Dynamical emergence of a Kosterlitz-Thouless transition in a 2D disordered Bose gas

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GdR Complexe Annual Workshop - Paris, December 7, 2022

Based on : Thibault Scoquart, Dominique Delande, and Nicolas Cherroret, Phys. Rev. A **106**, L021301 (2022)

Laboratoire Kastler Brossel
Physique quantique et applications

SCIENCES SORBONNE UNIVERSITÉ

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Introduction : Quench dynamics of isolated quantum systems

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$$\hat{H} = \hat{H}_0 + \hat{H}_q$$

Unitary evolution

$\hat{H}_0 \quad |\Psi(t=0)\rangle$

$$|\Psi(t)\rangle = e^{-i\hbar\hat{H}t}|\Psi(0)\rangle$$

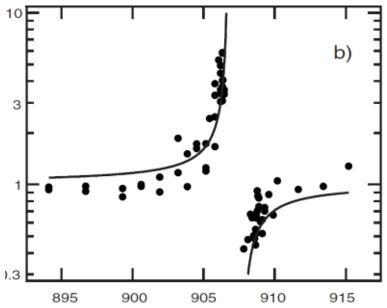
Equilibrium ?

$t = 0$ $\qquad t = \tau_{therm}$ $\qquad$ time t

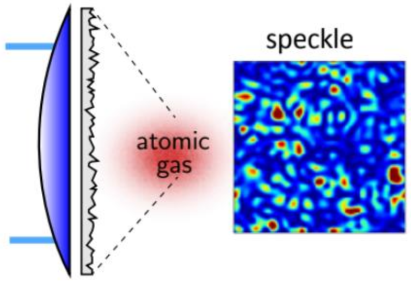
→ Dynamical evolution towards equilibrium ?

→ Effect of disorder ?

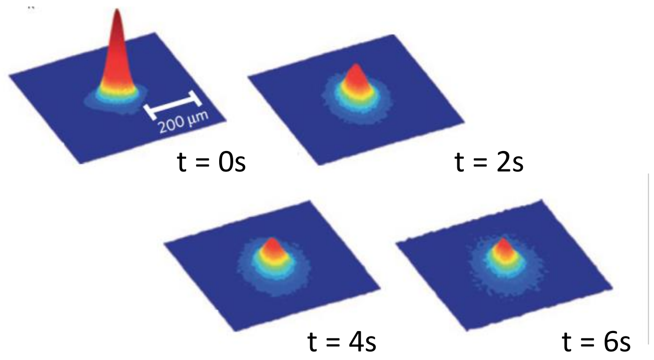
- Implemented in **cold atoms experiments** :



Tunable interactions



Tunable disorder (Speckle)



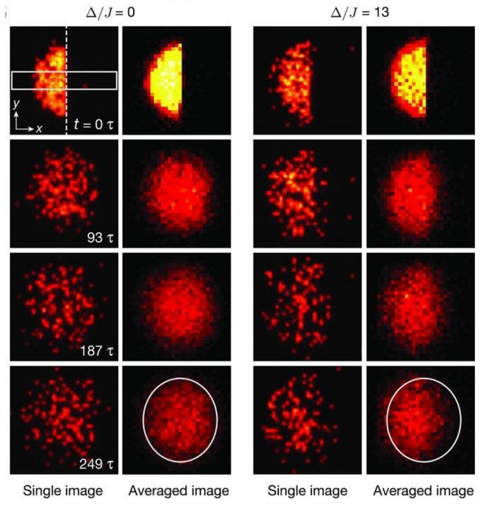
Time-resolved experiments

Chin et al. Rev. Mod. Phys. **82**, 1225

Jendrzejewski et al., Nature phys. **8**, 398 (2012)

Introduction : Quench dynamics of isolated quantum systems

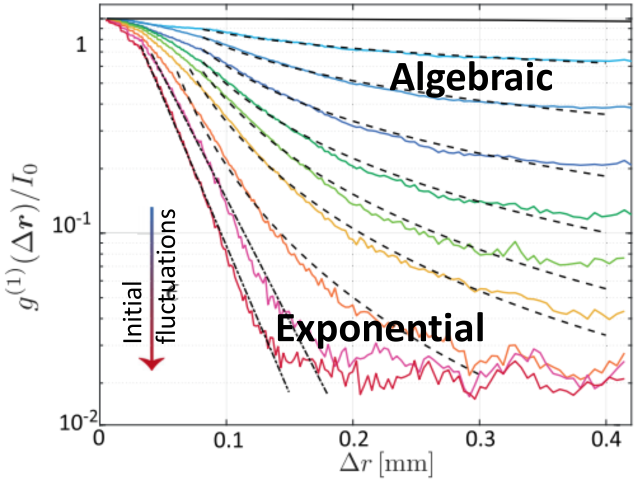
Many-body localization



Theory :
Gornyi *et al.*, PRL (2005)
Basko *et al.*, Ann. Phys. (2006)
Abanin *et al.*, Rev. Mod. Phys. (2019)

Choi, J *et al.*, Science (2018)*

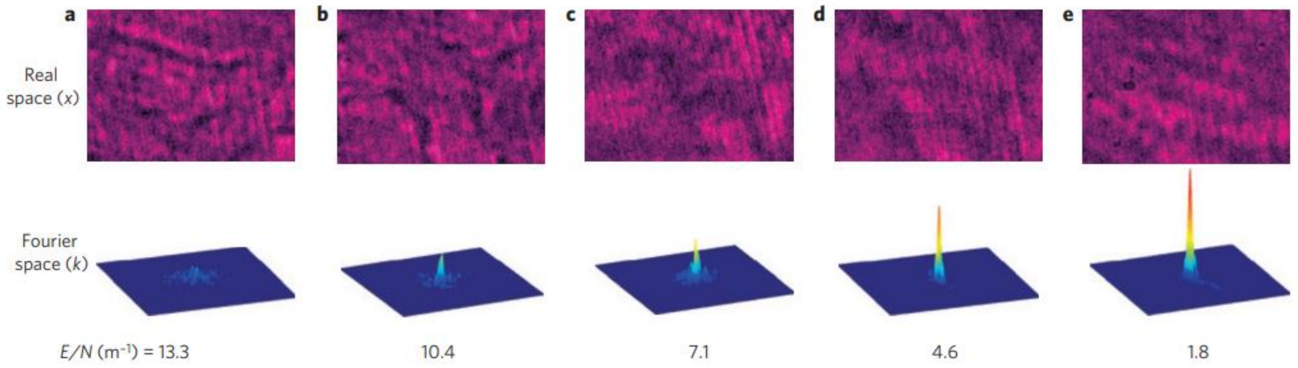
Prethermalization in near-integrable systems



Theory:
Mori *et al.*, (2018)
Berges *et al.*, Phys. Rev. Lett. (2004)

Experiments:
Gring *et al.*, Science (2012).
M. Abuzarli *et al.*, Phys. Rev. Lett. (2022)*

Dynamical emergence of condensates (nonlinear optics)



Theory:
Connaughton *et al.*, Phys. Rev. Lett. (2005).
Shukla and Nazarenko, Phys. Rev. A (2022).
In atomic vapor:
Šantic *et al.*, Rev. Lett. (2018).
In photorefractive crystal:
Sun *et al.*, Nat. Phys. (2012).*
In multimode fibers:
K. Baudin *et al.*, Phys. Rev. Lett. (2020).

Our system

- **Dilute (weakly interacting) ultracold Bose gas** : **Mean-field** treatment of interactions

Nonlinear Schrödinger equation $(\hbar = 1)$

$$i\partial_t \psi = -\frac{\nabla^2 \psi}{2m} + gN|\psi|^2\psi$$

For the **classical** field $\psi(\boldsymbol{r}, t)$ describing collective atomic behavior.

- **Gaussian uncorrelated** disorder potential :

$$\begin{cases} \overline{V(\boldsymbol{r})} = 0 \\ \overline{V(\boldsymbol{r})V(\boldsymbol{r'})} = \gamma\delta(\boldsymbol{r} - \boldsymbol{r'}) \end{cases}$$

γ sets the disorder strength

- Interaction time scale as a reference :

$$\tau_g = \frac{1}{g\rho_0}$$

$(\rho_0 = N/V)$

- System of **classical nonlinear waves in disorder:** not only ultracold Bose gases but also **nonlinear optics**…

Quench protocol

Add disorder $V(\boldsymbol{r})$

Initial plane wave $|\boldsymbol{k}_0\rangle$

Eigenstate → no evolution

$t = 0$

time t

Quench protocol

Initial plane wave $|\boldsymbol{k}_0\rangle$

Eigenstate → no evolution

Add disorder $V(\boldsymbol{r})$

$t = 0$

Out-of-eq. evolution with:

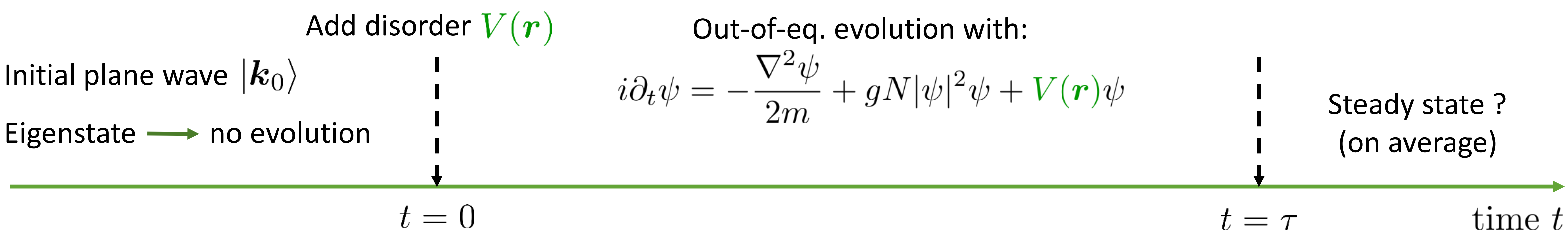
$$i\partial_t \psi = -\frac{\nabla^2 \psi}{2m} + gN|\psi|^2\psi + V(\boldsymbol{r})\psi$$

$t = \tau$

Steady state ?
(on average)

time t

Quench protocol



- For a given initial state $(\boldsymbol{k}_0, \gamma, g)$:
  - **Simulations** of the **full time-evolution** of the gas on a **2D grid** of step a: $|\psi(t)\rangle$ for all t
  - Look at **time-resolved, disorder-averaged quantities:**

Quench protocol

Initial plane wave $|\boldsymbol{k}_0\rangle$

Eigenstate ⟶ no evolution

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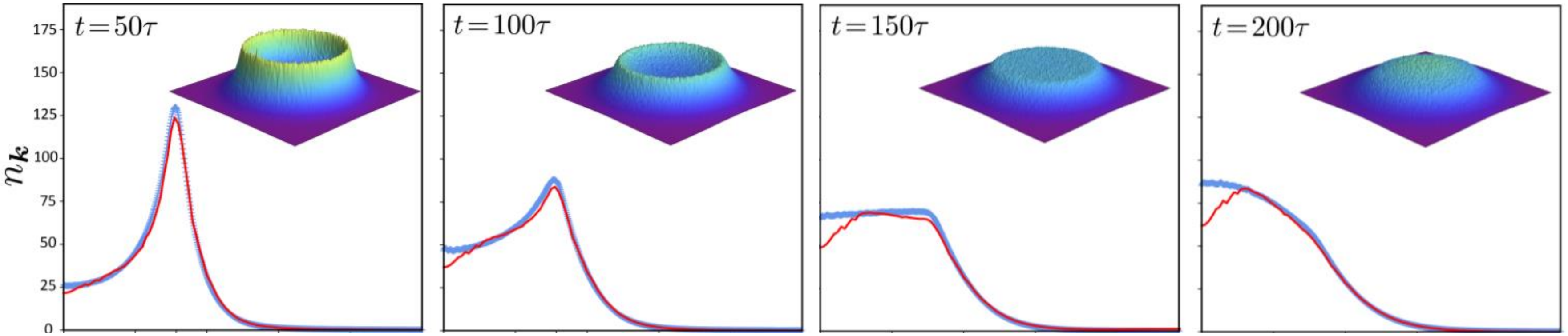
e.g. avg momentum space distrib.

$$n_{\boldsymbol{k}}(t) \equiv \overline{|\psi(\boldsymbol{k},t)|^2}$$



Nicolas Cherroret, Thibault Scoquart, Dominique Delande, Annals of Physics **435,** 168543 (2021)

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Properties of dynamical evolution ?

