Institut Langevin
ONDES ET IMAGES

Learning from the disorder in multimode fibers

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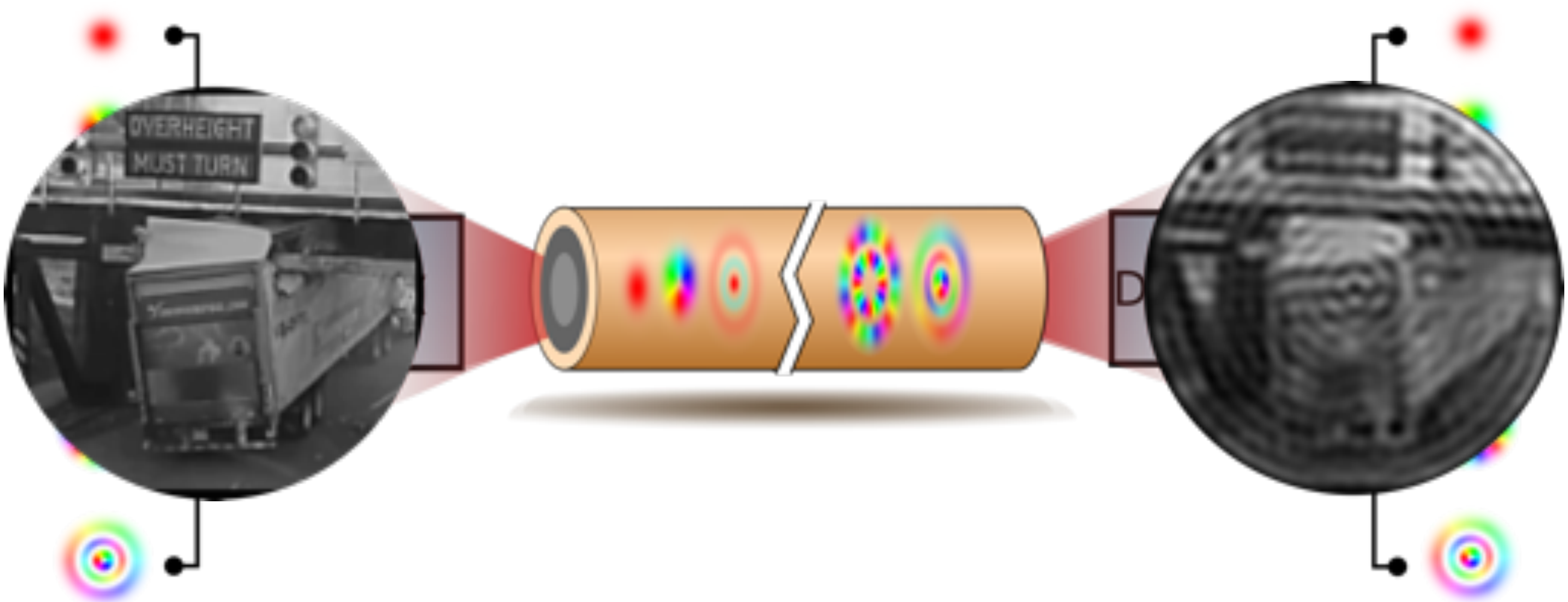
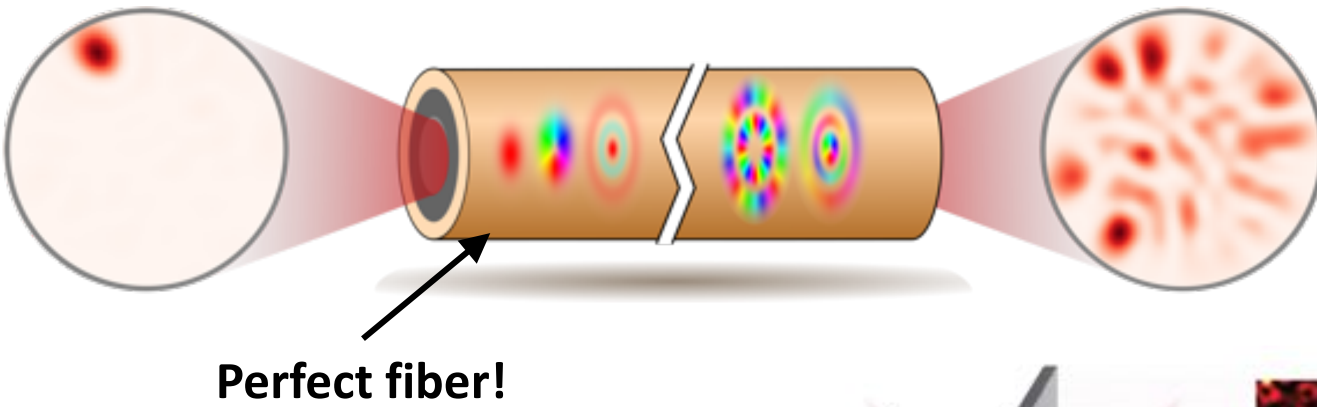
Multimode fibers

- Generally simple and symmetric systems so **no disorder!**
- **Finite** number of guided modes can be computed
- **Separate channels** encoded in the spatial degree of freedom
- Increase the number of channels for **telecommunication**
- Minimally invasive endoscopic **imaging**

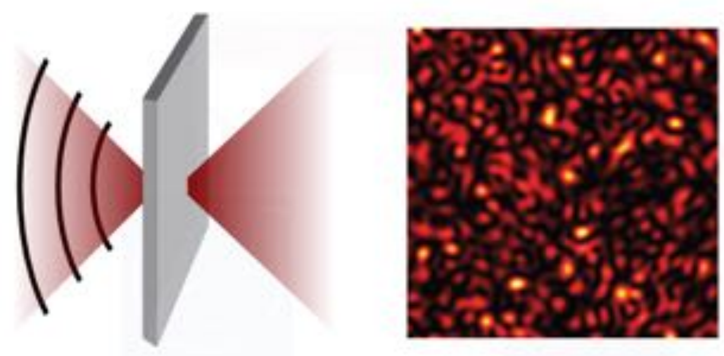


Multimode fibers VS scattering media

- **But** in reality out comes a speckle!



