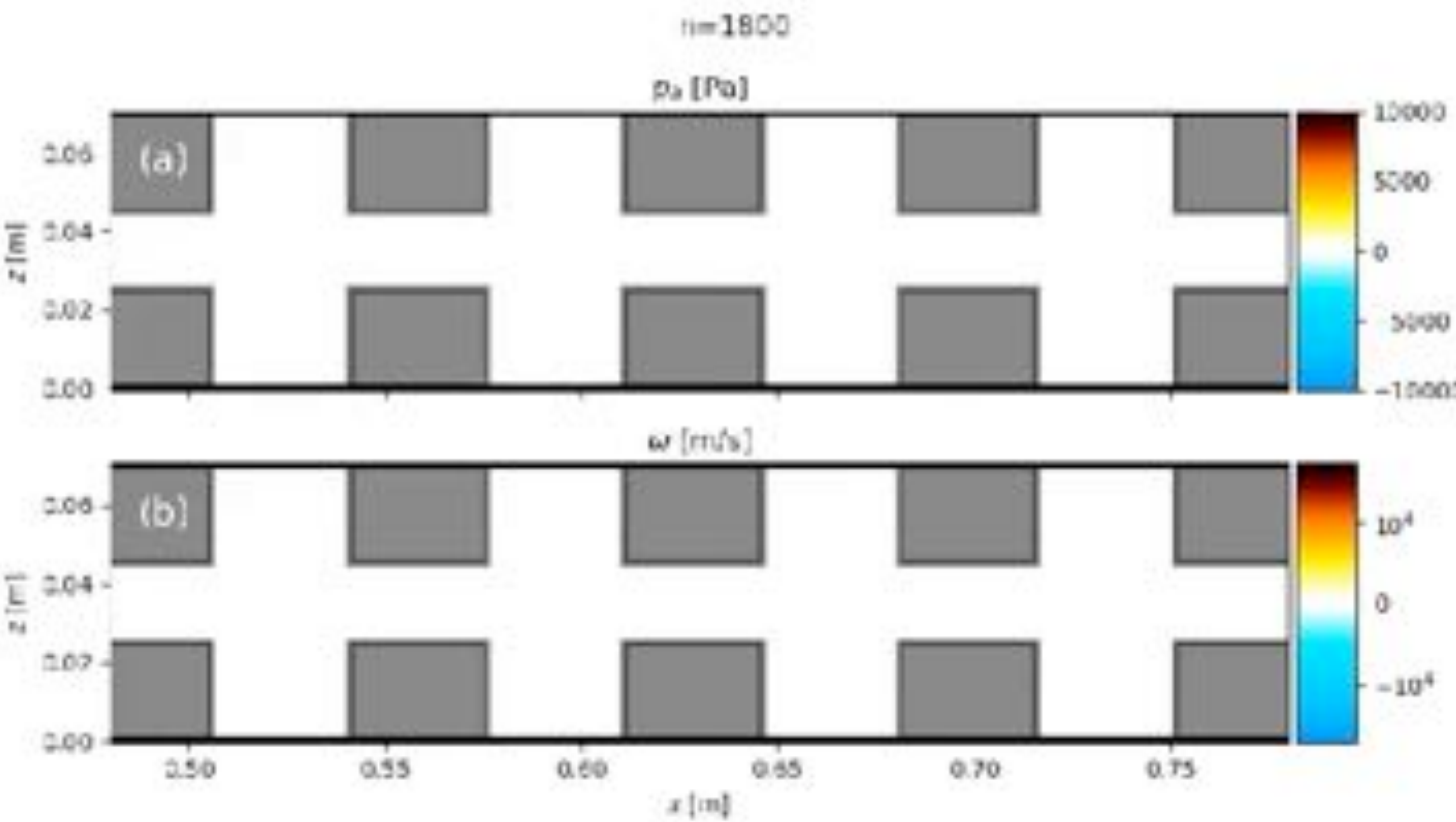

Experiments and the effect of losses

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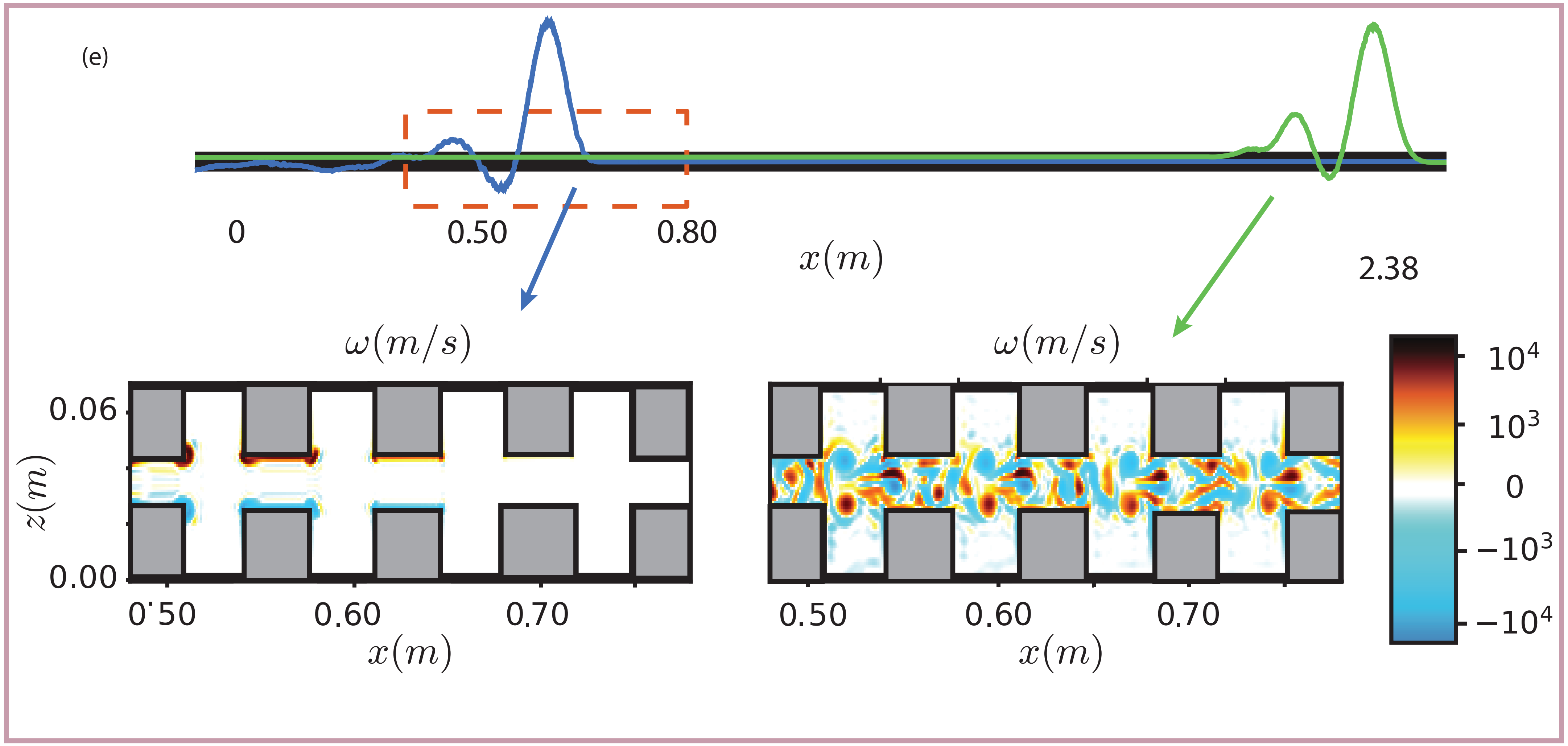
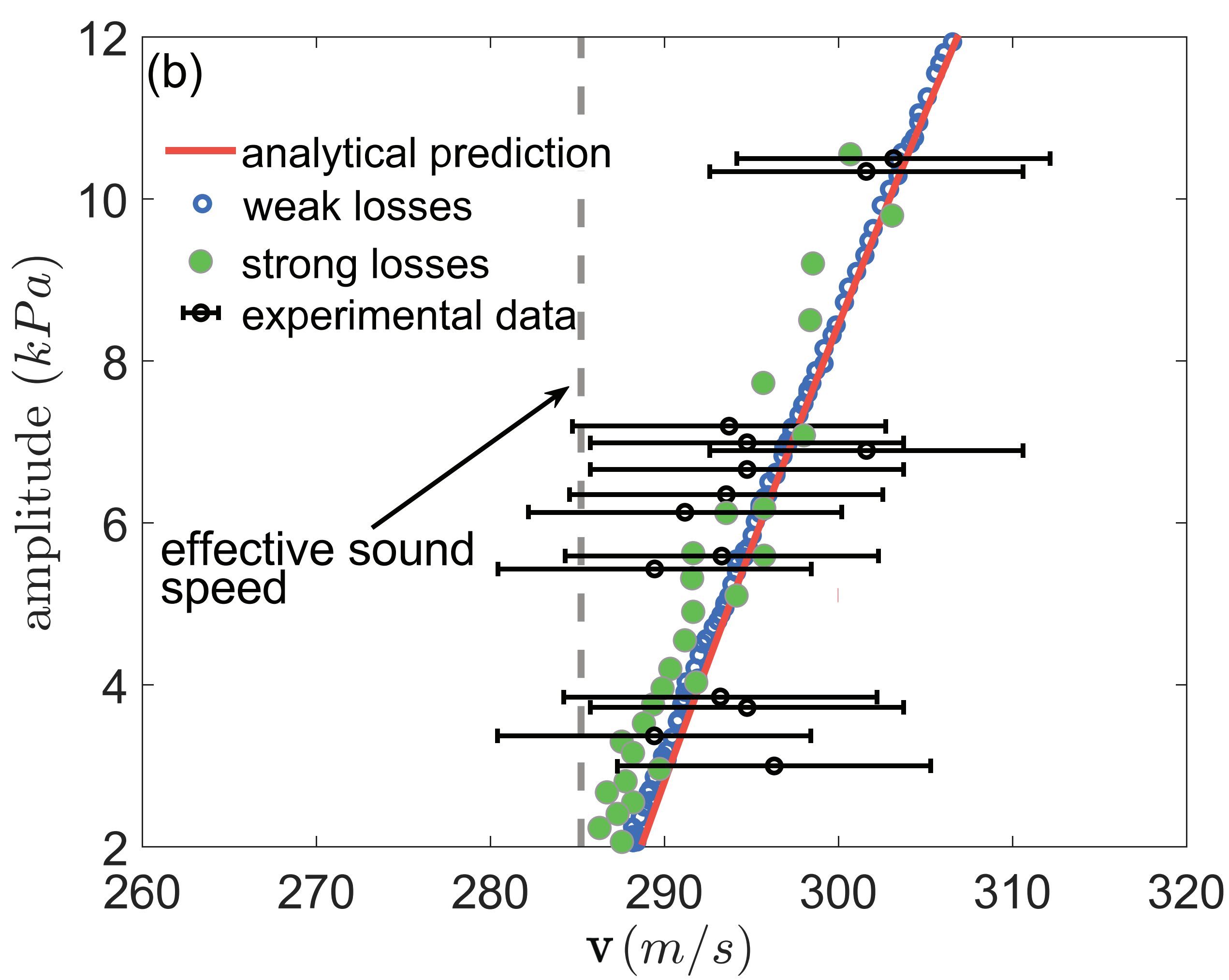
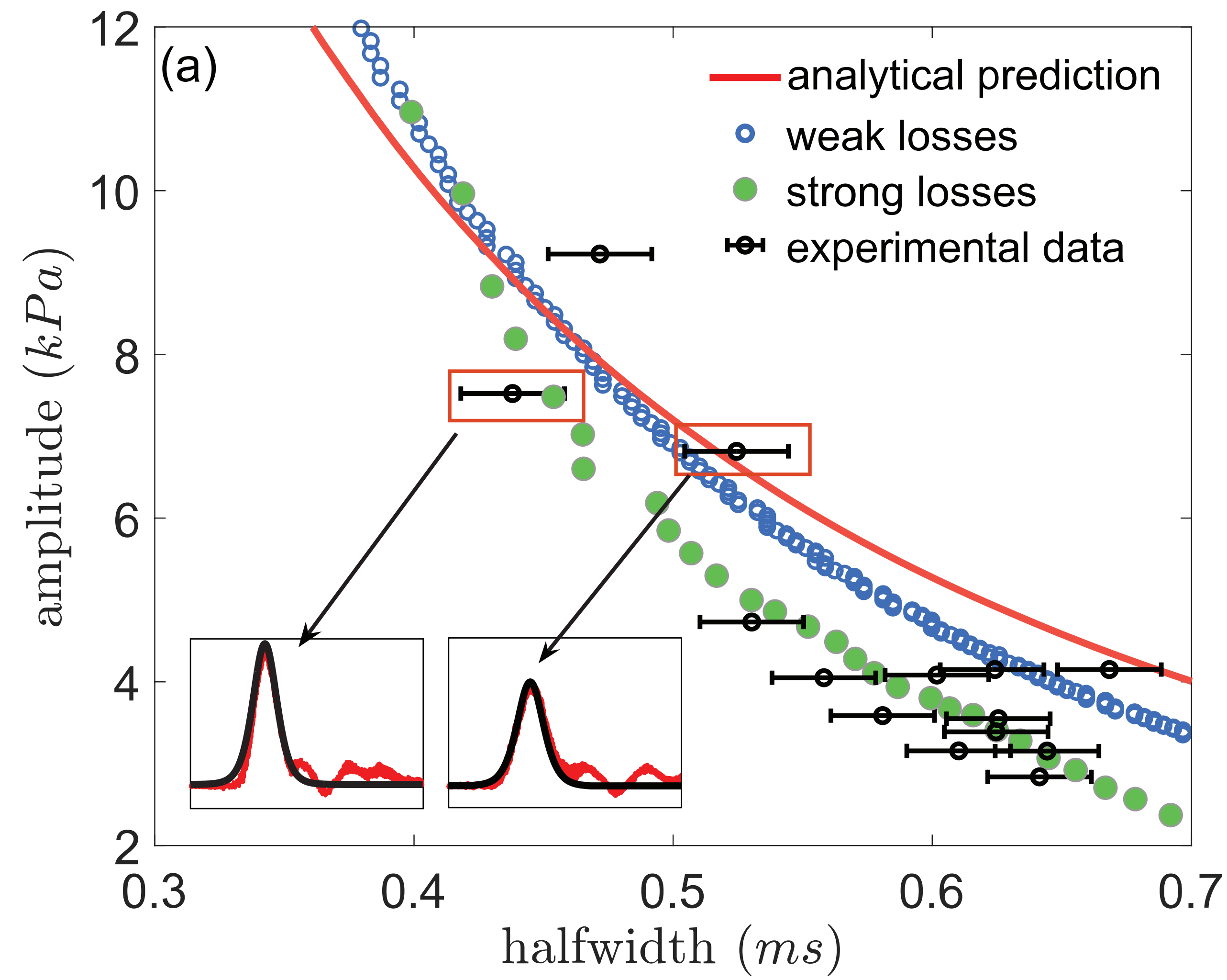


Experiments comparison with the soliton

Comparison with theoretical prediction

$$p(x,t) = A\,\mathrm{sech}^2\left(w\left(x-\nu t\right)\right) \qquad A = \frac{3\left(\nu^2-\tilde{c}^2\right)}{2b\nu^2}, \quad w = \left((\nu^2-\tilde{c}^2)/4(\beta_x+\beta_m\nu^2\right)^{1/2}$$