Steady state ?
(on average)

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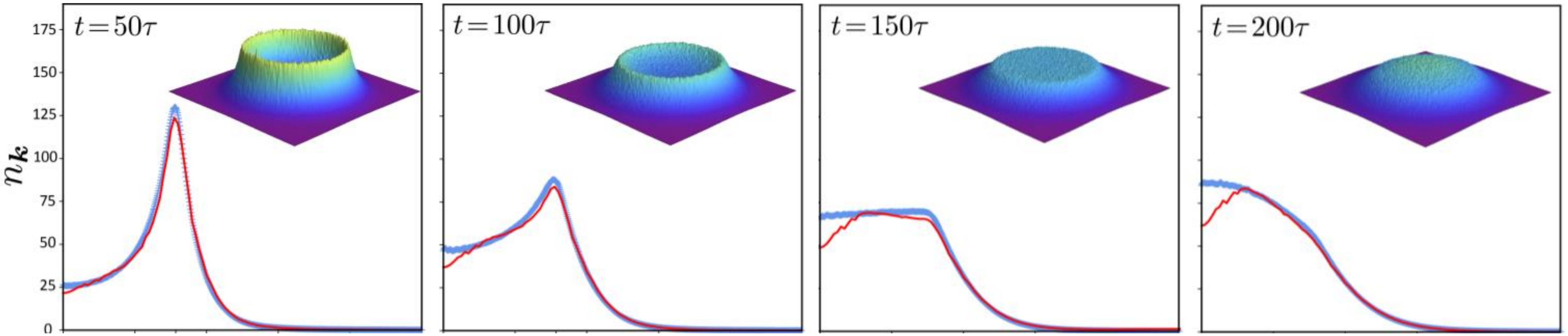

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Initial plane wave $|\boldsymbol{k}_0\rangle$

$$i\partial_t\psi = -\frac{\nabla^2\psi}{2m} + gN|\psi|^2\psi + V(\boldsymbol{r})\psi$$

Thermal state ?

Eigenstate ⟶ no evolution

Properties of dynamical evolution ?

$t = 0$ $\qquad$ $\tau_{\text{therm}} = ?$ $\qquad$ time t

- For a given initial state $(\boldsymbol{k}_0, \gamma, g)$:
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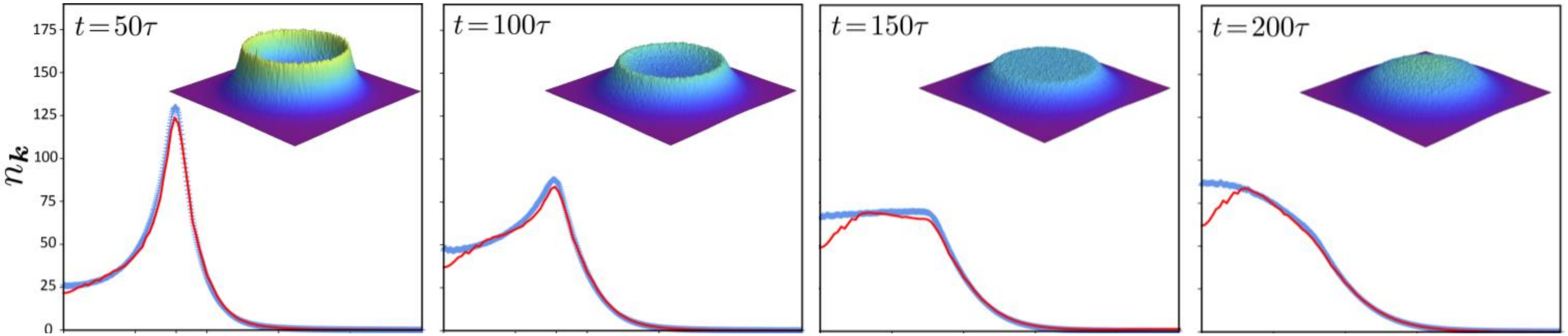

Nicolas Cherroret, Thibault Scoquart, Dominique Delande, Annals of Physics **435,** 168543 (2021)

Out-of-equilibrium dynamics

Based on :
T. Scoquart, D. Delande, and N. Cherroret, Phys. Rev. A **106**, L021301 (2022).
T. Scoquart, P-É. Larré, D. Delande and N. Cherroret, *EPL* **132** 66001 (2020).

Out-of-equilibrium dynamics

- For various initial states $(\boldsymbol{k}_0, \gamma, g)$, look at the **spatial coherence function of the gas**:

$$g_1(\boldsymbol{r}, t) \equiv \overline{\psi^*(0,t)\psi(\boldsymbol{r},t)}$$

→ **2 distinct regimes**

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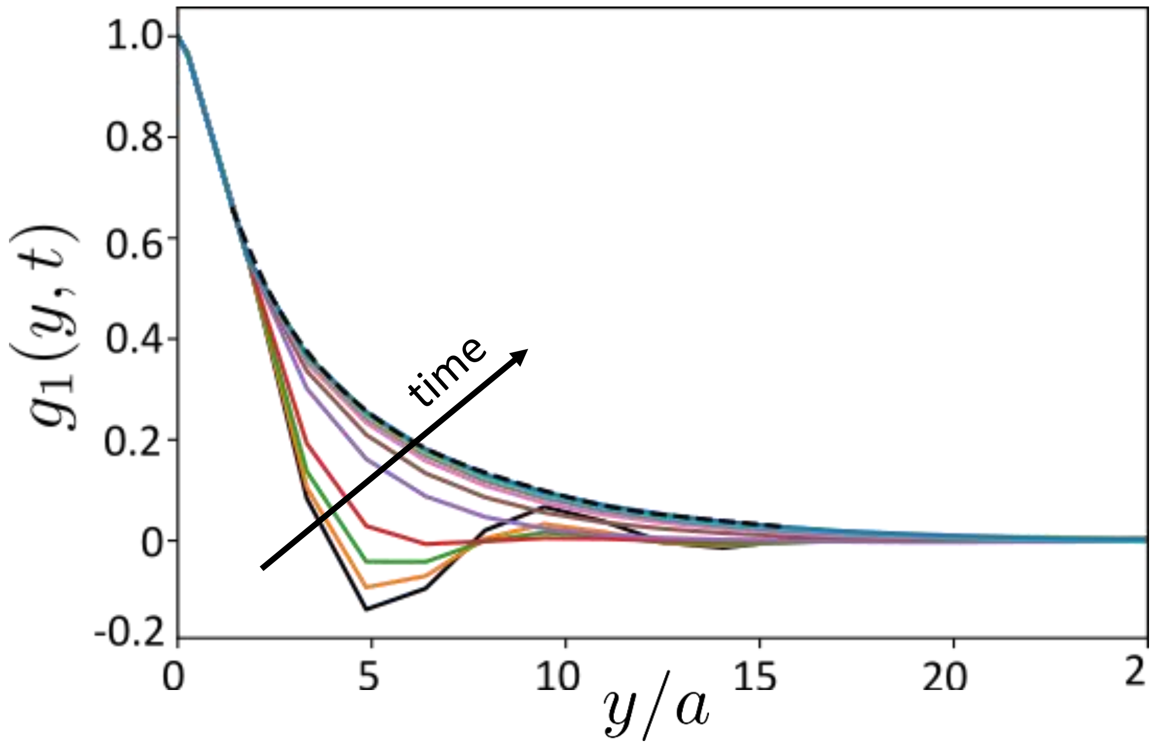
1) Multiple scattering regime $\gamma \gg g$:



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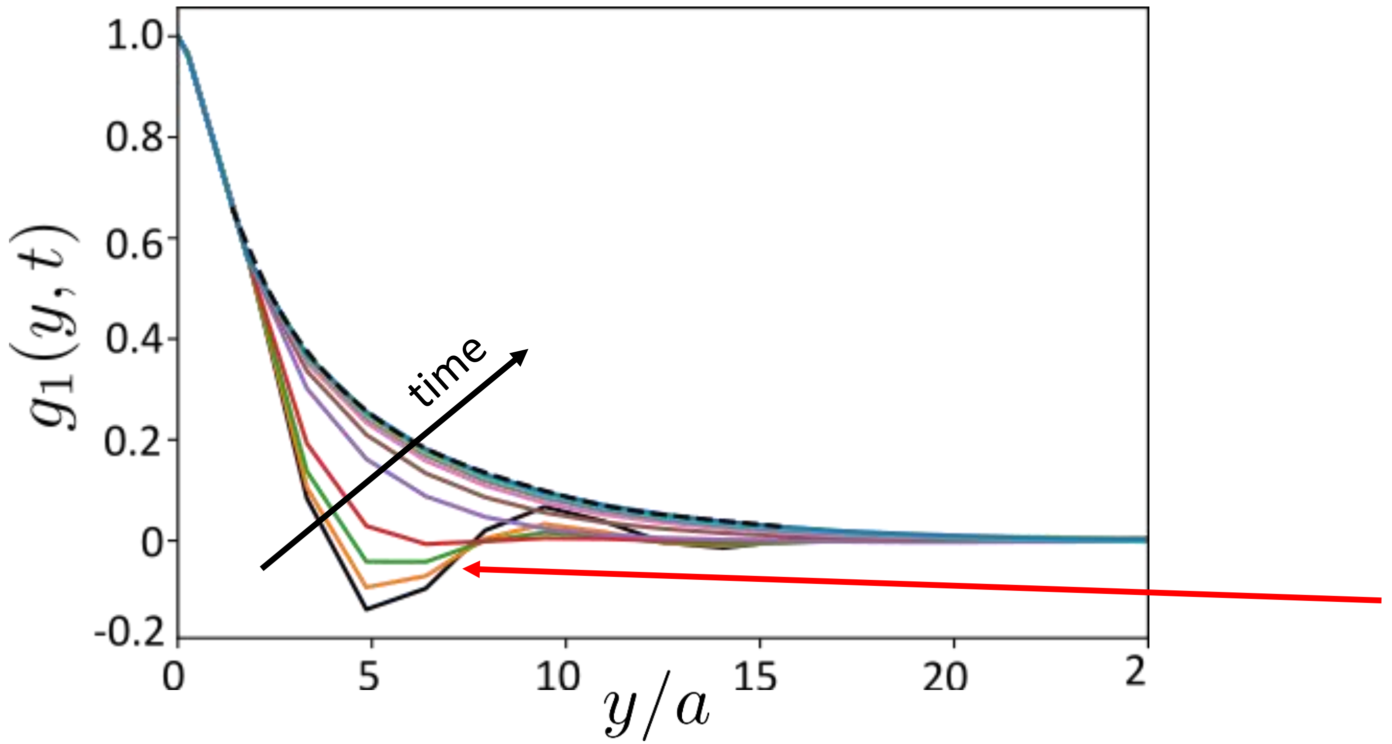

Disorder dominates at short times:
short-range coherence

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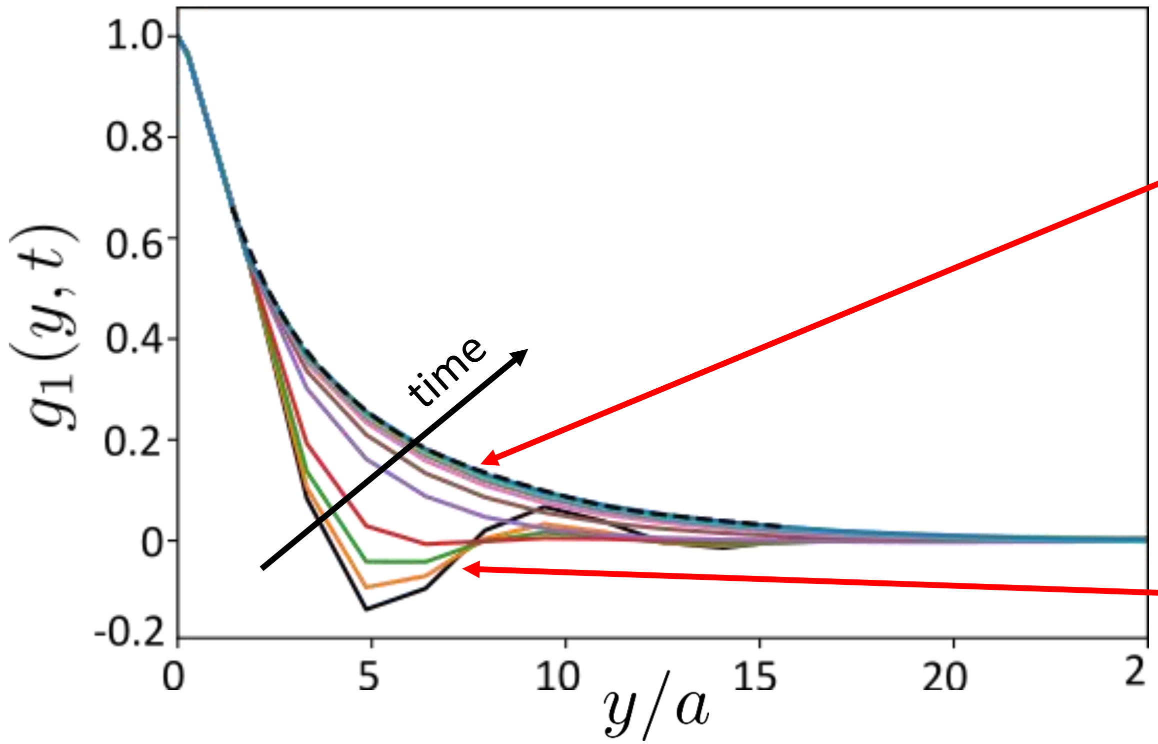