- Similar to scattering media



- Origin of speckle is different:
  1. Dispersion
  2. Mode coupling

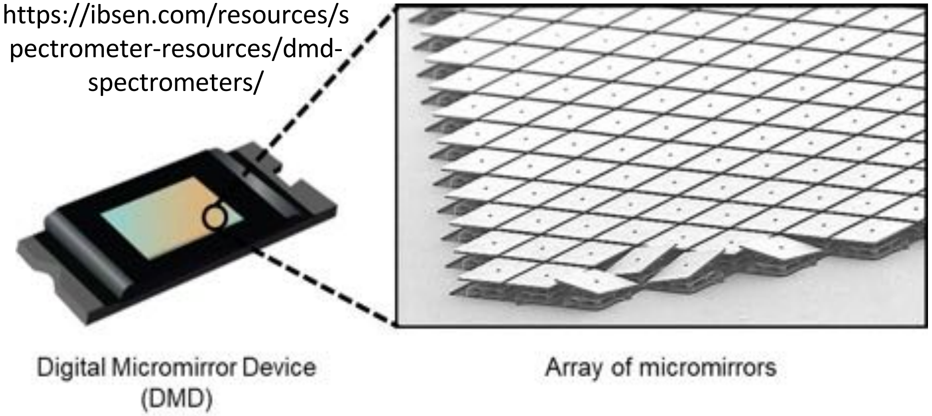
Tools of the trade

- Use same techniques
  e.g. the transmission matrix

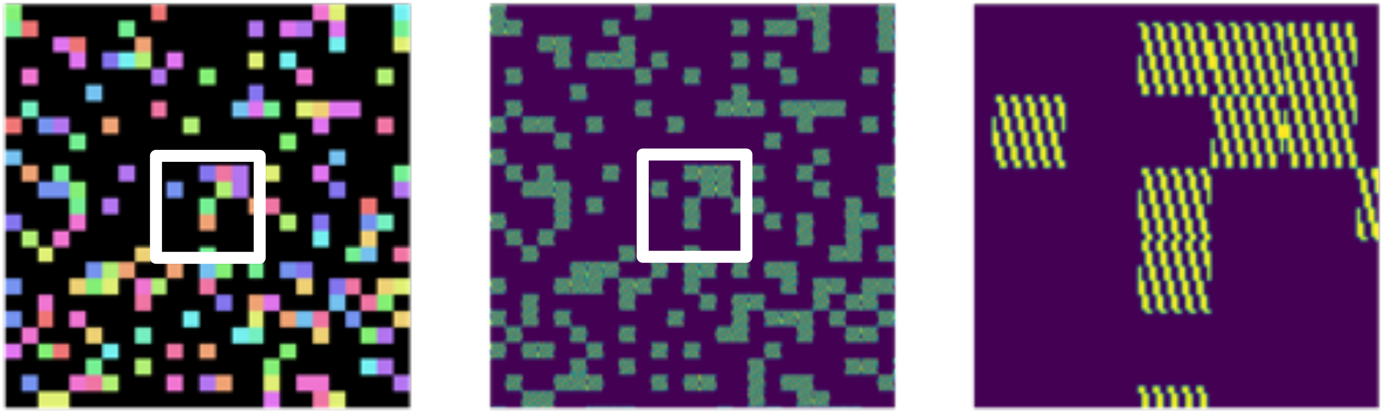
$$\mathbf{E}_{\mathrm{out}} = \mathbf{H} \cdot \mathbf{E}_{\mathrm{in}}$$

- Use **SLM** to shape input light
- Our tool of choice:
  Micromirror device (DMD)
  ➤ Cheap and fast
  ➤ Binary amplitude modulation