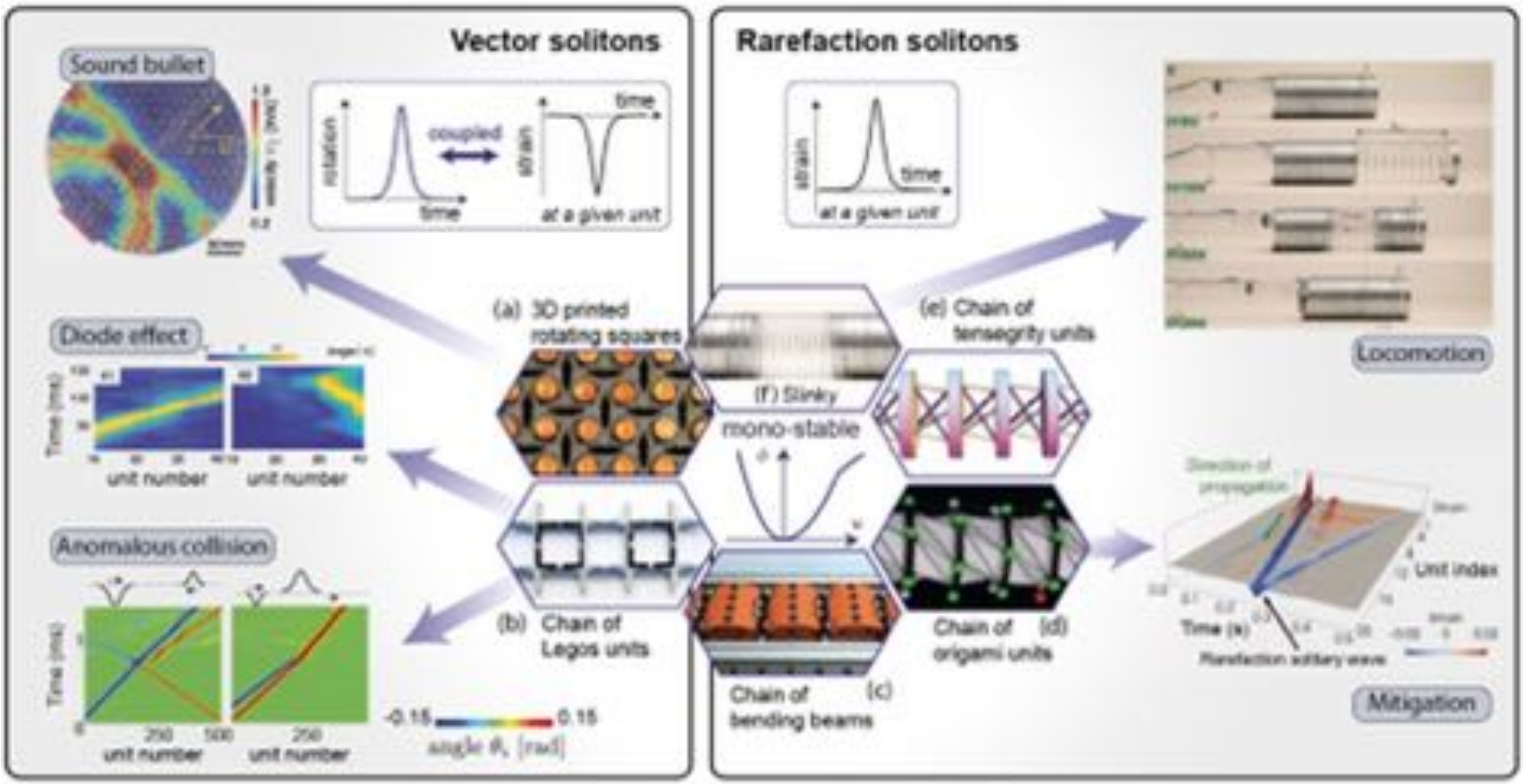


Journal of Sound and Vibration **546** 117433 (2023)

Part 2: Modulation Instability in FlexEM

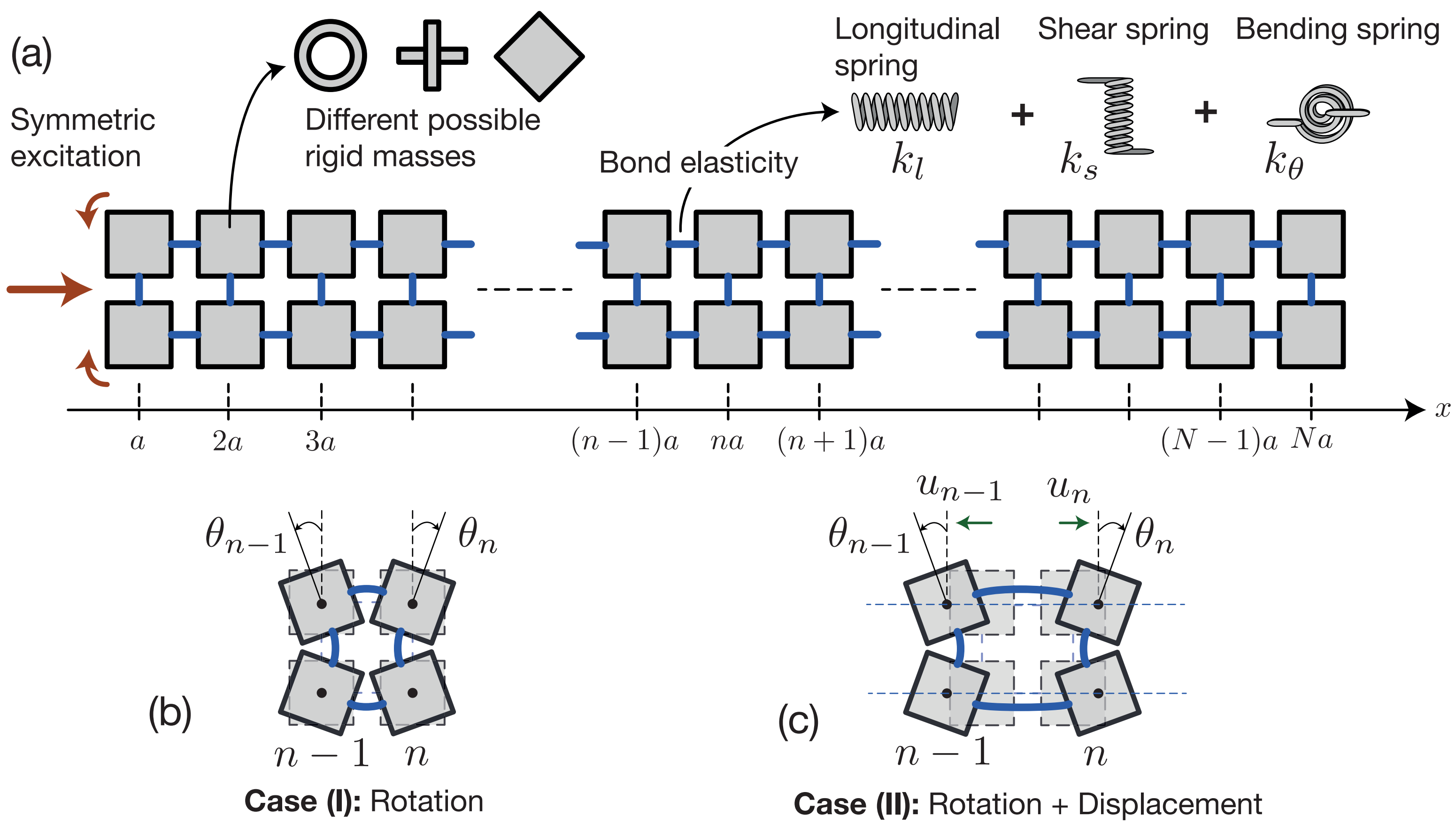
Flexible Elastic Metamaterials

Pulse excitations



B. Deng *et al*, J. Appl. Phys. **130,** 040901 (2021)

Part 2: Modulation Instability in FlexEM

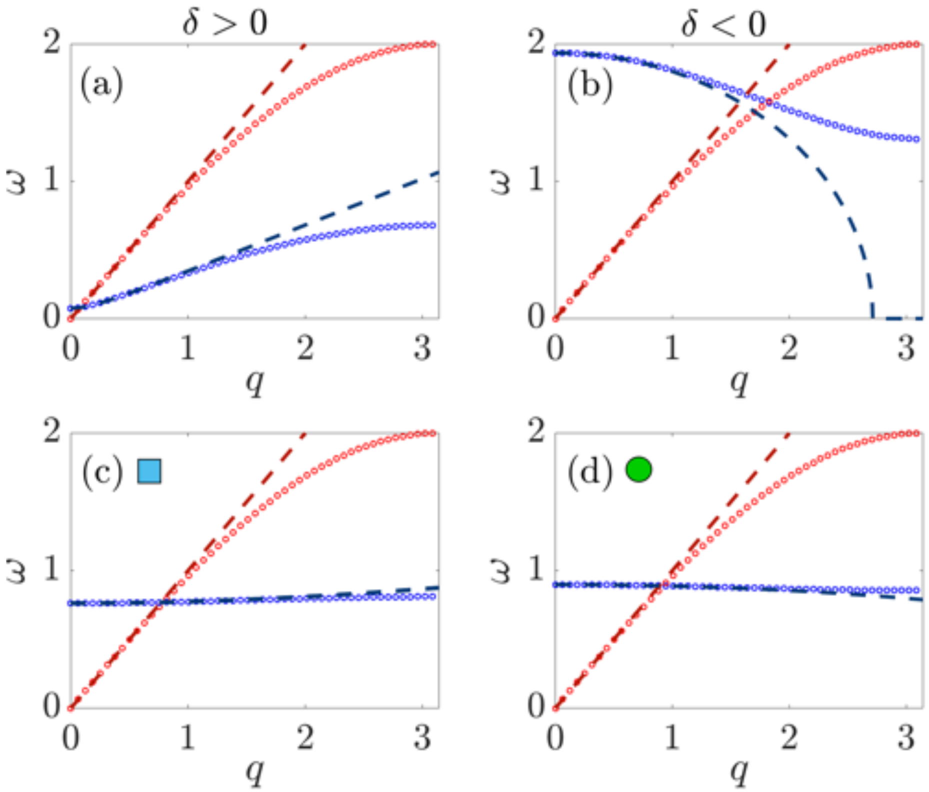


$$\frac{\partial^2 U_n}{\partial T^2} = U_{n+1} - 2U_n + U_{n-1} - \frac{\cos\theta_{n+1} - \cos\theta_{n-1}}{2}$$

$$\frac{1}{\alpha^2}\frac{\partial^2 \theta_n}{\partial T^2} = -K_\theta\left(\theta_{n+1} + 4\theta_n + \theta_{n-1}\right) + K_s\cos\theta_n\left[\sin\theta_{n+1} + \sin\theta_{n-1} - 2\sin\theta_n\right]$$

$$- \sin\theta_n\left[2\left(U_{n+1} - U_{n-1}\right) + 4 - 2\cos\theta_n - \cos\theta_{n+1} - \cos\theta_{n-1}\right]$$