Fast evolution towards exponential coherence:

$$g_1(\boldsymbol{r}, t) \sim e^{-r/\mathcal{L}(t)}$$

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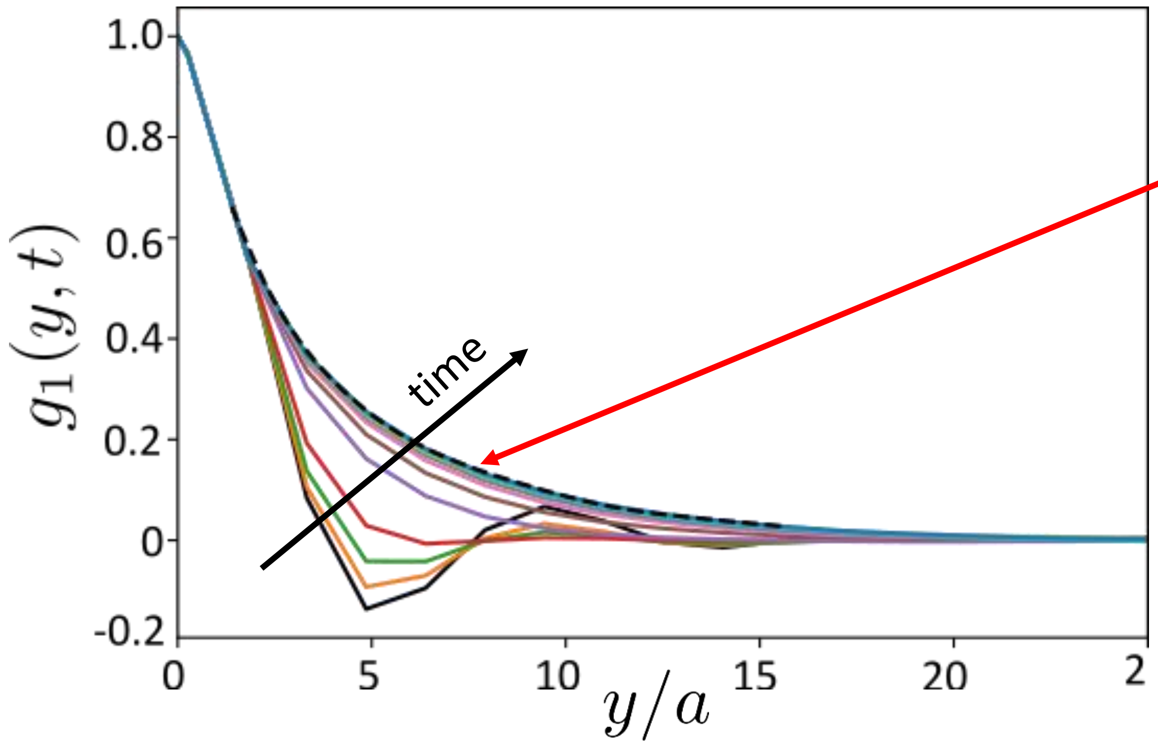
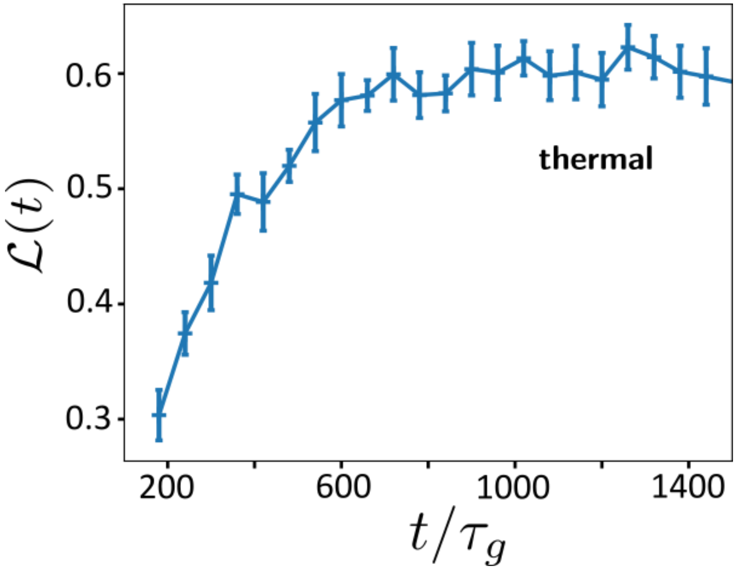


Fast evolution towards exponential coherence:

$$g_1(\boldsymbol{r}, t) \sim e^{-r/\mathcal{L}(t)}$$

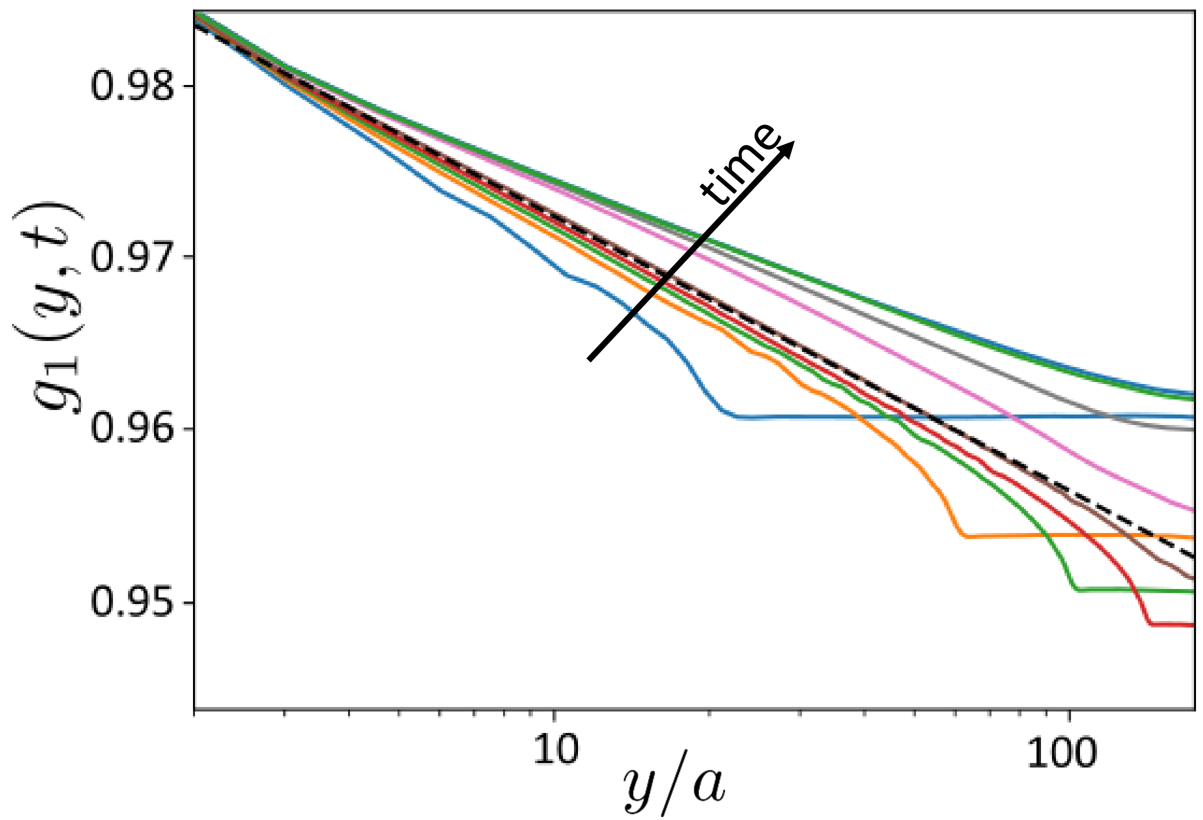
Quickly reaches a **thermal** state

$$\tau_{therm} \sim 10^2 \tau_g$$



Out-of-equilibrium dynamics

2) Weak disorder regime $g \gg \gamma$:



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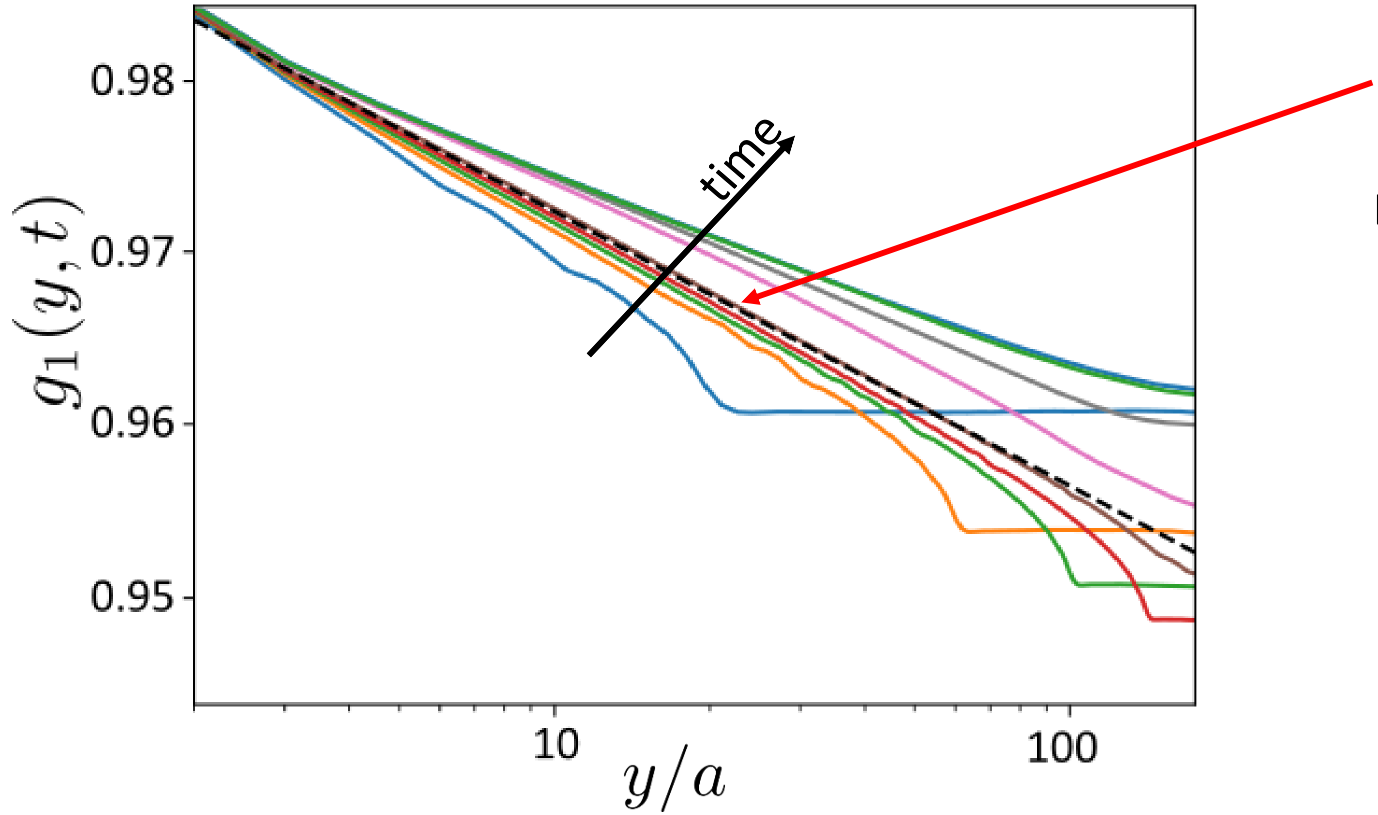