https://ibsen.com/resources/spectrometer-resources/dmd-spectrometers/

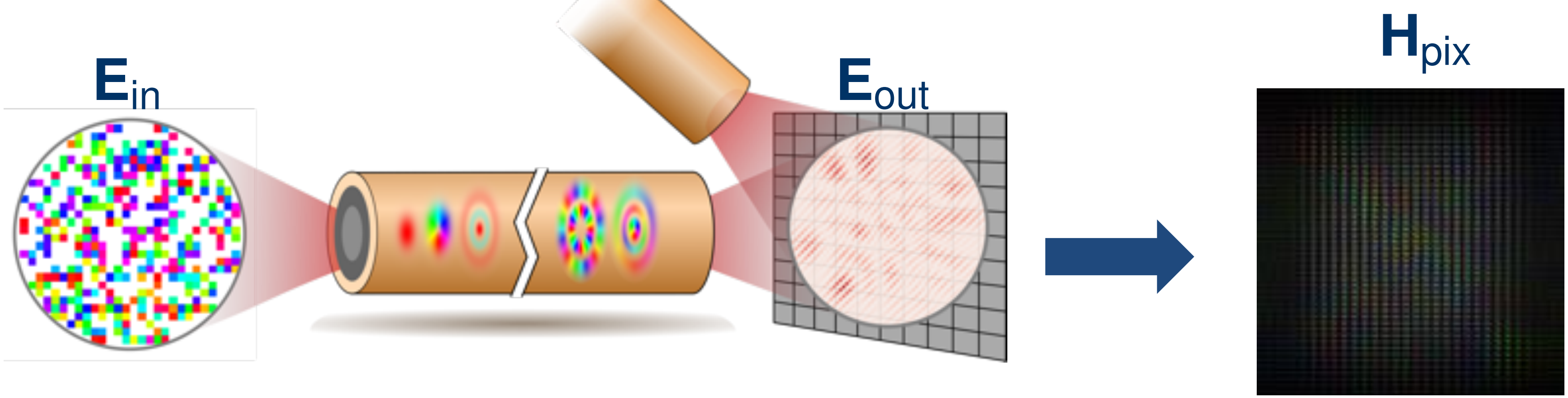


- **Lee holograms** to shape the input field in phase and amplitude

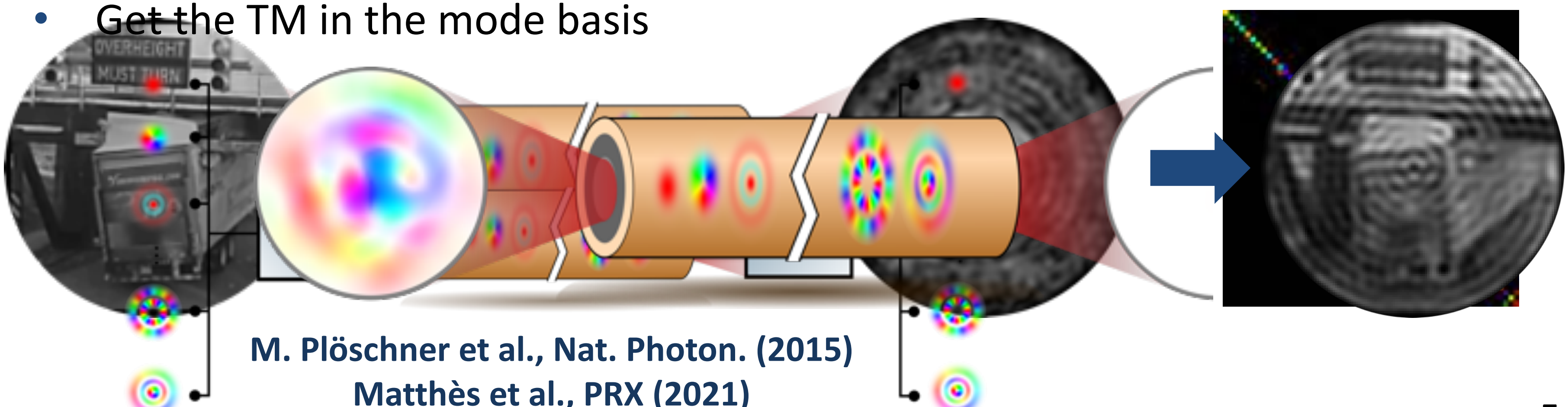


TM in pixel basis

- Send random inputs and measure outputs with off-axis holography



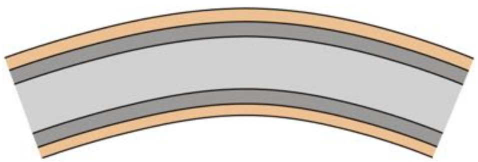
- Focus at the output
- Direct imaging
- Get the TM in the mode basis

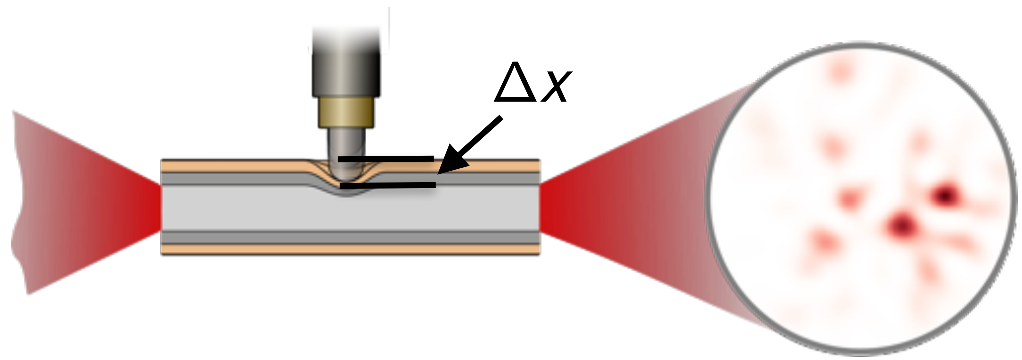


M. Plöschner et al., Nat. Photon. (2015)
Matthès et al., PRX (2021)

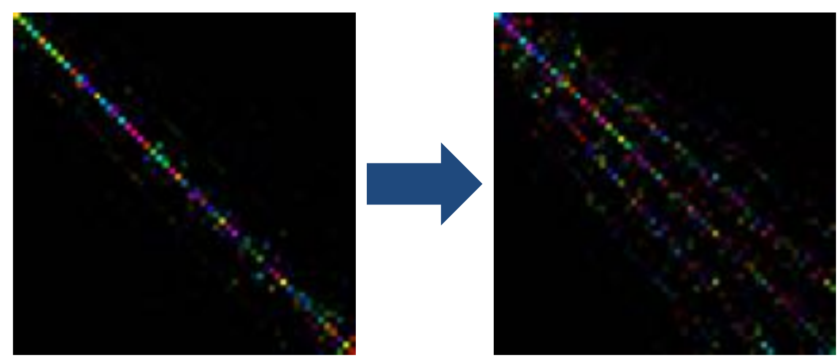
Addition of perturbations

- They can be curved and deformed during use





- Changes in the output
- Introduction of coupling
  - ➢ no independent channels
- **What can we do?**



To crash or no to crash?