Part 2: Modulation Instability in FlexEM

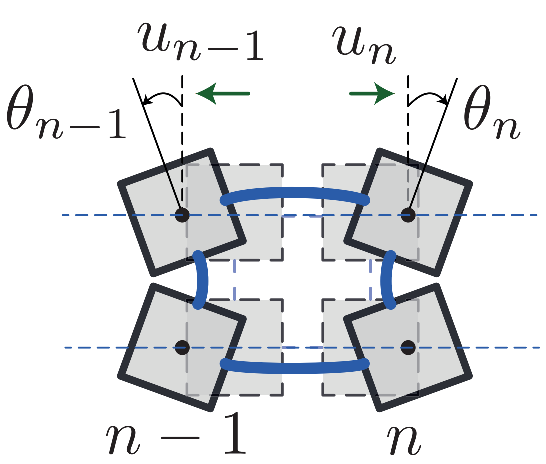


Decoupled in the linear limit

$$\omega^{(1)} = 2\sin\left(\frac{qa}{2}\right) ,$$

$$\omega^{(2)} = \pm\sqrt{4\alpha^2(K_s - K_\theta)\sin^2\left(\frac{qa}{2}\right) + 6\alpha^2 K_\theta}$$

$$\delta = K_s - K_\theta$$



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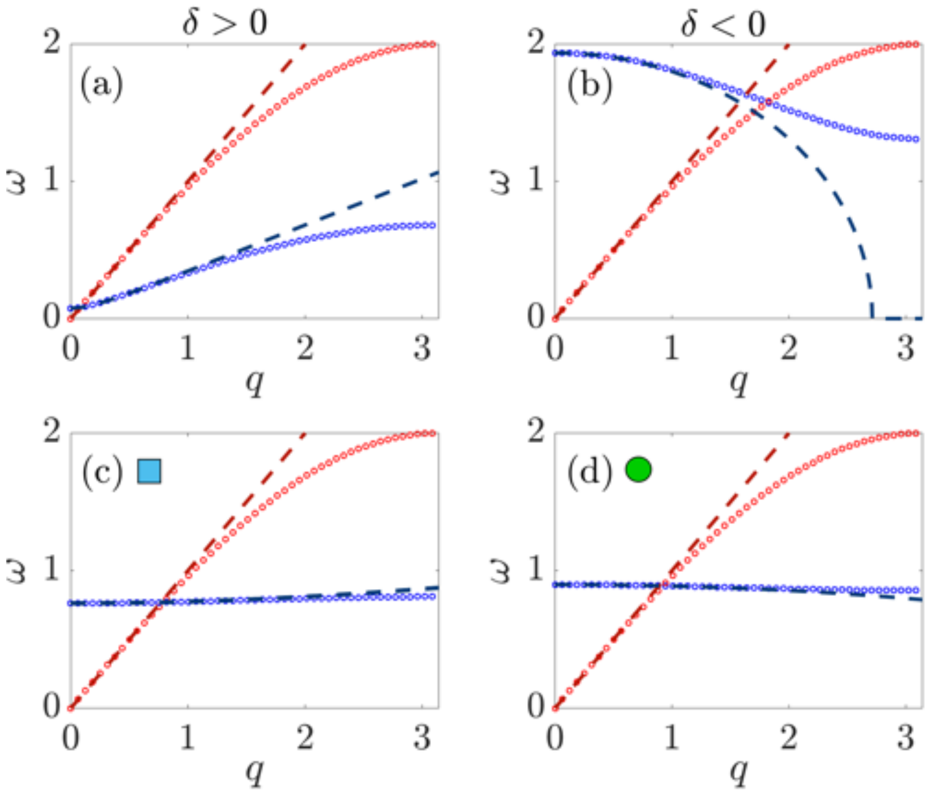

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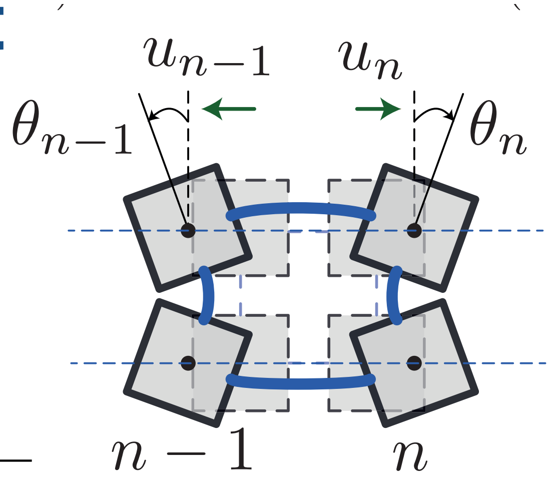

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$$\delta = K_s - K_\theta$$

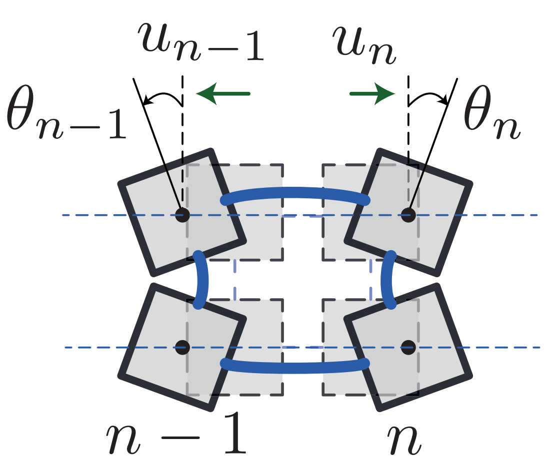
Effective PDE At long wavelengths $\lambda \gg a$

$$\frac{\partial^2 U}{\partial T^2} = \frac{\partial^2 U}{\partial X^2} + \theta\frac{\partial\theta}{\partial X},$$

$$\frac{\partial^2 \theta}{\partial T^2} = C_1\frac{\partial^2 \theta}{\partial X^2} - C_2\theta - C_3\theta^3 - C_4\theta\frac{\partial U}{\partial X}$$

Part 2: Modulation Instability in FlexEM

At long wavelengths $\lambda \gg a$



$$\frac{\partial^2 U}{\partial T^2} = \frac{\partial^2 U}{\partial X^2} + \theta \frac{\partial \theta}{\partial X},$$
$$\frac{\partial^2 \theta}{\partial T^2} = C_1 \frac{\partial^2 \theta}{\partial X^2} - C_2 \theta - C_3 \theta^3 - C_4 \theta \frac{\partial U}{\partial X}$$

Use multiple scales analyisis

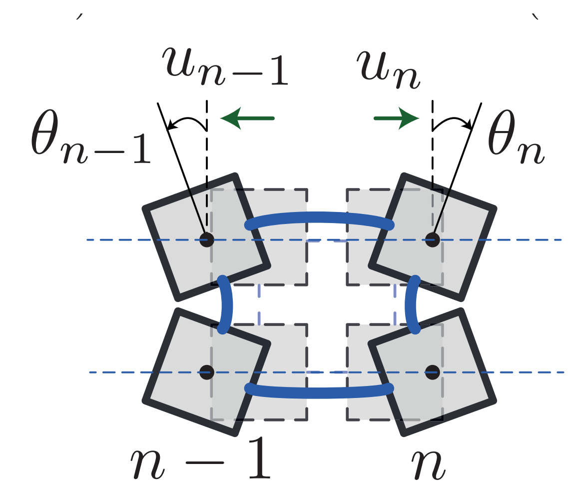
$$U = \epsilon u_1 + \epsilon^2 u_2 + \epsilon^3 u_3 + \ldots$$
$$\theta = \epsilon \theta_1 + \epsilon^2 \theta_2 + \epsilon^3 \theta_3 + \ldots$$

Part 2: Modulation Instability in FlexEM

At long wavelengths $\lambda \gg a$

$$\frac{\partial^2 U}{\partial T^2} = \frac{\partial^2 U}{\partial X^2} + \theta \frac{\partial \theta}{\partial X},$$

$$\frac{\partial^2 \theta}{\partial T^2} = C_1 \frac{\partial^2 \theta}{\partial X^2} - C_2\theta - C_3\theta^3 - C_4\theta \frac{\partial U}{\partial X}$$