Algebraic scaling of correlations:

$$g_1(\boldsymbol{r},t) \sim y^{-\alpha(t)}$$

Large-scale coherent structure: ‘Quasi-condensate’

⟶ **Superfluid** fraction at equilibrium

Out-of-equilibrium dynamics

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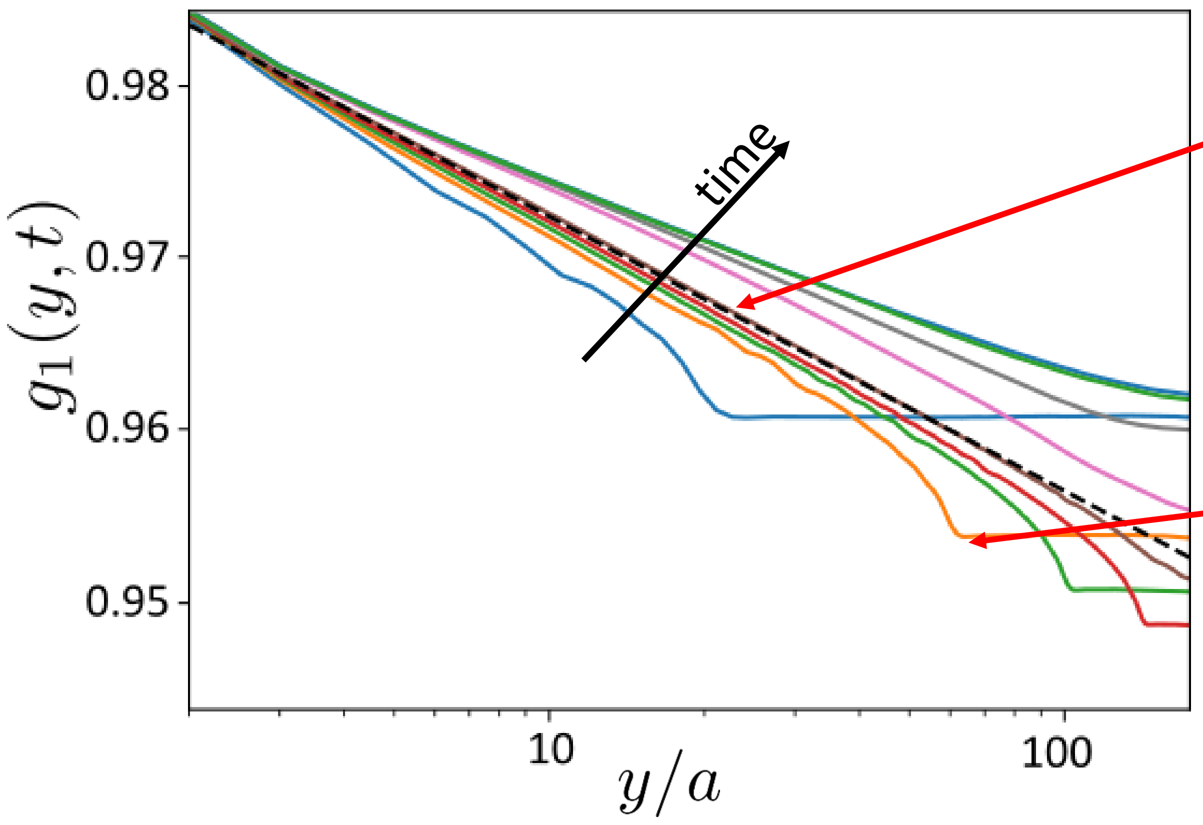

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Large-scale coherent structure: 'Quasi-condensate'

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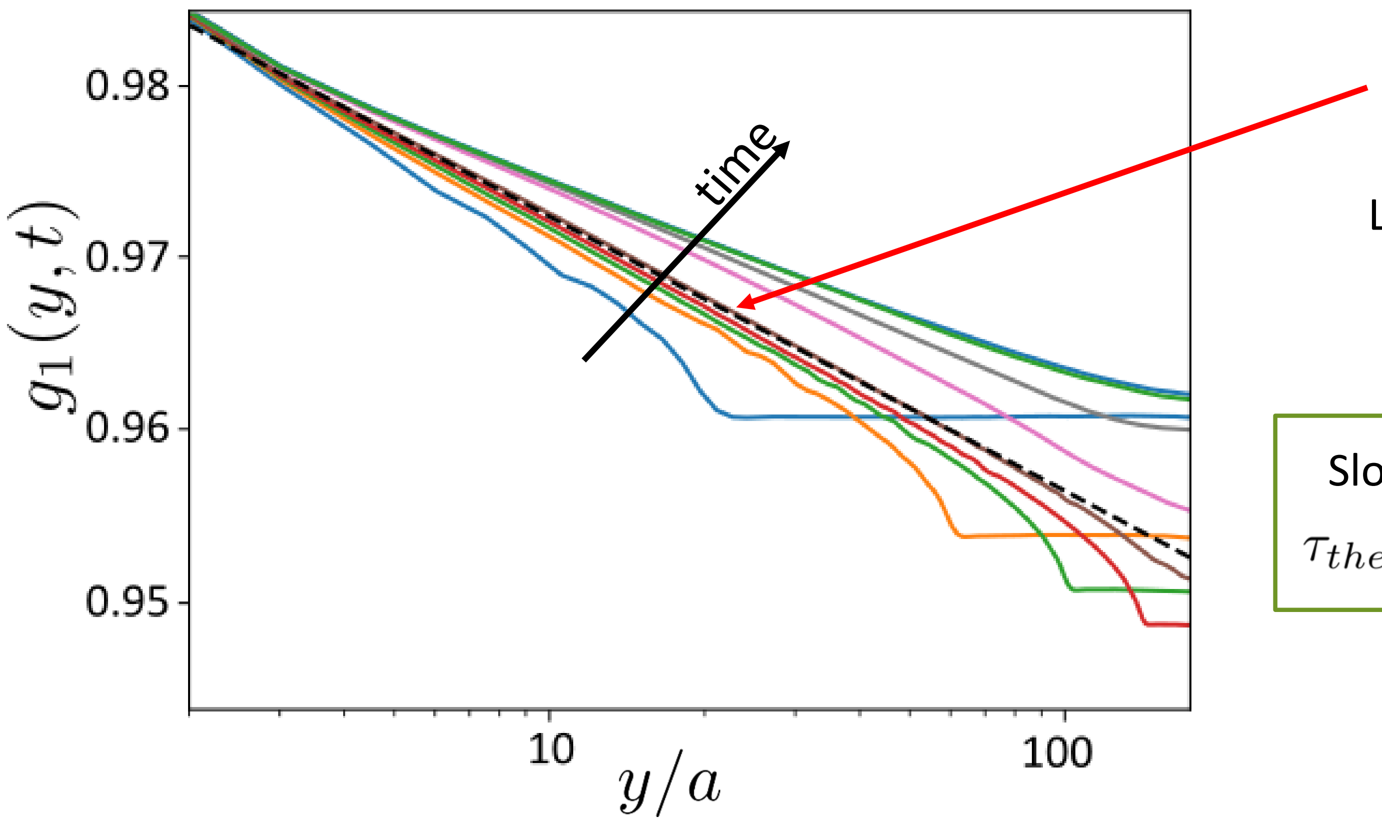
Slow dynamics, **light cone** spreading with sound velocity

$$c_s = \sqrt{\frac{g\rho_0}{m}}$$

Out-of-equilibrium dynamics

2) Weak disorder regime $g \gg \gamma$:



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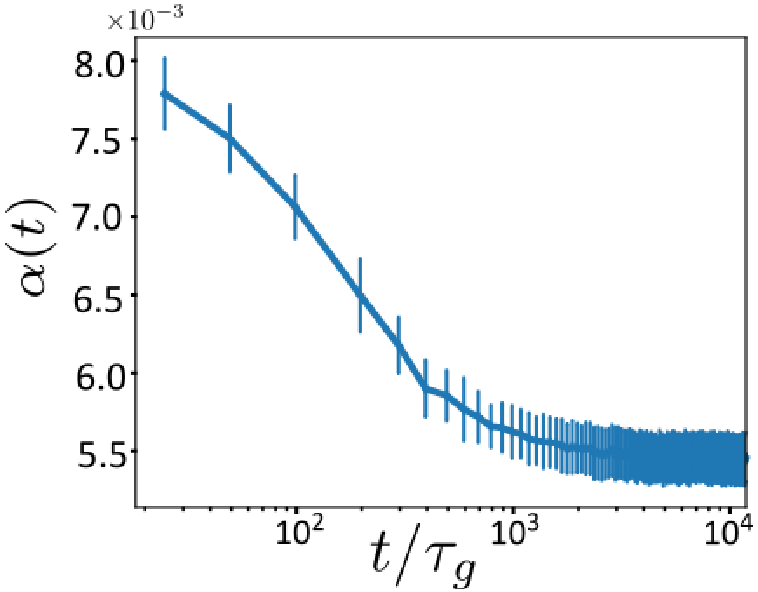
Slow dynamics

$$\tau_{therm} \sim 10^4 \tau_g$$



Out-of-equilibrium dynamics

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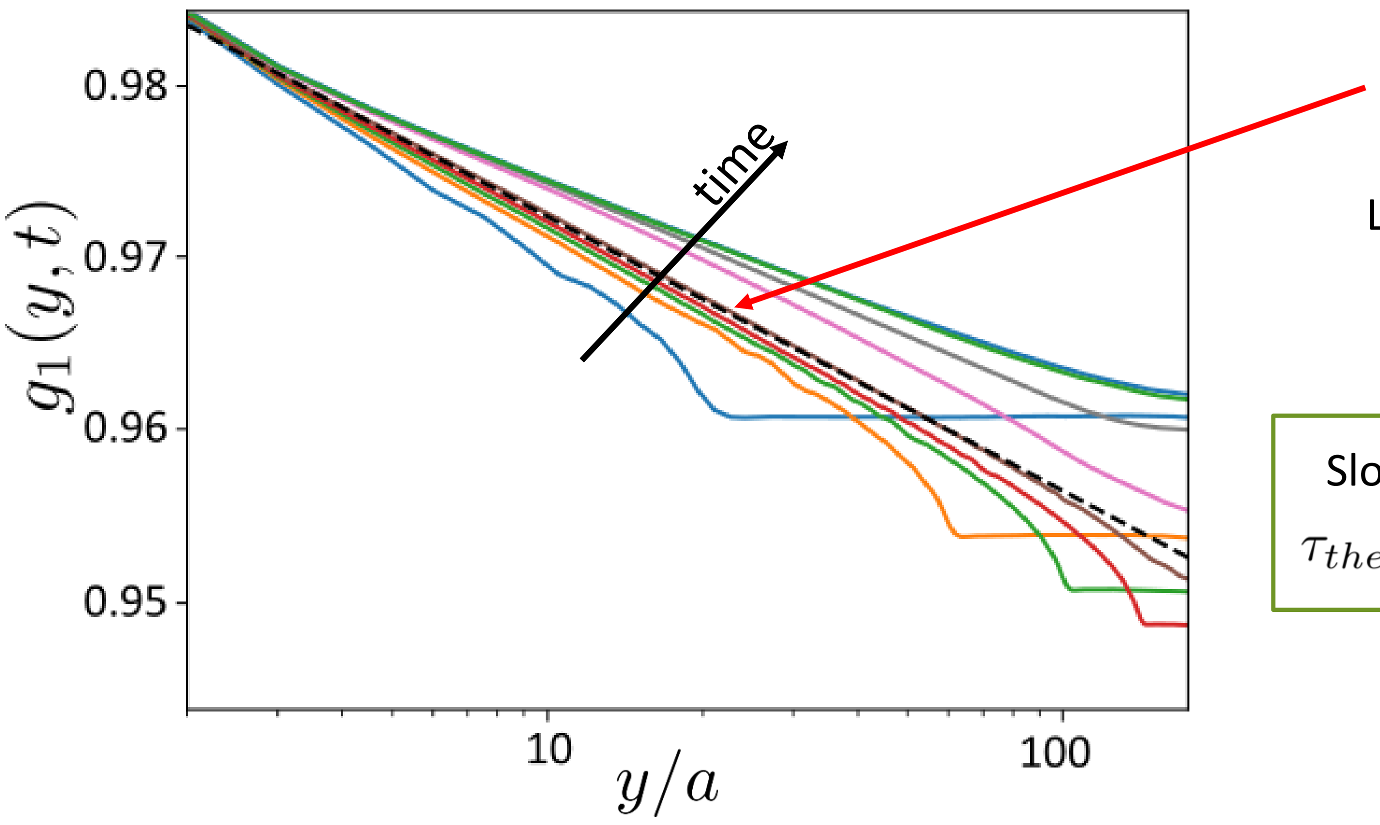