Perturbation

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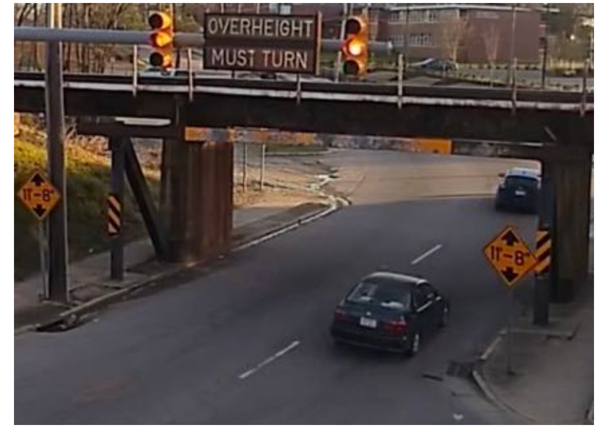
Outline

1. Not to crash
   - ➢ not sensitive to perturbation
   - ➢ ideal for imaging, telecom

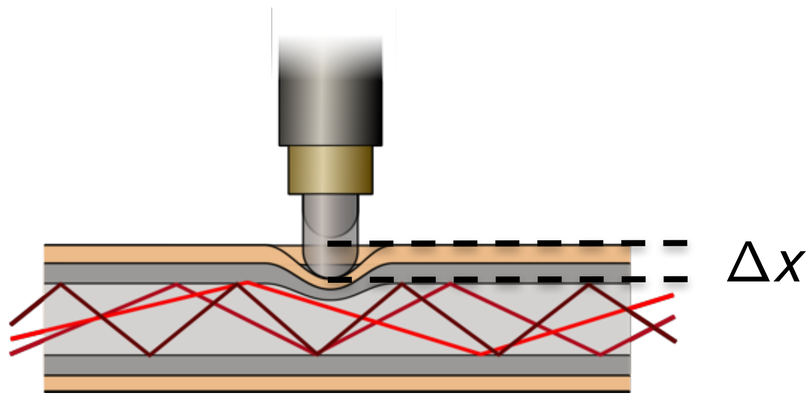


2. To crash
   - ➢ very sensitive to perturbation
   - ➢ good for sensing

3. Develop best strategies for estimating the perturbation

Not to crash (sensible answer)

- Analogous ray picture



- Solution: Generalized Wigner-Smith operator

$$\mathbf{W} = -i\mathbf{H}^{-1}\partial_{\Delta x}\mathbf{H}$$

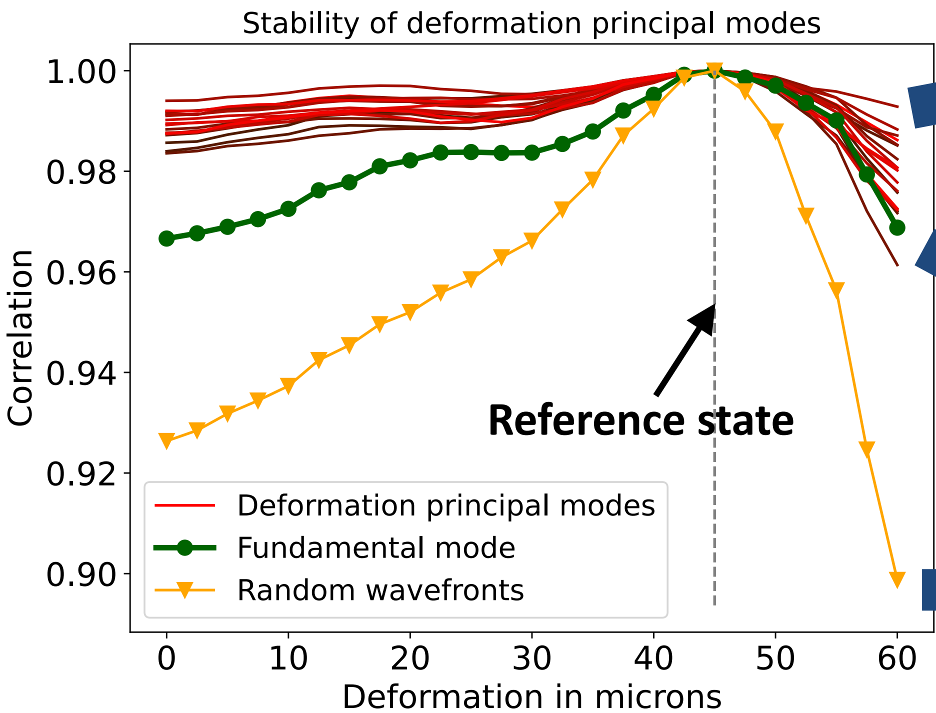
P. Ambichl al., PRL, 119 (2017)
M. Horodynski al., Nat. Photon., 14 (2020)