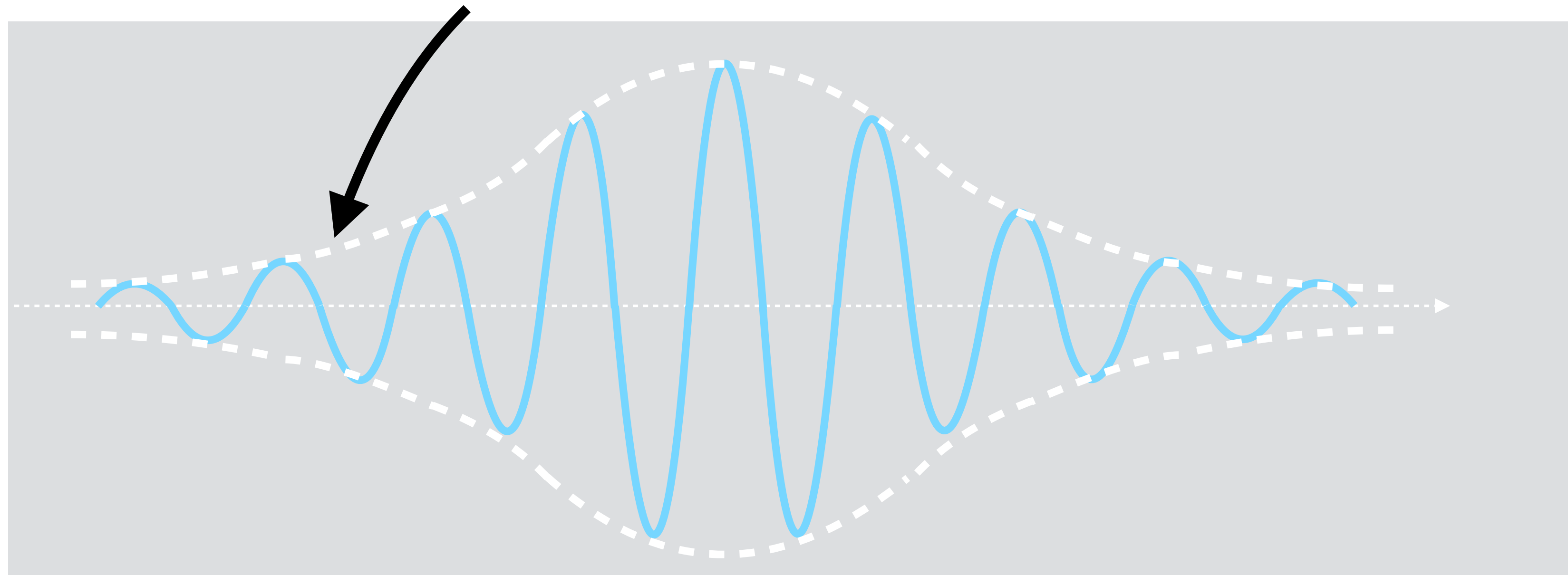
Use multiple scales analyisis

$$U = \epsilon u_1 + \epsilon^2 u_2 + \epsilon^3 u_3 + \ldots$$

$$\theta = \epsilon\theta_1 + \epsilon^2\theta_2 + \epsilon^3\theta_3 + \ldots$$

Larger rotations

$$\theta_1 = B(X_1, T_1, X_2, T_2, \ldots)e^{i(kX_0 - \omega T_0)} + \text{c.c}$$

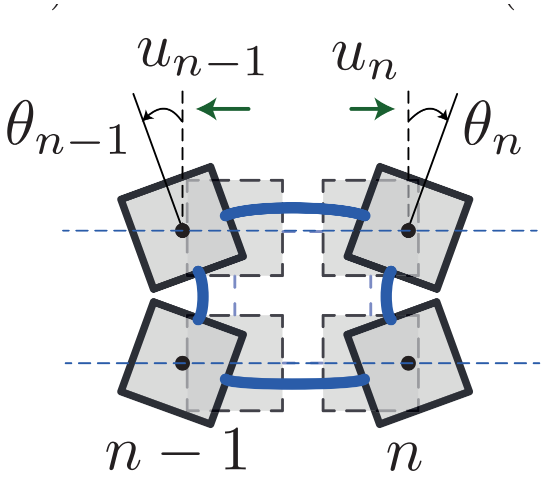


Part 2: Modulation Instability in FlexEM

Nonlinear Schrödinger Equation for the envelope

$$\theta = B(X_1, T_1, X_2, T_2, ...)e^{i(kX_0 - \omega T_0)} + \text{c.c}$$

$$i\frac{\partial B}{\partial \tilde{\tau}_2} + \frac{1}{2}\frac{\partial^2 B}{\partial \xi_1^2} + g|B|^2 B = 0$$



Derive conditions for Modulational Instability

$$B(\xi_1, \tilde{\tau}_2) = (A_0 + b(\xi_1, \tilde{\tau}_2))e^{i(k_0\xi_1 - \omega_0\tilde{\tau}_2 + \tilde{\theta}(\xi_1, \tilde{\tau}_2))}$$

$$b = f_1 e^{i(K\xi_1 - \Omega\tilde{\tau}_2)}, \quad \tilde{\theta} = f_2 e^{i(K\xi_1 - \Omega\tilde{\tau}_2)}$$

$$\Omega = Kk_0 \pm |K|\sqrt{\frac{K^2}{4} - gA_0^2}$$

If g > 0 and $K < |K_c| = 2A_0\sqrt{g}$

The perturbations grow with time

$$\Omega = \Omega_R \pm i\Omega_I$$

$$\Omega_I = |K|A_0\sqrt{g - \frac{K^2}{4A_0^2}}$$

Part 2: Modulation Instability in FlexEM

Nonlinear Scrödinger Equation for the envelope

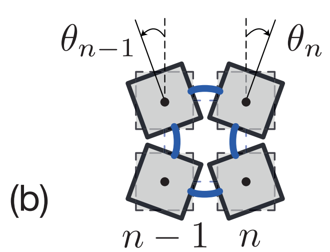
$$\theta = B(X_1, T_1, X_2, T_2, ...)e^{i(kX_0 - \omega T_0)} + \text{c.c}$$

$$i\frac{\partial B}{\partial \tilde{\tau}_2} + \frac{1}{2}\frac{\partial^2 B}{\partial \xi_1^2} + g|B|^2 B = 0$$

unstable if

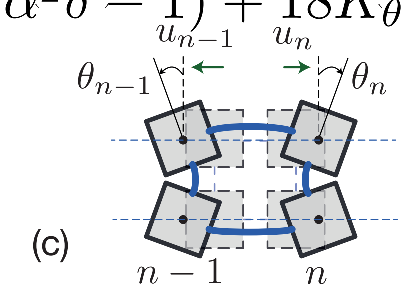
$$\Omega_I = |K| A_0 \sqrt{g - \frac{K^2}{4A_0^2}}$$

$$g = \frac{-3\alpha^2}{\delta\alpha^2 - v_g^2}$$

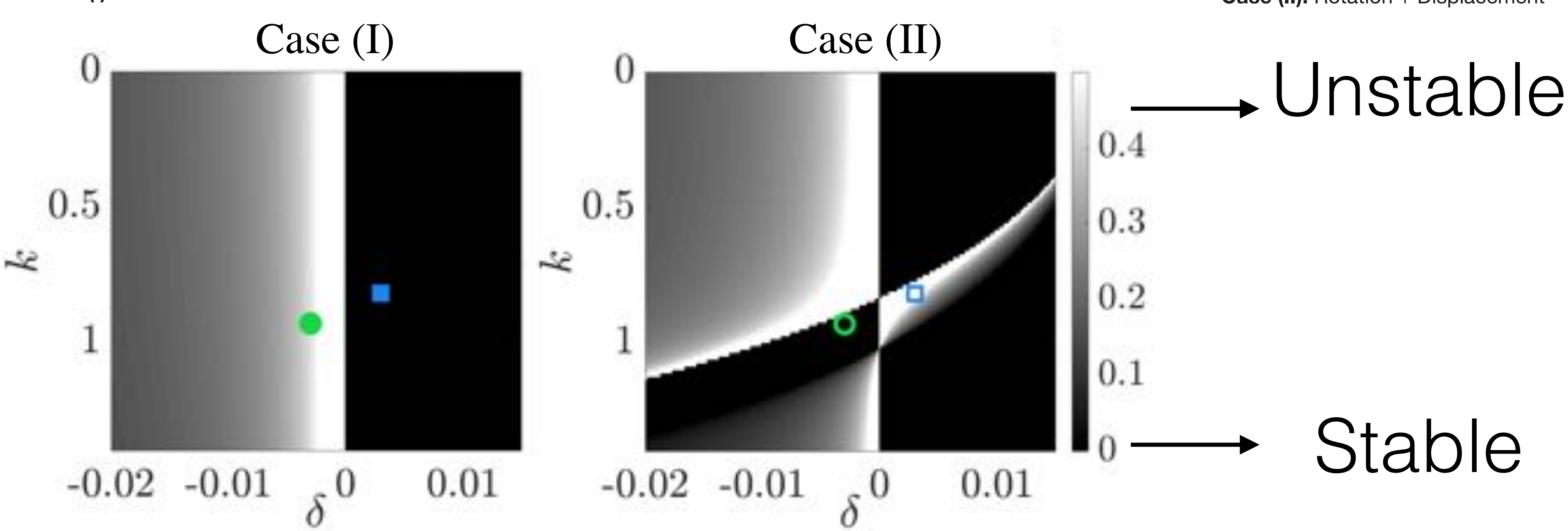


Case (I): Rotation

$$g = \frac{-3\alpha^2}{\delta\alpha^2 - v_g^2}\left(1 + \frac{k^2}{3k^2(\alpha^2\delta - 1) + 18K_\theta\alpha^2}\right)$$