Universal, algebraic scaling of correlations:

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Large-scale coherent structure: 'Quasi-condensate'

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Slow dynamics

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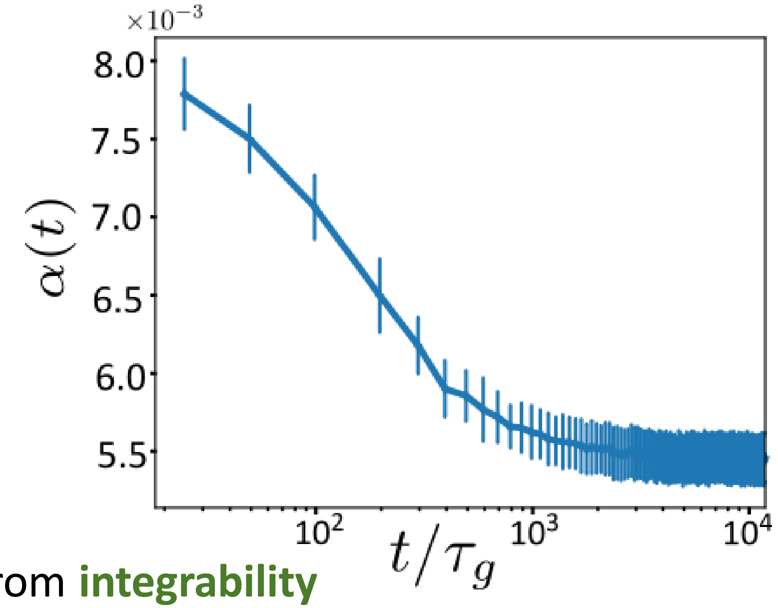
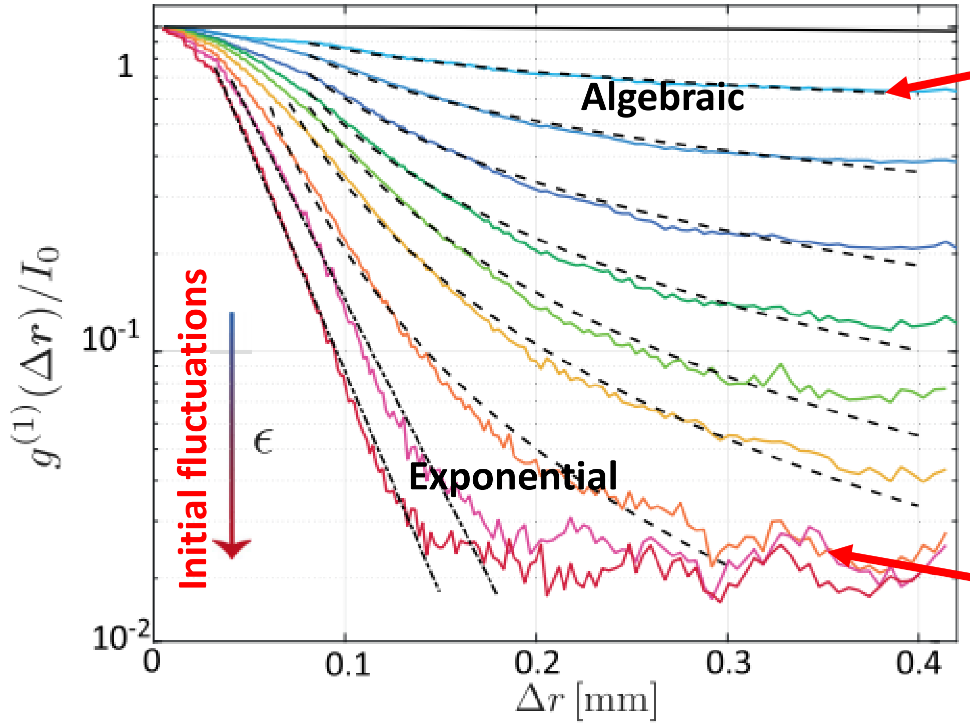


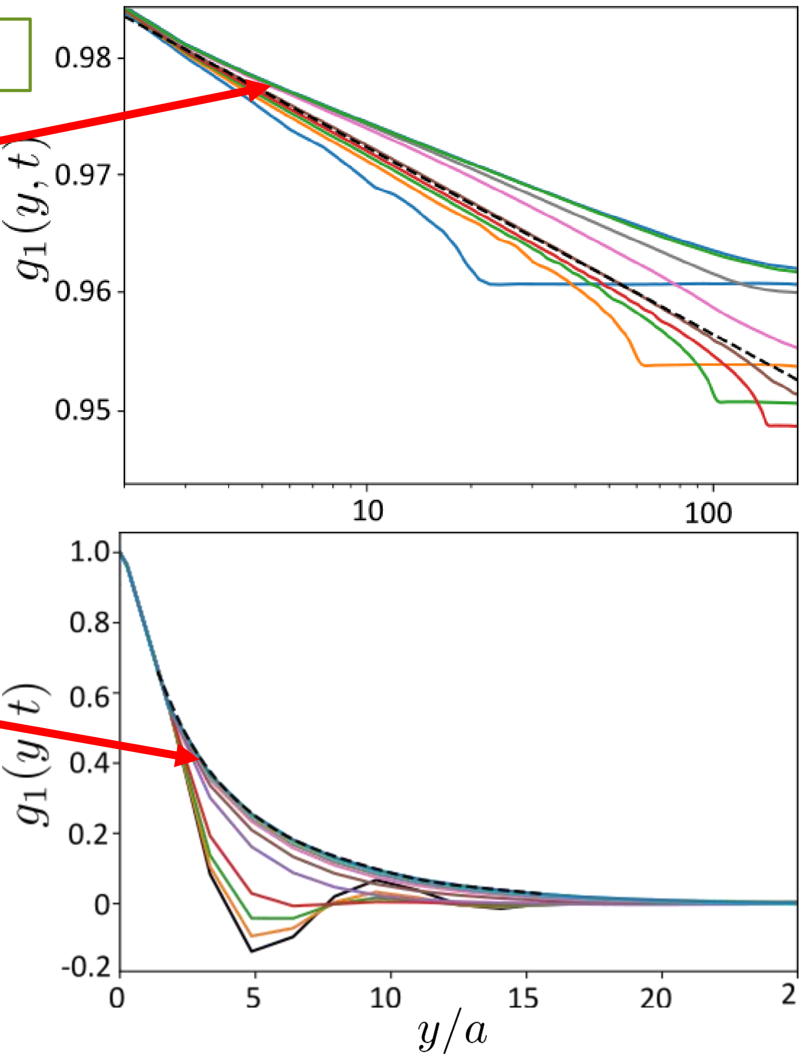
Prethermalization :

- Out-of-eq **metastable state** of systems weakly perturbed from **integrability**
- Clear **separation of time scales** (slow evolution far from equilibrium)

Comparison to experiment

Spatial coherence of **long-lived out-of-eq. state** in a **quantum fluid of light**





Disorder strength γ

System **weakly perturbed from integrability**:

Fluctuating initial state ⟷ Disorder potential in our case

Murad Abuzarli, Nicolas Cherroret, Tom Bienaimé, and Quentin Glorieux, Phys. Rev. Lett. **129**, 100602 (2022)

Kosterlitz-Thouless transition in disorder

Based on :
T. Scoquart, D. Delande, and N. Cherroret, Phys. Rev. A **106**, L021301 (2022).

Thermalization of a homogeneous dilute Bose gas

- Thermalization characterized by the **average energy distribution** of the gas:

$$f_\epsilon(t) \sim \frac{1}{\nu_\epsilon} \overline{\langle\psi(t)|\, \delta(\epsilon - H)\, |\psi(t)\rangle}$$

- For a **classical, homogeneous** system described by NLS, we expect:

$$f_\epsilon(t \to \infty) = \frac{T}{\epsilon - \mu}$$

Thermal **Rayleigh-Jeans (RJ)** distribution

→ $T(\boldsymbol{k}_0, g, \gamma)$ and $\mu(\boldsymbol{k}_0, g, \gamma)$ fully determined by CV of energy/particle number

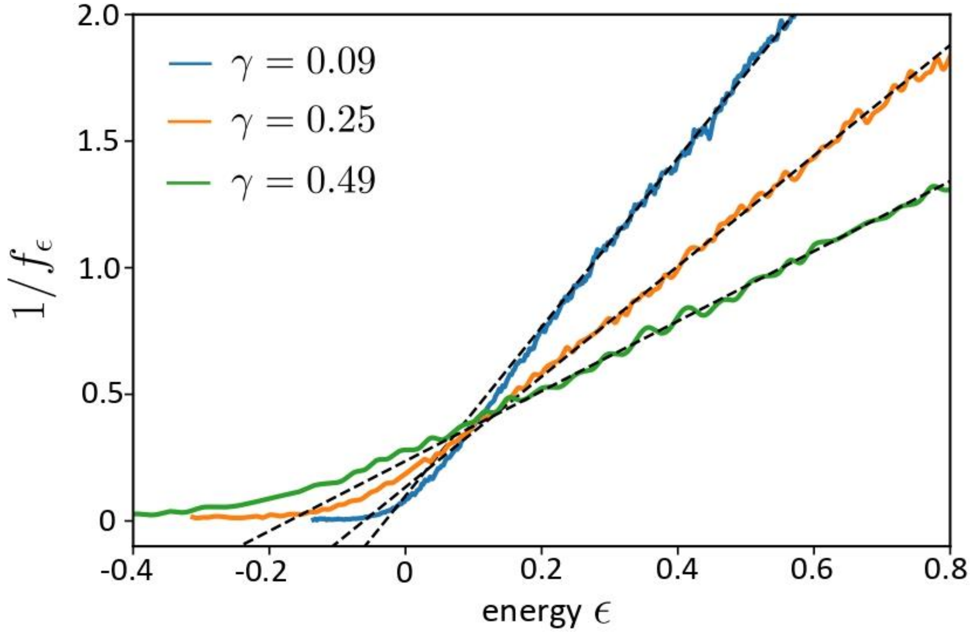
- But what happens with **disorder**, or **quasi-condensation ?** → **RJ** only describes the **high energies**

→ We have to determine $T(\boldsymbol{k}_0, g, \gamma)$ and $\mu(\boldsymbol{k}_0, g, \gamma)$ numerically

Characterizing the equilibrium

- **Rayleigh-Jeans** still valid a large energies
- Numerically compute $1/f_\epsilon(t \to \infty) \sim \frac{T}{\mu} - \frac{\epsilon}{\mu}$

⟶ $T(\boldsymbol{k}_0, g, \gamma)$ and $\mu(\boldsymbol{k}_0, g, \gamma)$ from a simple fit