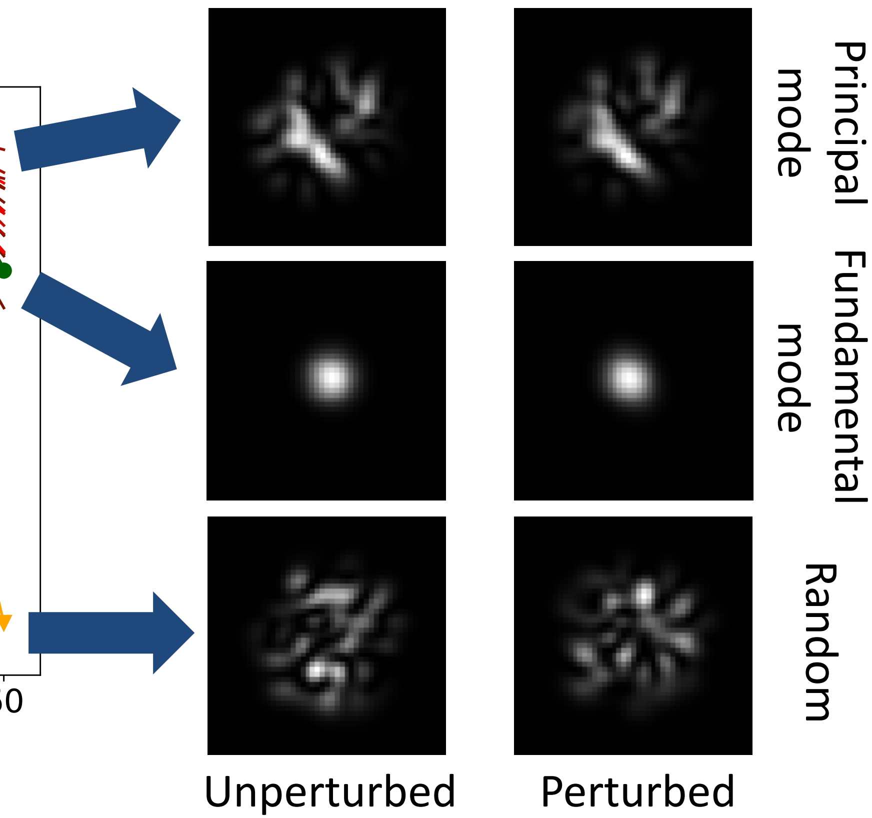
Eigenmodes are insensitive to first-order variations of Δx

Generalized principal modes (not to crash)

1. Random input
2. Fundamental mode
3. Generalized principal mode

Get basis insensitive to disorder!

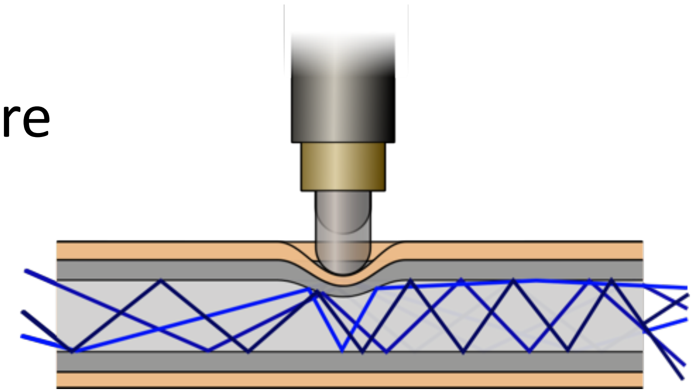




Matthès et al., PRX (2021)

Crashing modes (to crash)

- Again ray picture



- Idea: define **crashing modes** as those that maximize the change in intensity (**information** about perturbation)

$$\mathcal{J} = \sum_n (\partial_{\Delta x} |E_{\mathrm{out},n}|^2)^2$$

equivalent to minimizing intensity correlation

- Can be solved by brute force **but is there another way?**

Anti-principal modes (to crash)

- Working out the math…

$$\mathcal{J} = \sum_{n,m,m'} \partial_{\Delta x}(H_{nm}H^*_{nm'}) \left(\sum_{l,l'} \partial_{\Delta x}(H_{nl}H^*_{nl'}) E_{\text{in},l} E^*_{\text{in},l'} \right) E_{\text{in},m} E^*_{\text{in},m'}$$

Third-order tensor

vector

- Rank-one tensor approximation maximizes

$$\mathcal{J} = \langle \mathbb{W}^{(3)}, \mathbf{u} \otimes \mathbf{E}_{\text{in}} \otimes \mathbf{E}^*_{\text{in}} \rangle$$

- For matrices the solution is given by the singular value decomposition

$$\mathbf{H} = \sum_i \sigma_i \mathbf{u}_i \otimes \mathbf{v}^*_i = \mathbf{U\Sigma V}^\dagger$$

diagonal

Reminder

$$(\mathbf{u} \otimes \mathbf{v})_{ij} = u_i v_j$$

- **For tensor a bit more complicated**

Higher-order SVD

- A tensor of order L can be decomposed as the outer product of orthonormal singular vectors

$$\mathbb{T} = \sum_{i_1=1}^{I_1} \cdots \sum_{i_L=1}^{I_L} s_{i_1 \ldots i_L} \mathbf{u}_{i_1}^{(1)} \otimes \cdots \otimes \mathbf{u}_{i_L}^{(L)}$$

- S_{ijk} are the entries of the core tensor which satisfies the properties of

1. All orthogonality $\langle \mathbb{S}_{i_n=\alpha}, \mathbb{S}_{i_n=\beta} \rangle = 0 \qquad \alpha \neq \beta$
2. Ordering $\|\mathbb{S}_{i_n=1}\| \geq \|\mathbb{S}_{i_n=2}\| \geq \ldots$