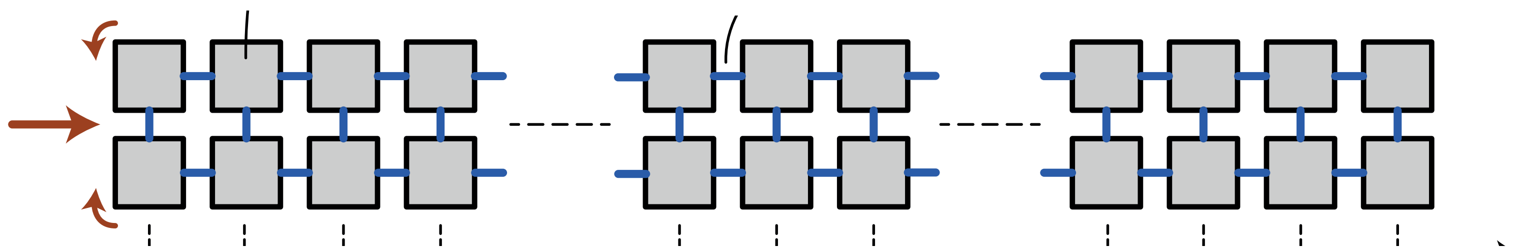


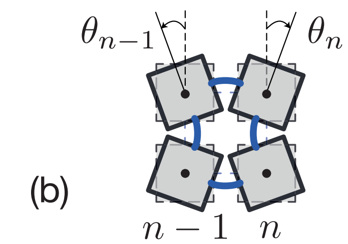
Case (II): Rotation + Displacement



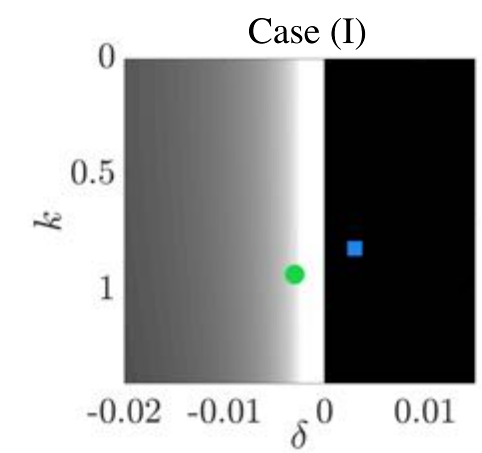
Part 2: Modulation Instability in FlexEM

Case (I)



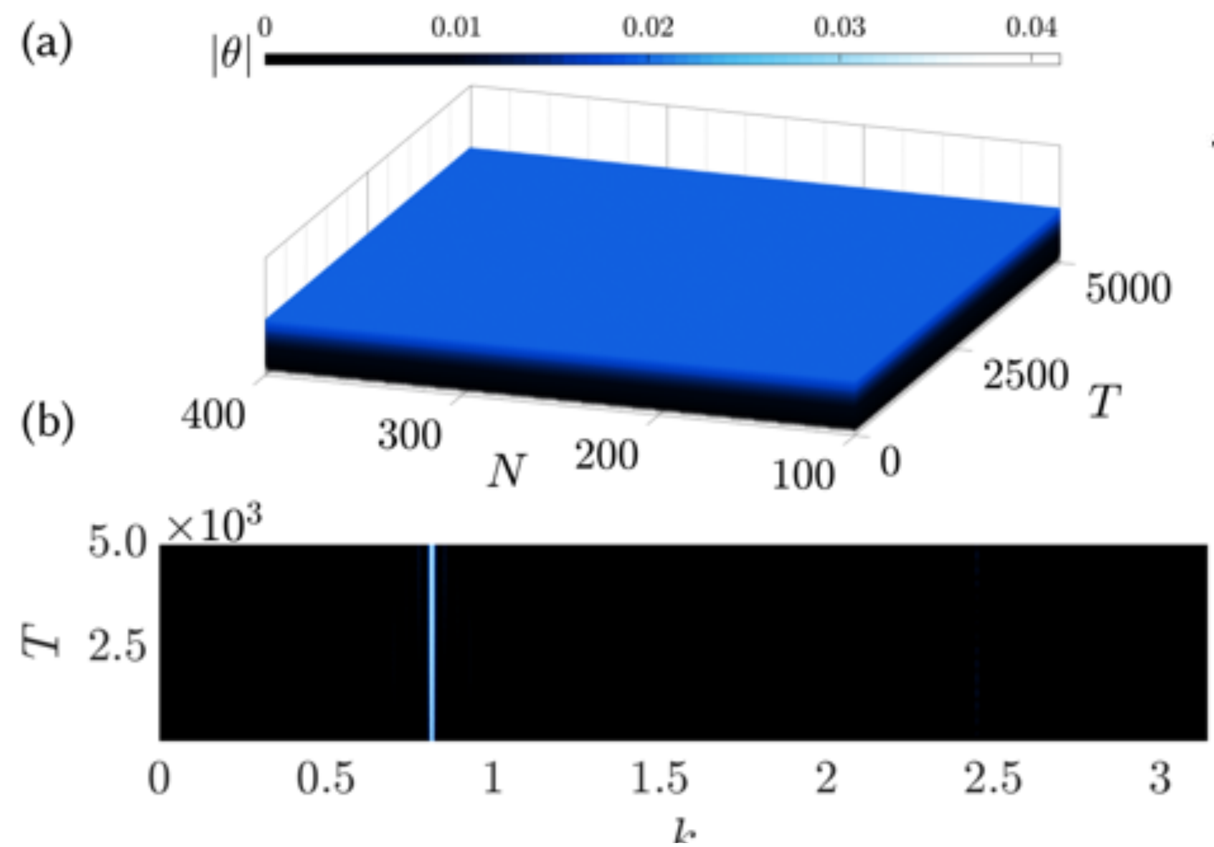


Case (I): Rotation



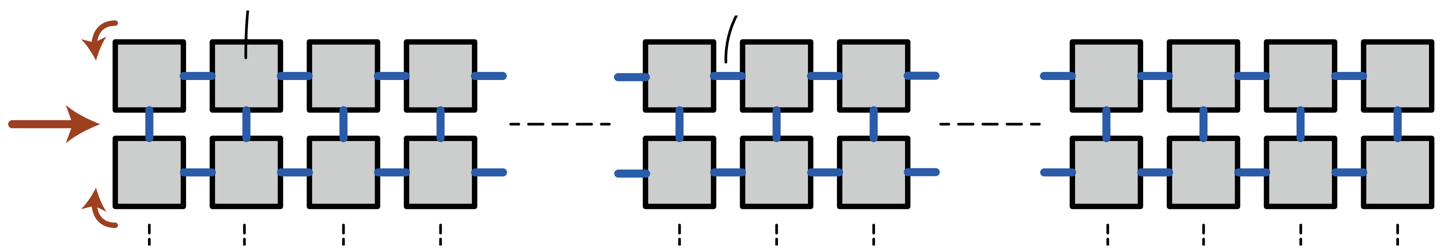
I. C. Modulated plane wave

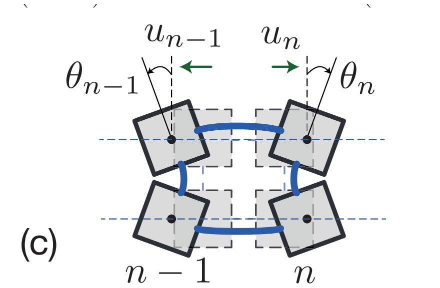
$$\theta(n,0) = 2\epsilon(1+b_0)\cos(kn)\,,$$
$$\dot{\theta}(n,0) = 2\epsilon\omega(k)(1+b_0)\sin(kn)$$



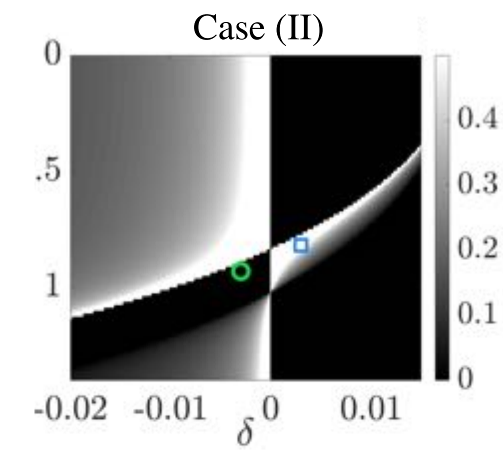
Part 2: Modulation Instability in FlexEM

Case (II)