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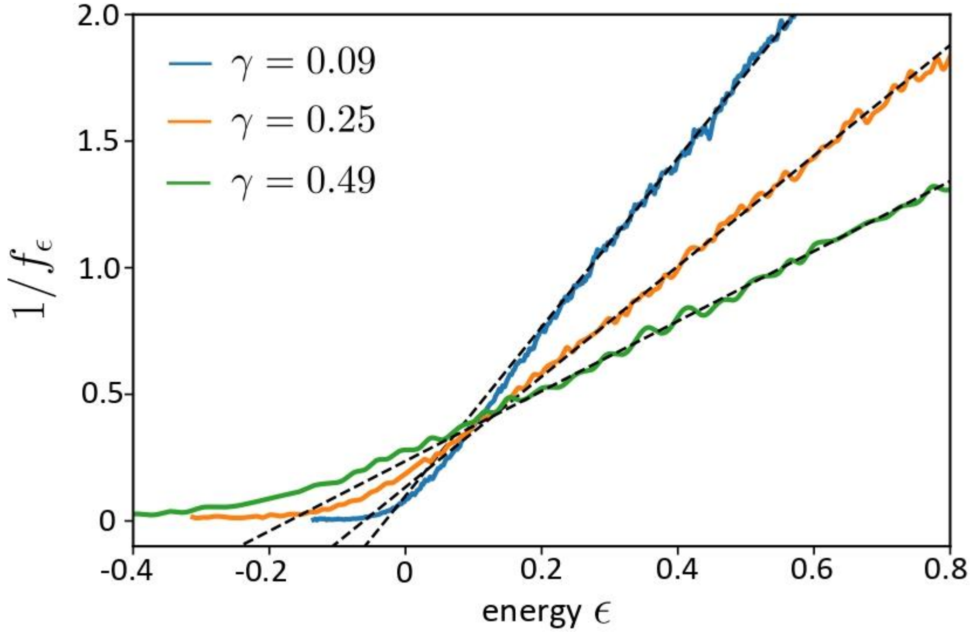
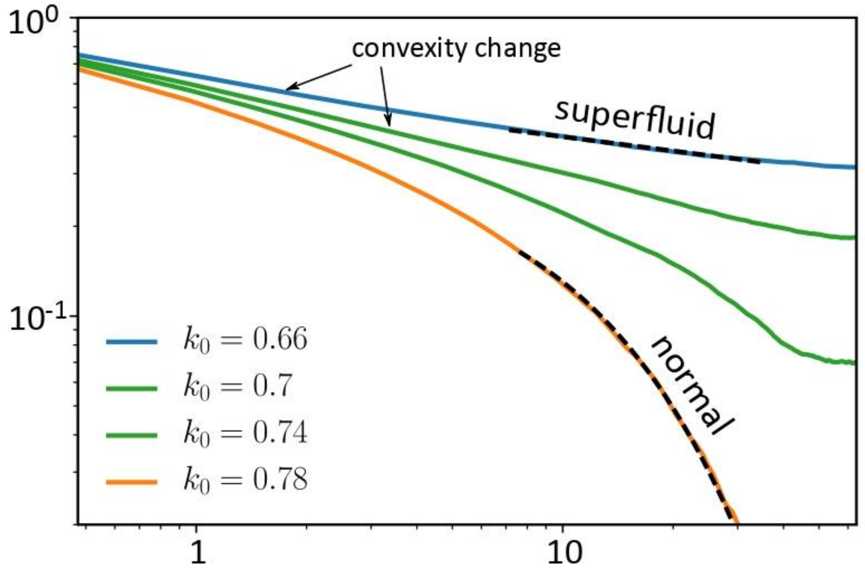




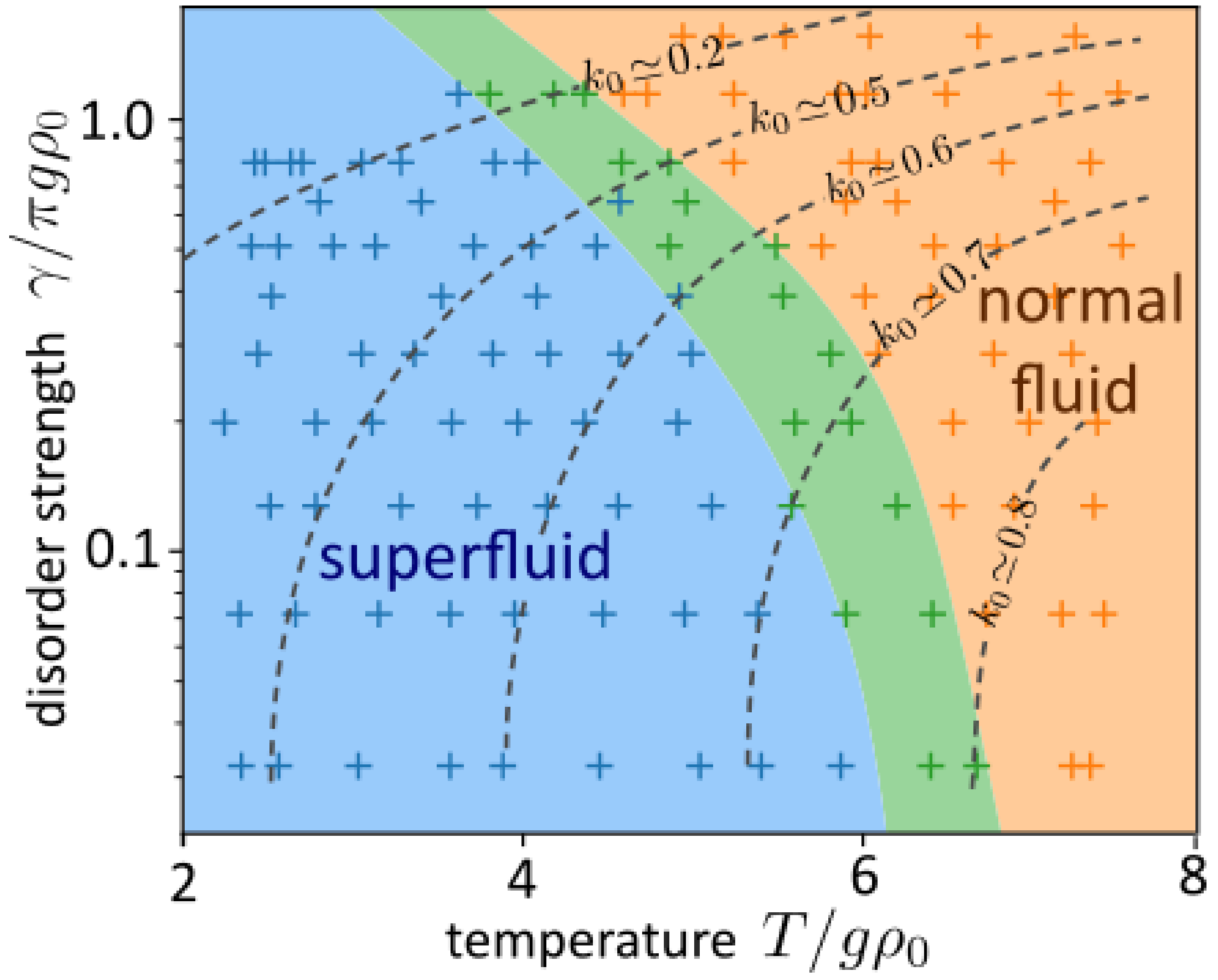
- To discriminate the **phase** we are in : $g_1(\boldsymbol{r}, t \to \infty)$

Exponential → **Normal fluid** region

Algebraic → **Superfluid** region

- In between : Critical region for the **KT transition** !

Phase diagram of the KT transition



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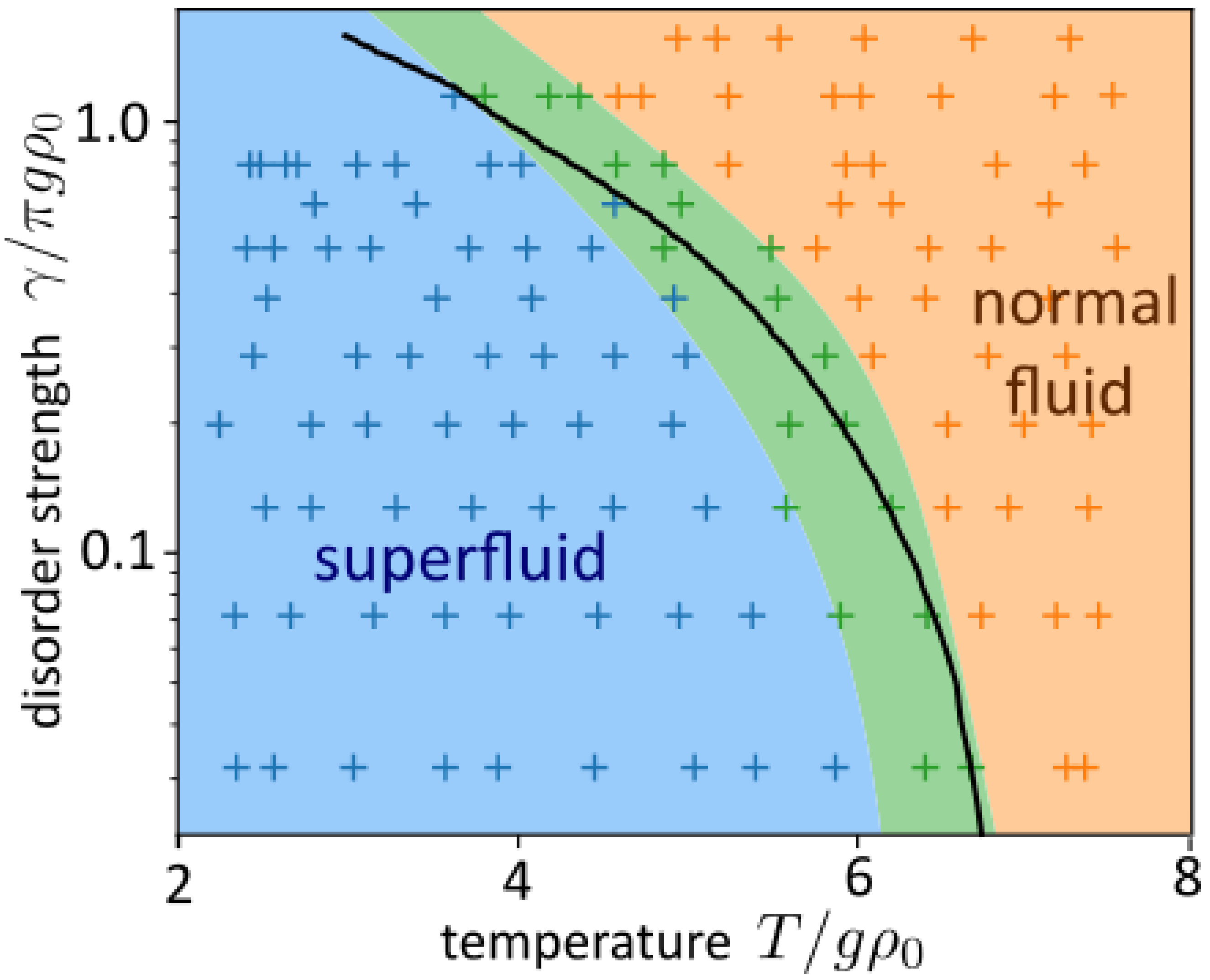


Shows how the **Kosterlitz-Thouless transition (2D)** Is affected by disorder

↓

Yields estimate of $T_{\mathrm{KT}}(\gamma)$

↓

In good agreement with our theory: ———

$$\gamma^* = \frac{4\pi}{mI_1}\left[1 - \frac{T_{\mathrm{KT}}}{T_d}(4 - I_2)\right]$$

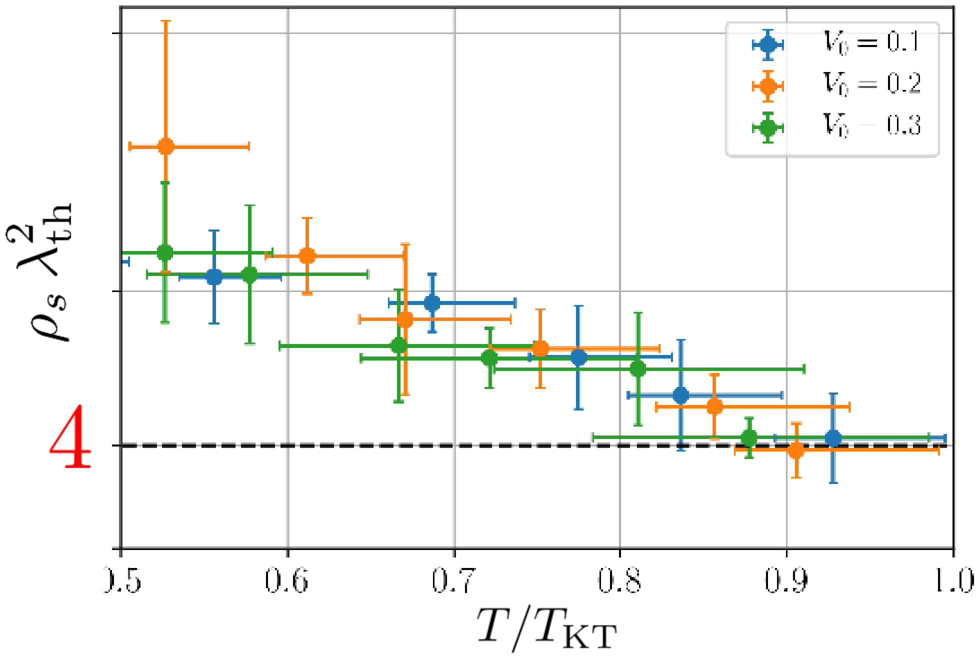
where I_1 and I_2 are numerical constants depending on the initial state $(\boldsymbol{k}_0, \gamma, g)$