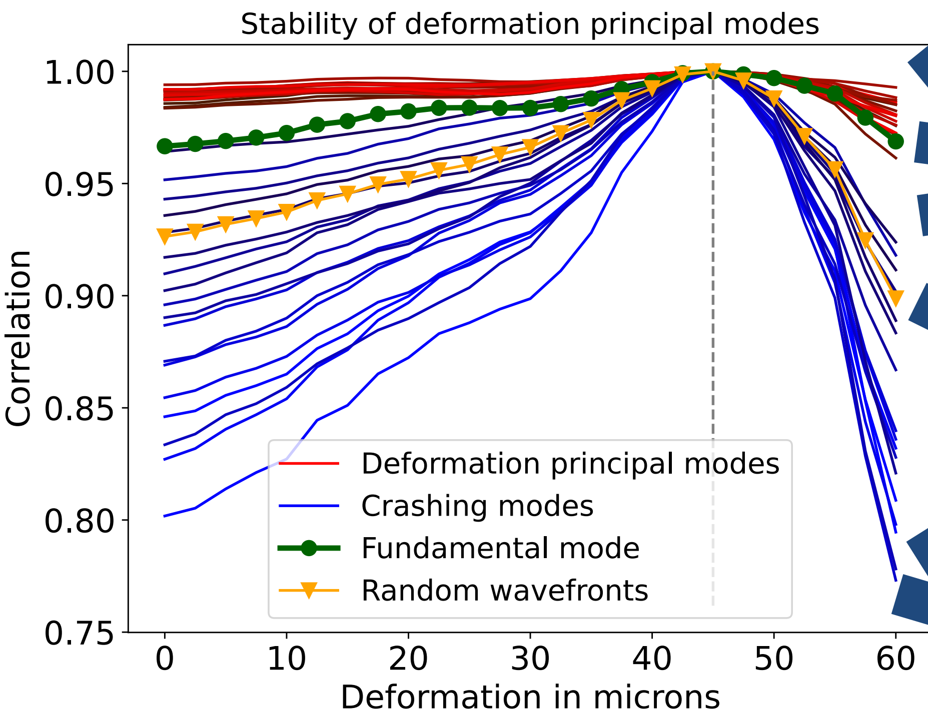
- Truncating it does **not** give the best lower-rank approximation **but** a really good guess

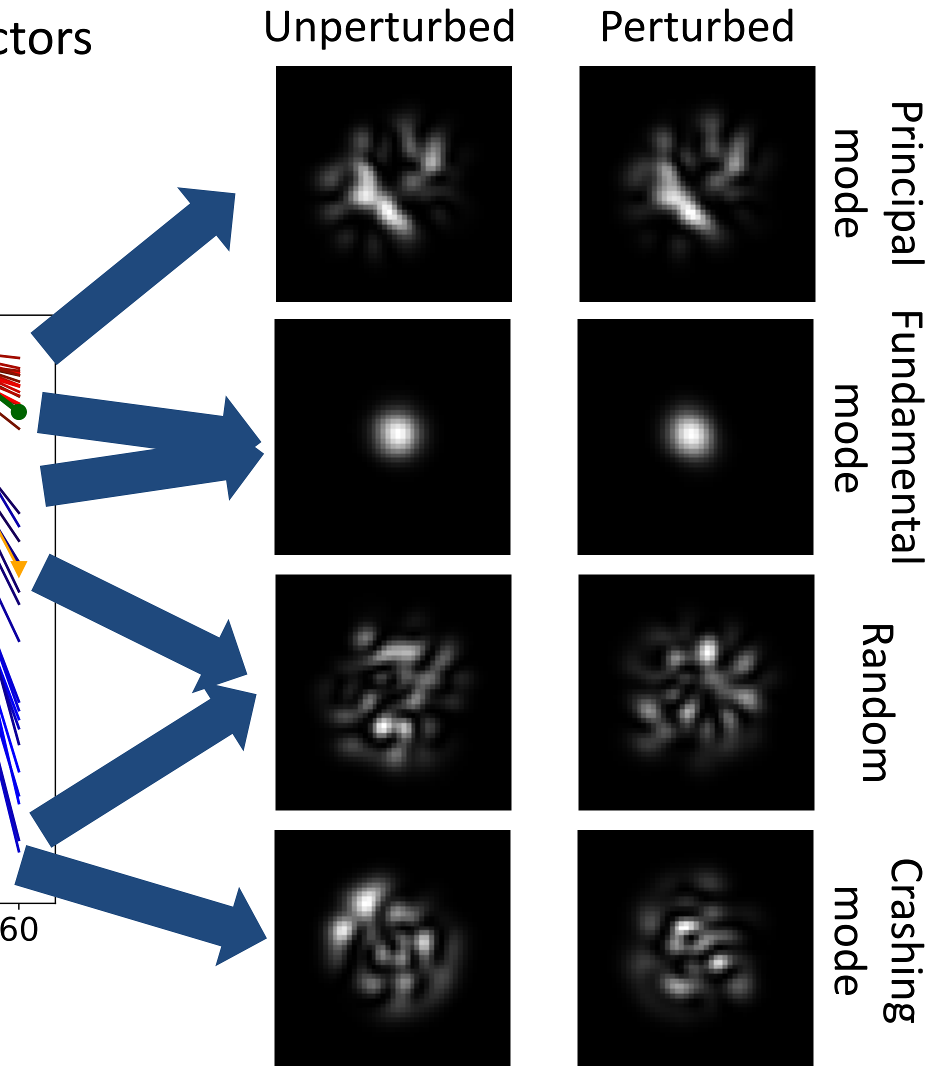
Tucker, Psychometrika (1996)
Kolda and Bader, SIAM Review (2009)

Crashing modes

- Crashing modes as HO singular vectors

$$\mathbb{W}^{(3)} = \sum_{i,j,k} s_{ijk} \mathbf{u}_i \otimes \mathbf{v}_j^* \otimes \mathbf{v}_j$$