Case (II): Rotation + Displacement

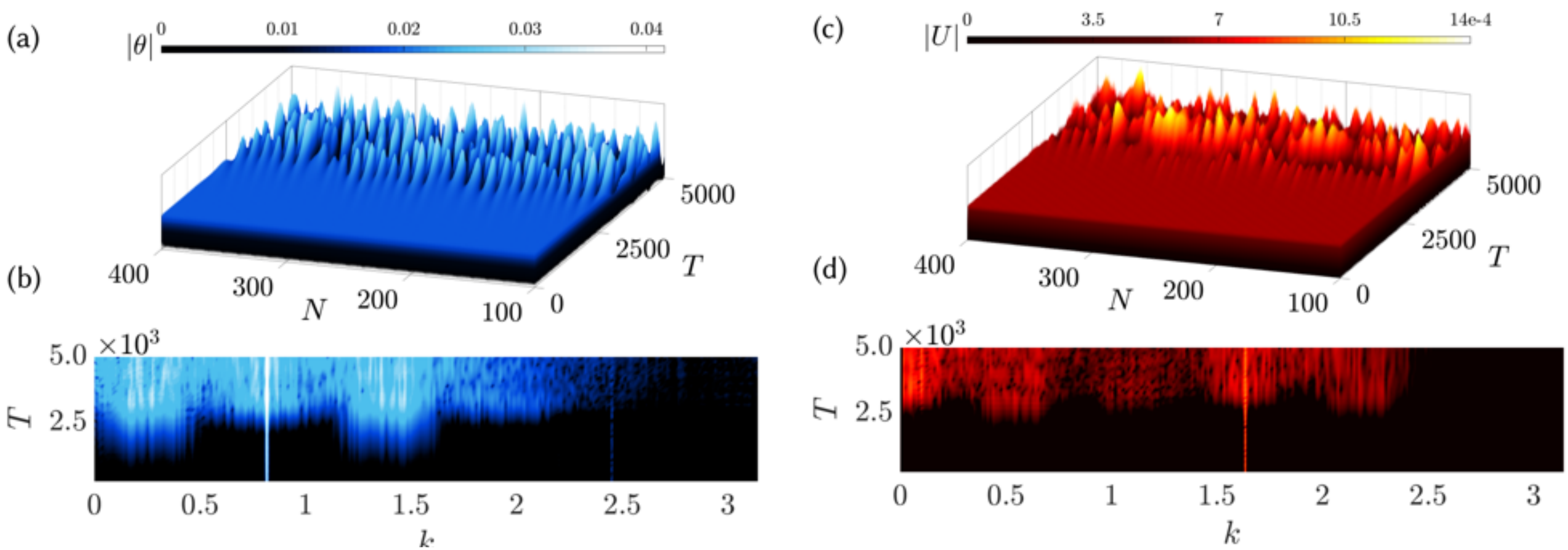


I. C. Modulated plane wave

$$\theta(n,0) = 2\epsilon(1+b_0)\cos(kn)\,,$$
$$\dot{\theta}(n,0) = 2\epsilon\omega(k)(1+b_0)\sin(kn)$$

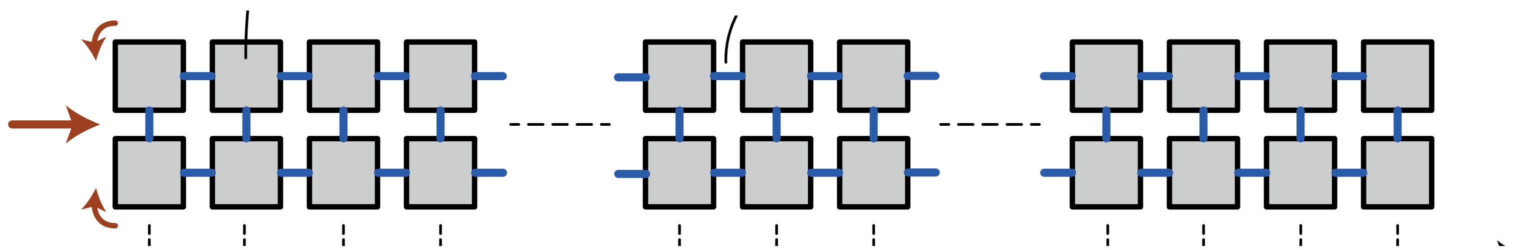
Coupling induces the instability

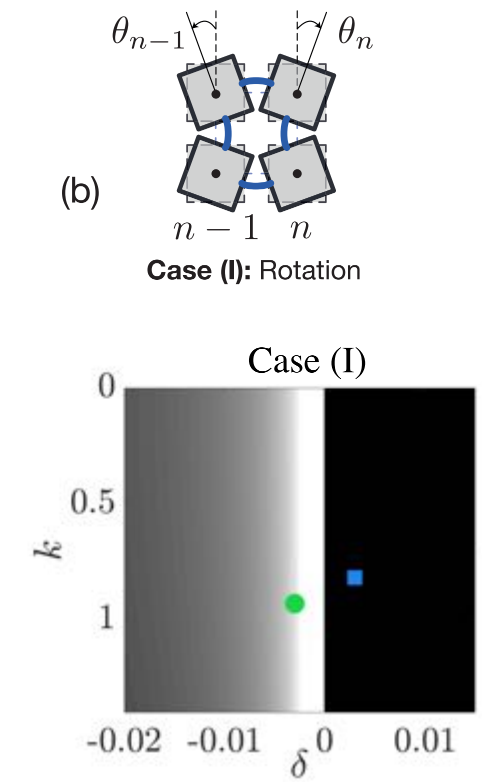
new wavenumbers
localised structures appear in the long time



Part 2: Modulation Instability in FlexEM

Case (I)

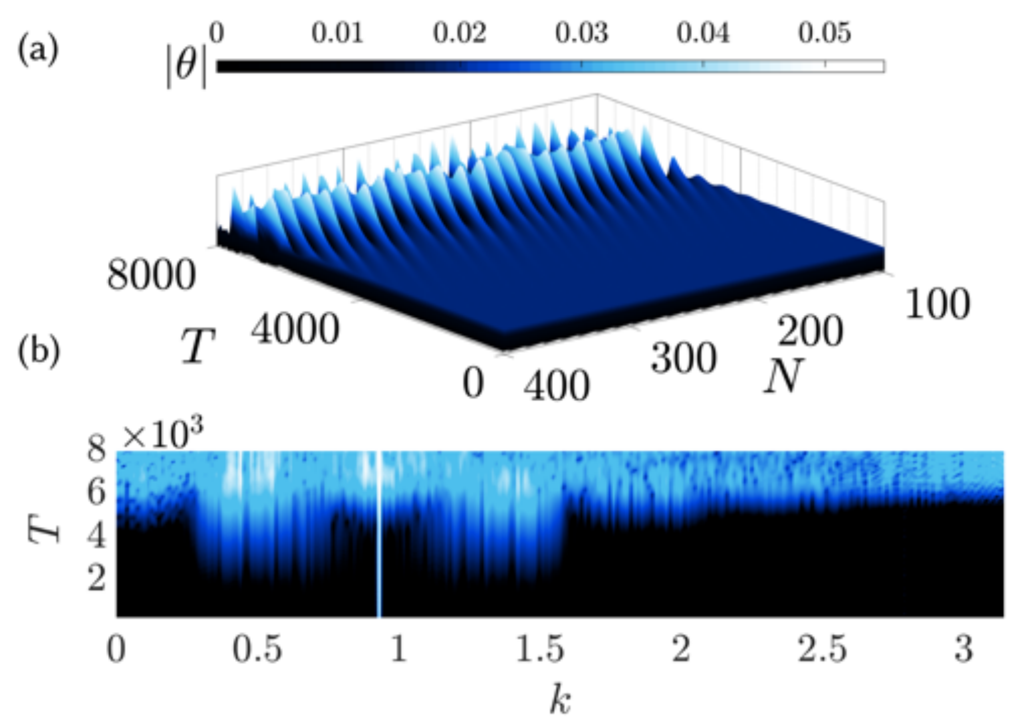




I. C. Modulated plane wave

$$\theta(n,0) = 2\epsilon(1+b_0)\cos(kn)\,,$$
$$\dot{\theta}(n,0) = 2\epsilon\omega(k)(1+b_0)\sin(kn)$$

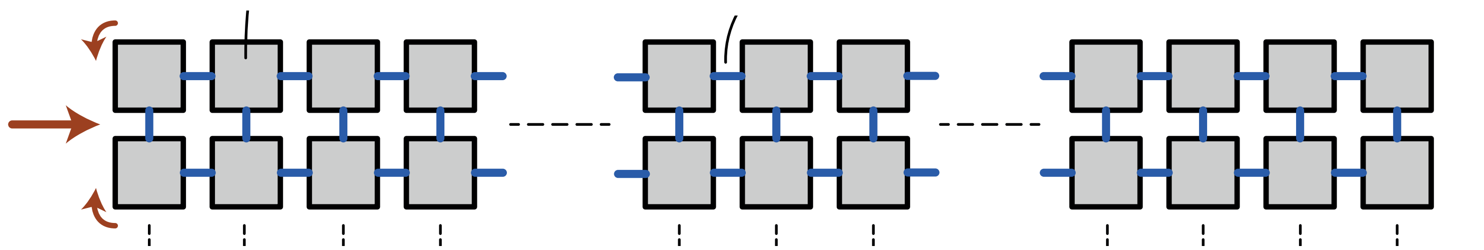
Rotations alone are unstable

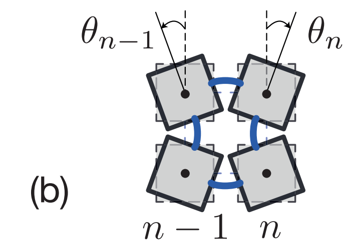


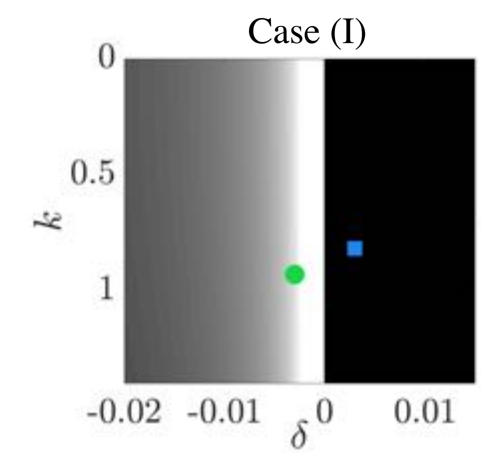
A. Demiguel, V. Achilleos, G. Theocaris, V. Tournat , arXiv: 2211.08531