Theory adapted from:
G. Bertoli *et al.*, Phys. Rev. Lett. **121**, 030403 (2018)
V. I. Yukalov *et al*, Physical Review A **75**, 023619 (2007)

Universal properties of the KT transition

2 **essential properties** of the KT transition still verified with **disorder**

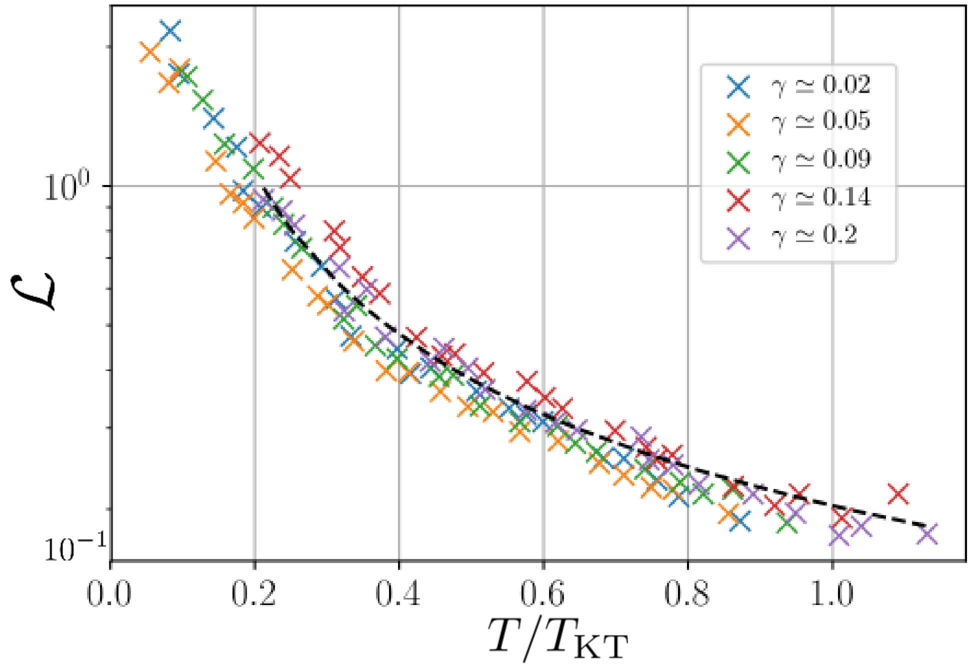
Superfluid phase: $g_1(r) \sim r^{1/(\rho_s \lambda_{\rm th}^2)}$



Universal jump of g_1 **algebraic exponent**

$$\rho_s(T_{\rm KT})\,\lambda_{\rm th}^2(T_{\rm KT}) = 4$$

Normal phase: $g_1(r) \sim e^{-r/\mathcal{L}}$

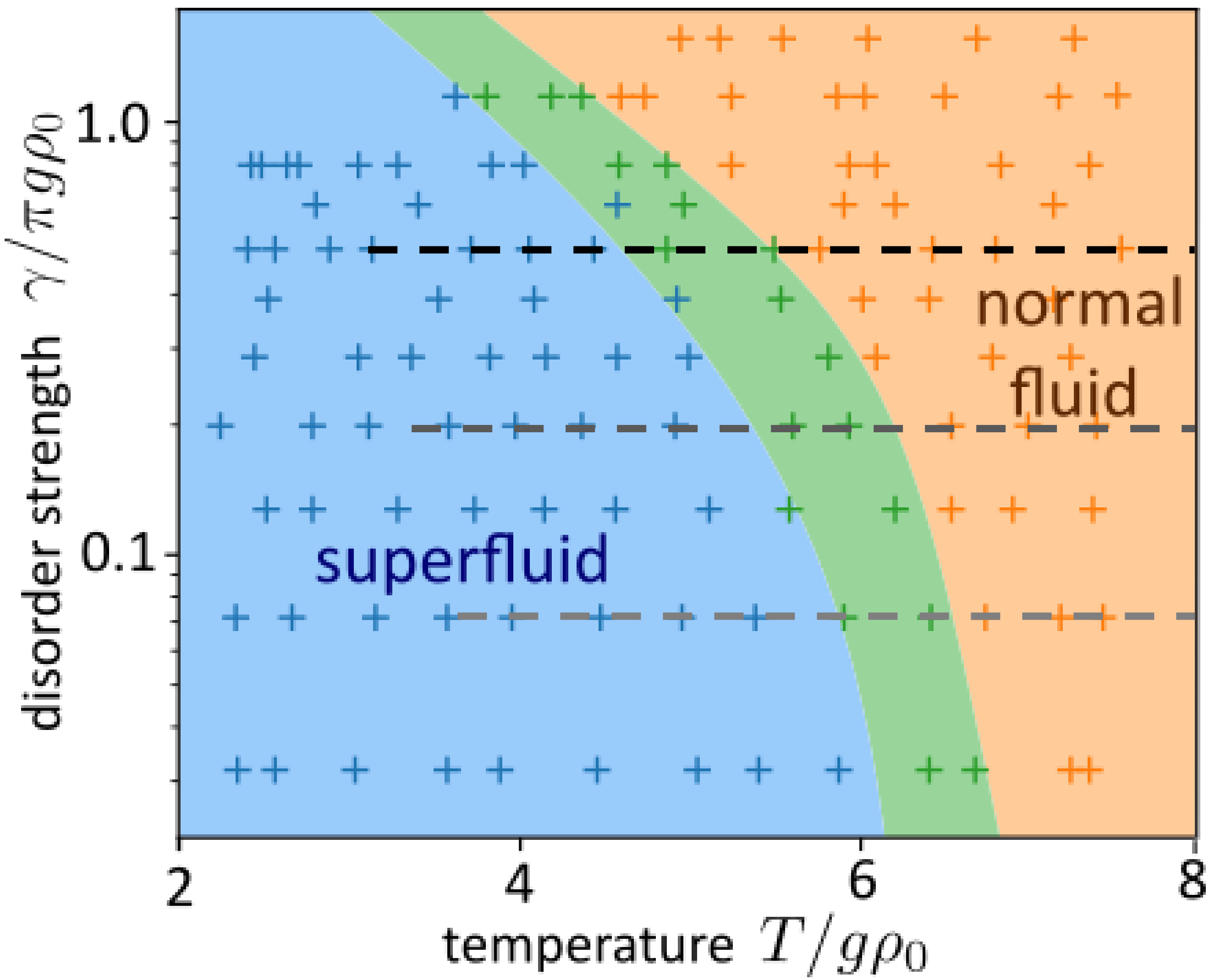


$$\mathcal{L}(T > T_{\rm KT}) \sim \lambda_T \exp\left\{\frac{\sqrt{\zeta T_{\rm KT}}}{\sqrt{T - T_{\rm KT}}}\right\}$$

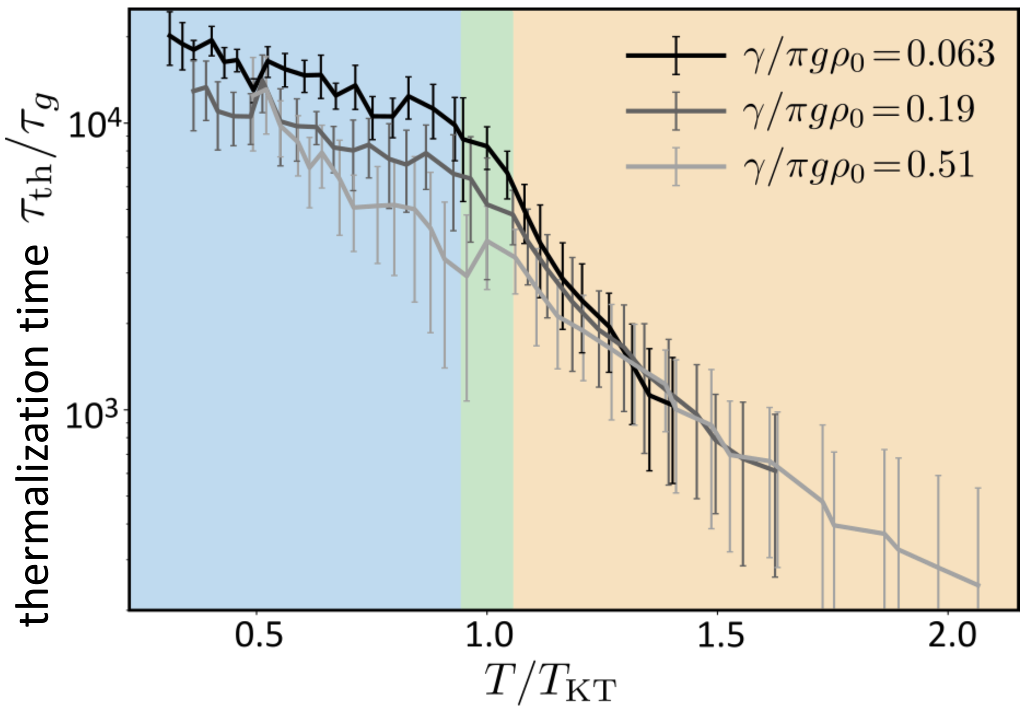
D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201–1205 (1977).

Z. Hadzibabic and J. Dalibard, La Rivista del Nuovo Cimento **34**, 389–434 (2011).

Thermalization times ?



- Coming from out-of-equilibrium, we can look at $\tau_{\rm th}$

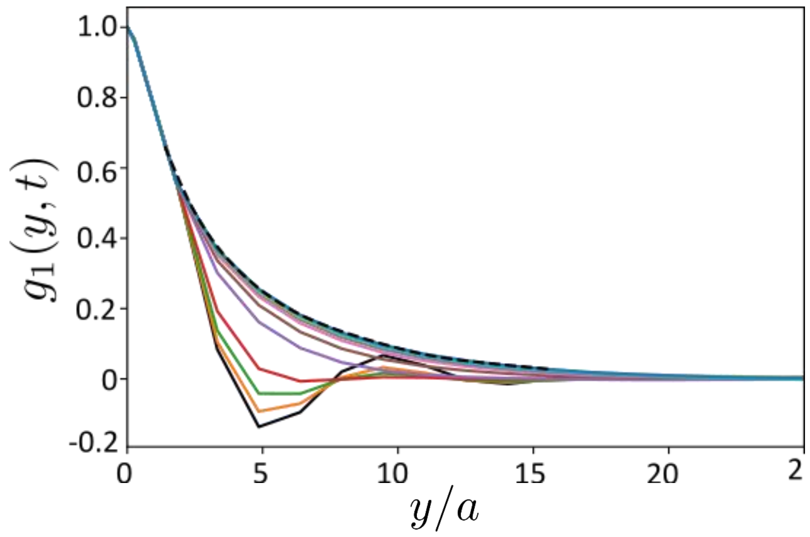


- As seen before, much **larger** in the superfluid region
- No **critical slowing down** at the transition