Sensing: Estimating the height of the bridge

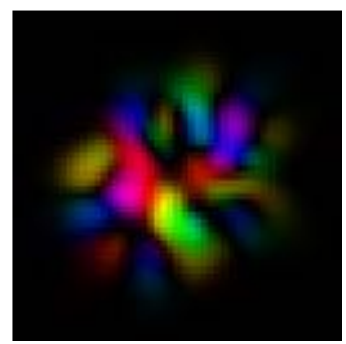
- **Best** state to estimate the deformation?
- With bridge and ray analogies answer seems obvious… **but** it depends on what we measure

Measure complex field:

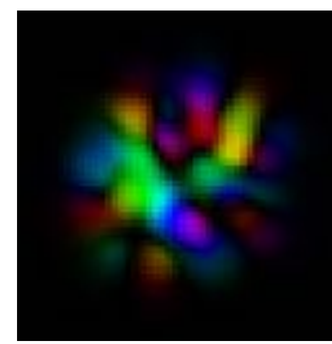
- Optimal states are the generalized principal modes, *Bouchet et al., Nat. Physics (2021)*
- Even the principal modes are affected by the disorder

$$\mathbf{E}_{\mathrm{PM}}(x + \Delta x) \approx \mathbf{E}_{\mathrm{PM}}(x) e^{\mathrm{i}\phi(\Delta x)}$$

Unperturbed



Perturbed



Why crash with disorder?

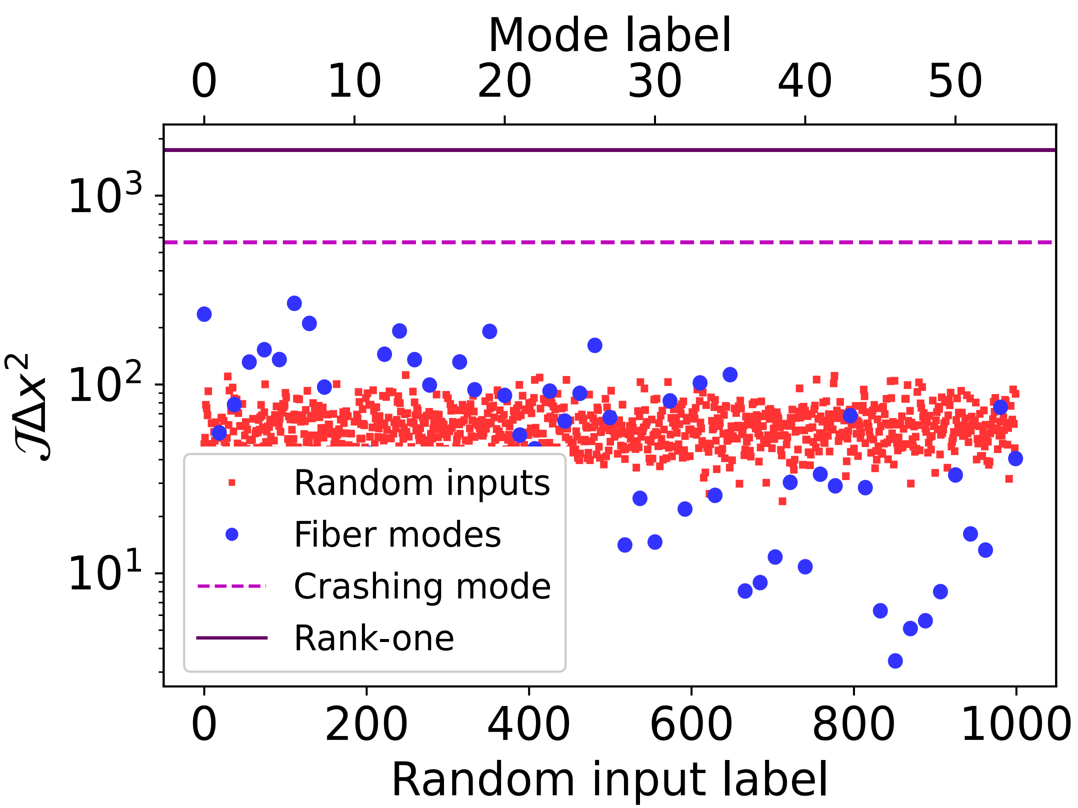
- To measure field we need interferometry:
  - Complicates the setup
  - Requires stability
- **Alternative:** measure intensity
- **Need to crash with disorder!**