Part 2: Modulation Instability in FlexEM

Case (I)



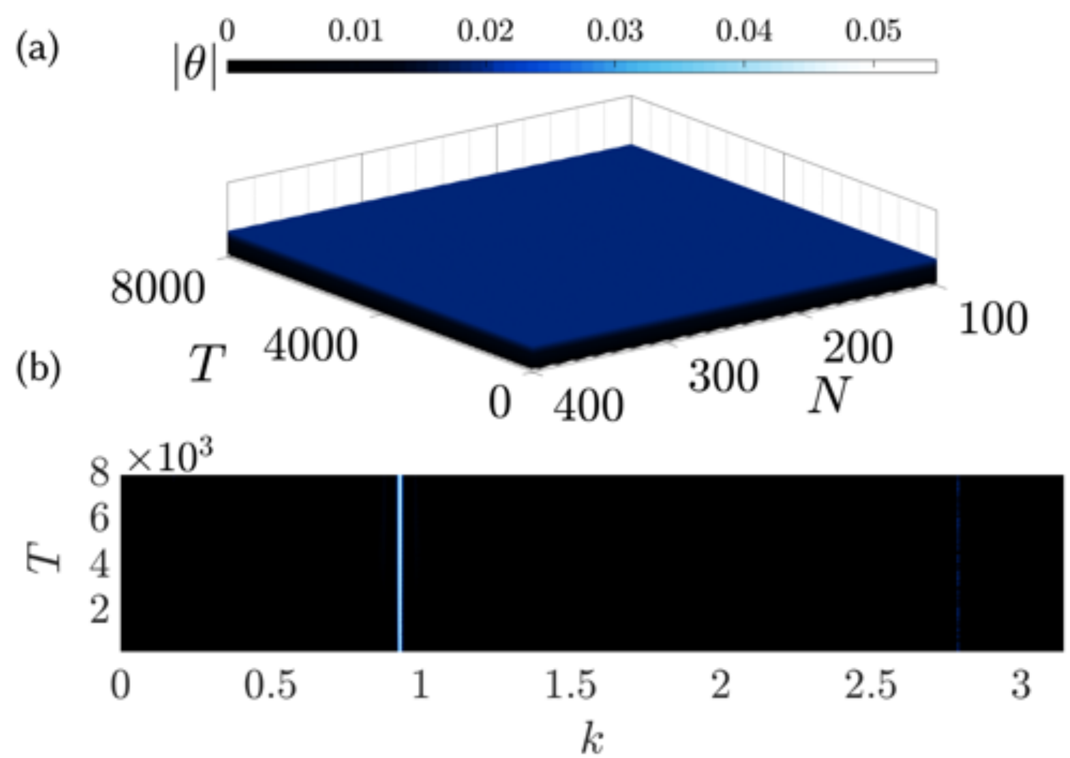




I. C. Modulated plane wave

$$\theta(n,0) = 2\epsilon(1+b_0)\cos(kn)\,,$$
$$\dot{\theta}(n,0) = 2\epsilon\omega(k)(1+b_0)\sin(kn)$$

Coupling stabilises the system



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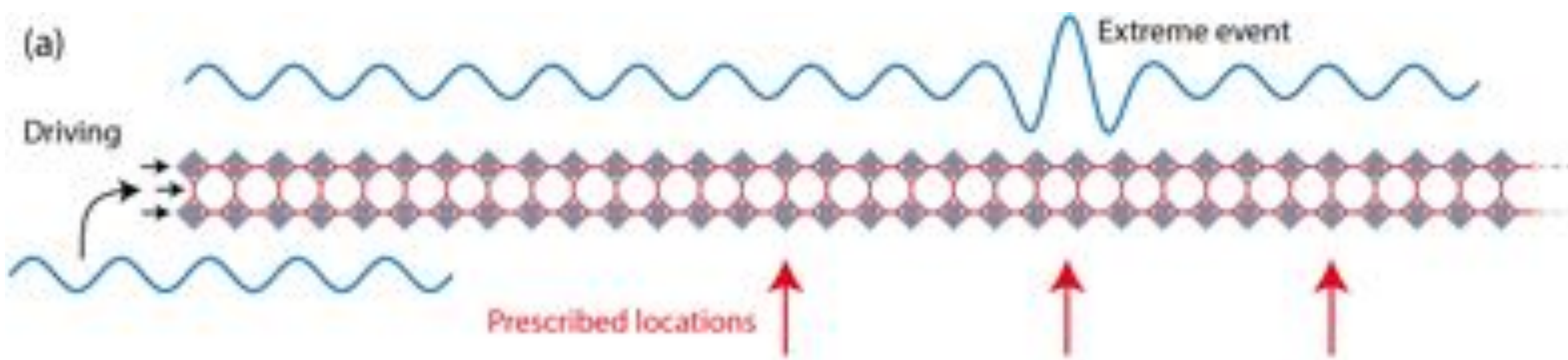
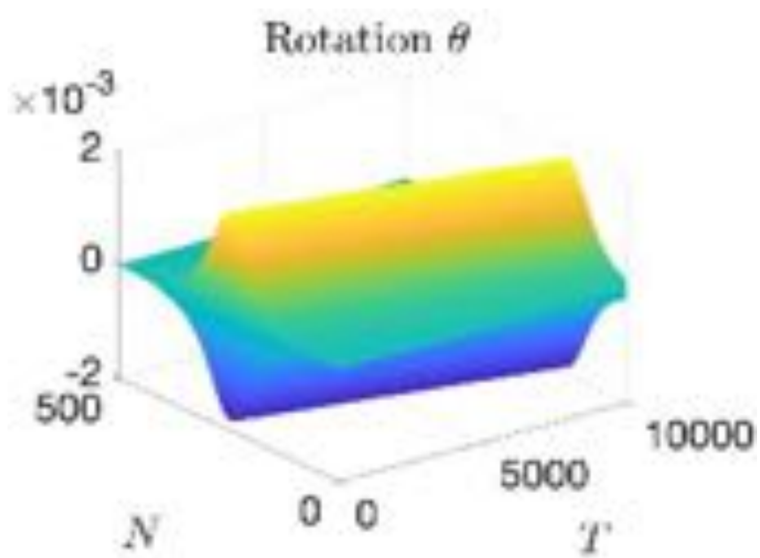
Part 2: Modulation Instability in FlexEM

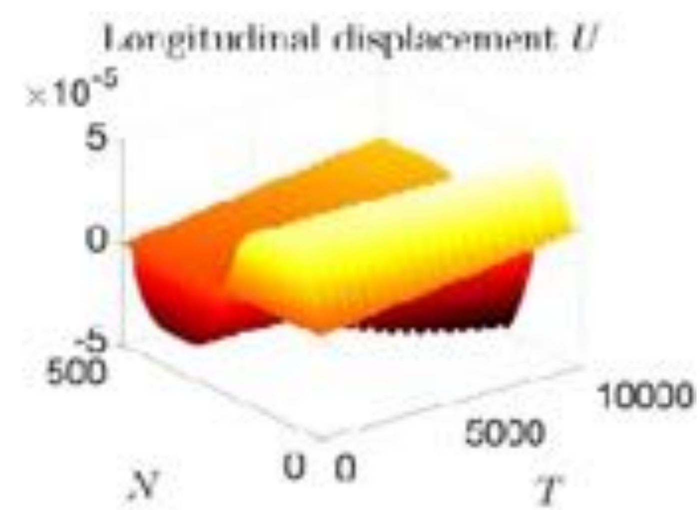
Main perspectives

1. Instability in smaller structures, driven dumped problem



2. Formation of localized nonlinear waves and rogue waves





Post-doc positions

Experiments with nonlinear waves in flexible elastic metamaterials

ANR project **ExFLEM :** collaboration LAUM (Le Mans), supméca (Paris)

Absorption of nonlinear waves using passive and active scatterers

ERC - StG project **NASA:** LAUM (Le Mans)