Summary and outlook

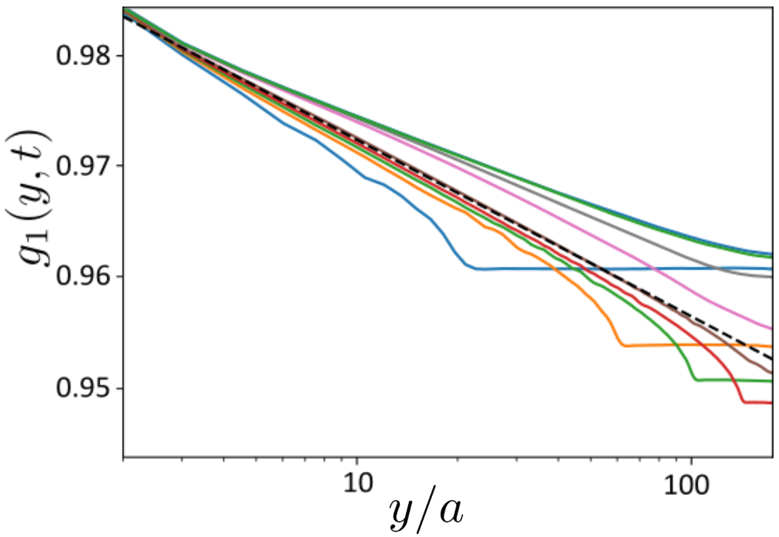
- Characterized numerically the **out-of-eq. emergence of the KT transition**, simulating a quench protocol:

Disorder > Interactions



Fast thermalization to a **normal fluid thermal state** : Rayleigh-Jeans

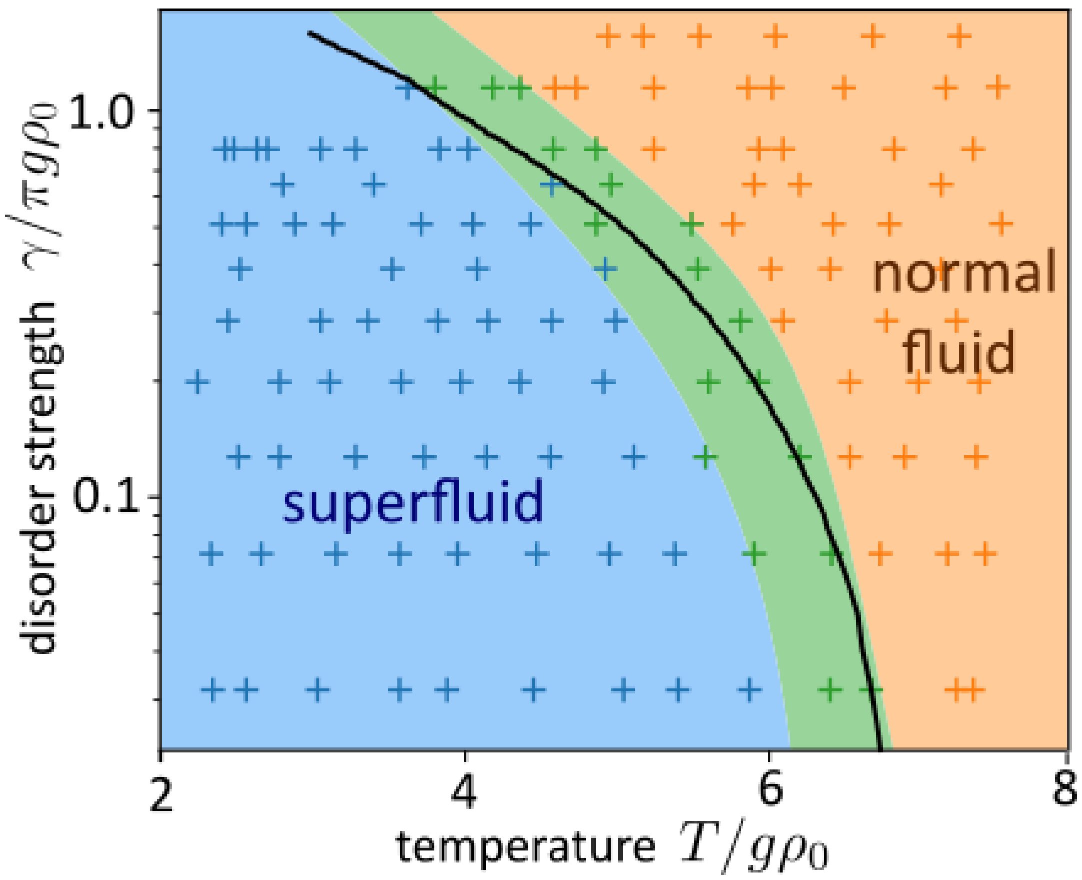
Interactions > Disorder



Slow dynamics: **prethermal regime** displays the emergence of a **superfluid** fraction

Summary and outlook

- At equilibrium: build the **phase diagram of the KT-transition** in the presence of **disorder**



Critical disorder/Temperature in agreement with theory : $\gamma^* = \frac{4\pi}{mI_1}\left[1 - \frac{T_{\mathrm{KT}}}{T_d}(4 - I_2)\right]$

Summary and outlook

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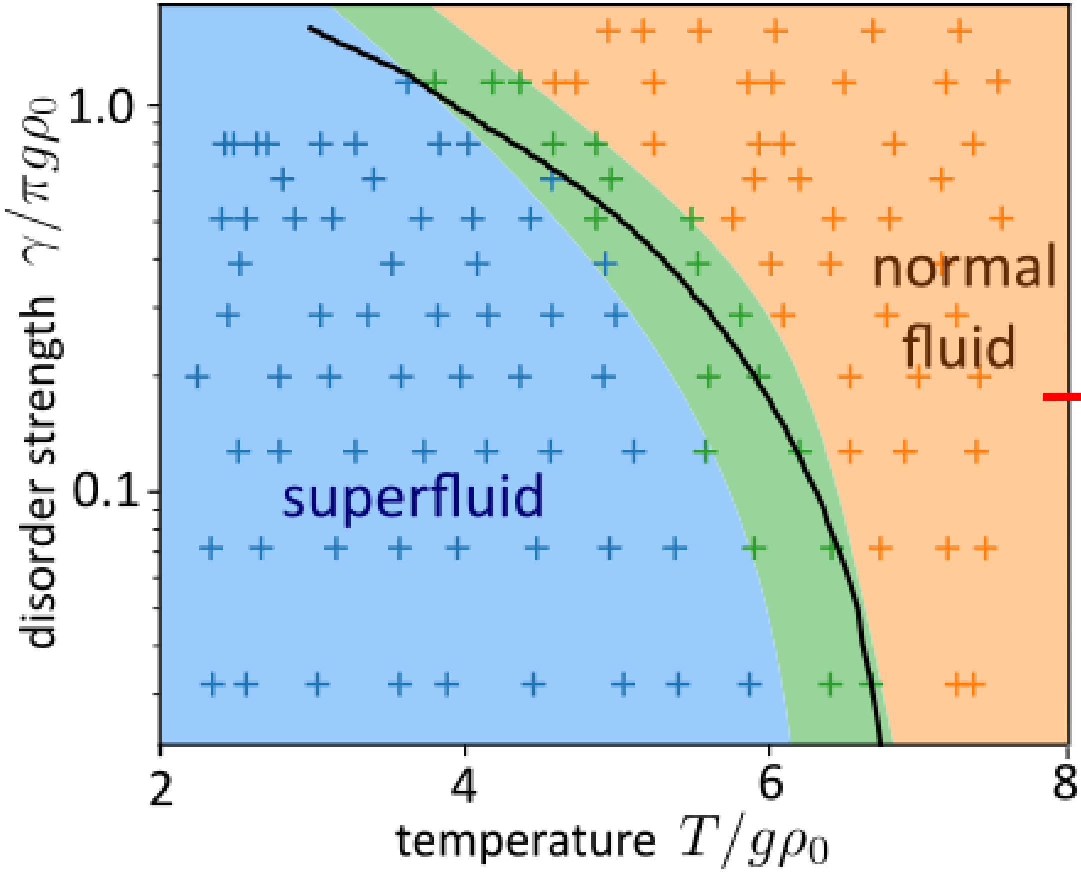

Width of the critical region = f(t) ?

Thermalization times τ_{th}?

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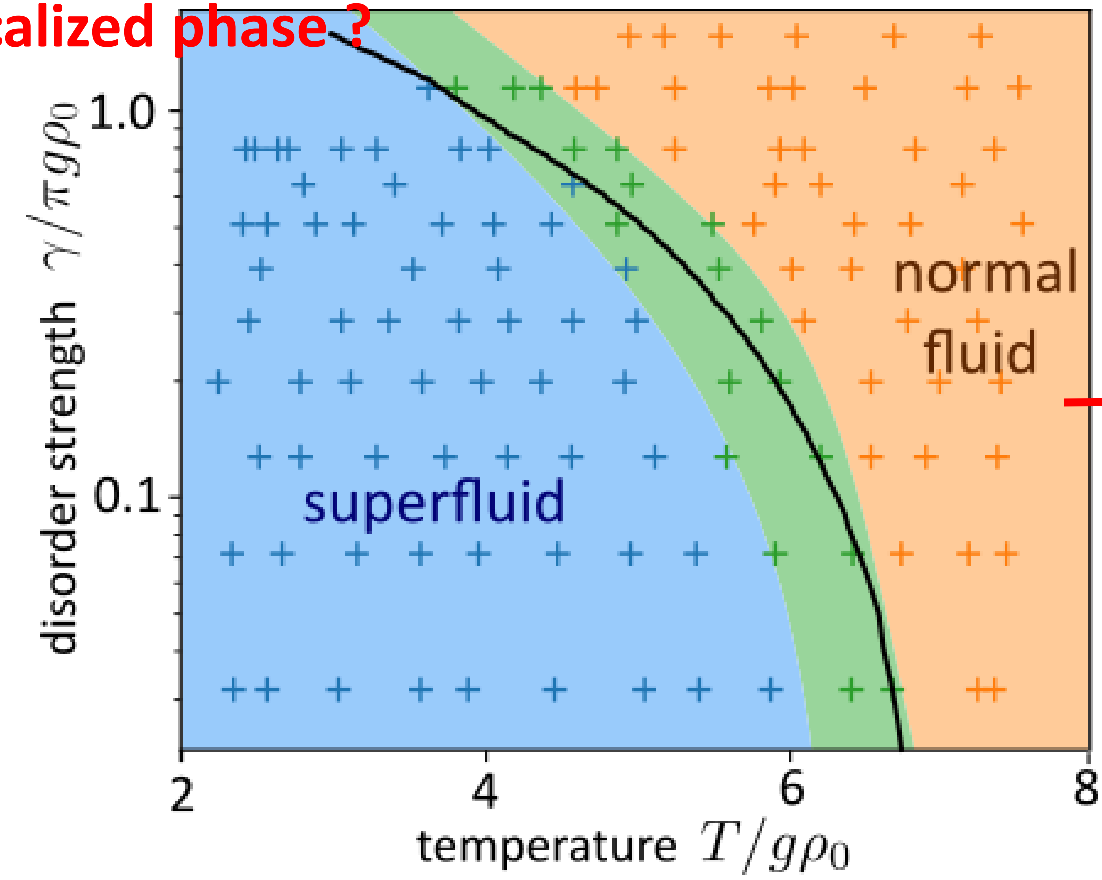
Summary and outlook

- At equilibrium: build the **phase diagram of the KT-transition** in the presence of **disorder**

Localized phase ?



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Thank you for listening !

Based on :

T. Scoquart, D. Delande, and N. Cherroret, Phys. Rev. A **106**, L021301 (2022).
T. Scoquart, P-É. Larré, D. Delande and N. Cherroret, *EPL* **132** 66001 (2020).

Characterizing the equilibrium

- For a given initial state $(\boldsymbol{k}_0, \gamma, g)$, the **equilibrium temperature** and **chemical potential** should follow from:

$$E = \frac{k_0^2}{2m} + g\rho_0 = \int_{\mu}^{+\infty} \mathrm{d}\epsilon\, \nu_\epsilon \epsilon \frac{T}{\epsilon - \mu} \quad \text{Conservation of energy}$$

$$N = \int_{\mu}^{+\infty} \mathrm{d}\epsilon\, \nu_\epsilon \frac{T}{\epsilon - \mu} \quad \text{Conservation of particle number}$$

DOS ?

BUT :

- Divergence at $\epsilon \to +\infty$: Ultraviolet catastrophe ? No issue here bc **discrete space** simulations
- Divergence at $\epsilon \to \mu$: Rayleigh-Jeans does not describe well the **low energy limit**:

Because of **disorder** (Lifshitz tail), and also if there is **condensation.**

→ We determine $T(\boldsymbol{k}_0, g, \gamma)$ and $\mu(\boldsymbol{k}_0, g, \gamma)$ numerically