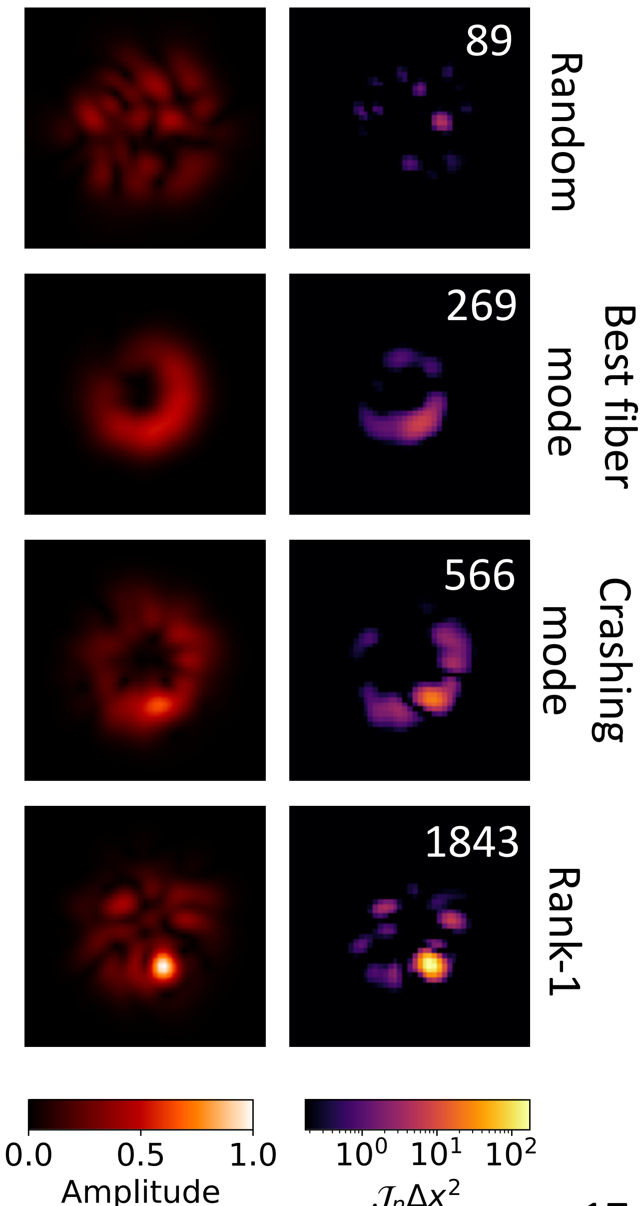
- Use crashing modes as a starting guess to find optimal field

$$\mathcal{J} = \langle \mathbb{W}^{(3)}, \mathbf{u} \otimes \mathbf{E}_{\text{in}} \otimes \mathbf{E}_{\text{in}}^{*} \rangle$$

Optimal mode for intensity

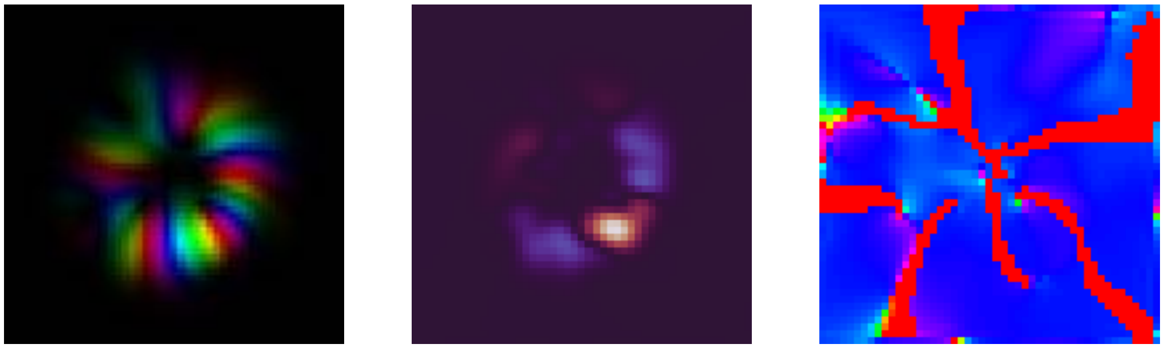
- The crashing mode outperforms random inputs and all the modes of the fiber but is not the best





What about the output projection?

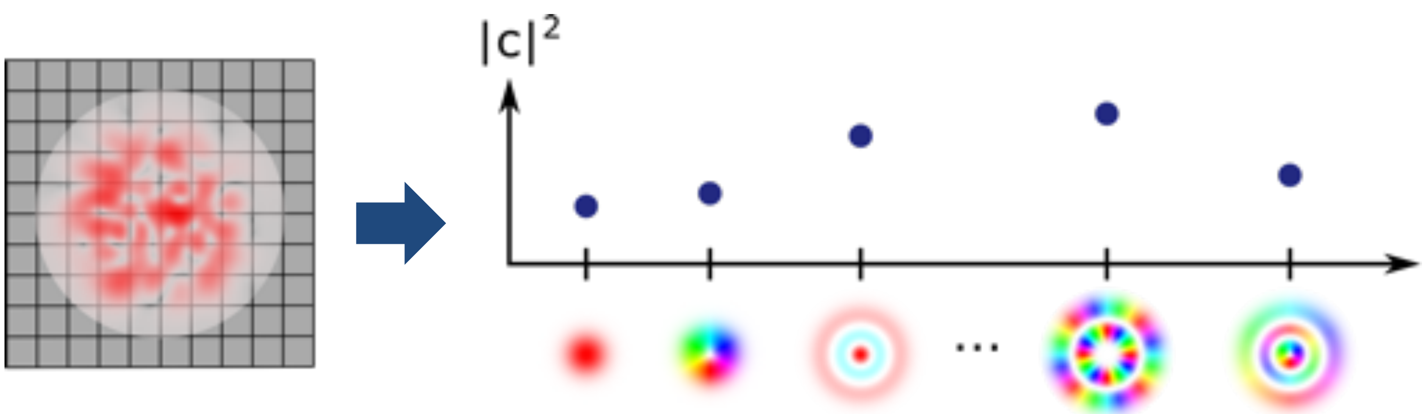
- We did a projection in the pixel basis
- Get change in intensity but loose relative phase between pixels



$$\Delta\phi = \phi_0 + \phi_n$$

- Constant global phase gone
- Change the output projection $\mathbf{E}_P = \mathbf{P}^\dagger \mathbf{E}_{\mathrm{out}}$
- Same information in less channels then **SNR increases**

Fontaine et al., Opt. Express (2012)
Labroille et al., Opt. Express (2014)
Tsang et al., PRX (2016)



Optimal output projection

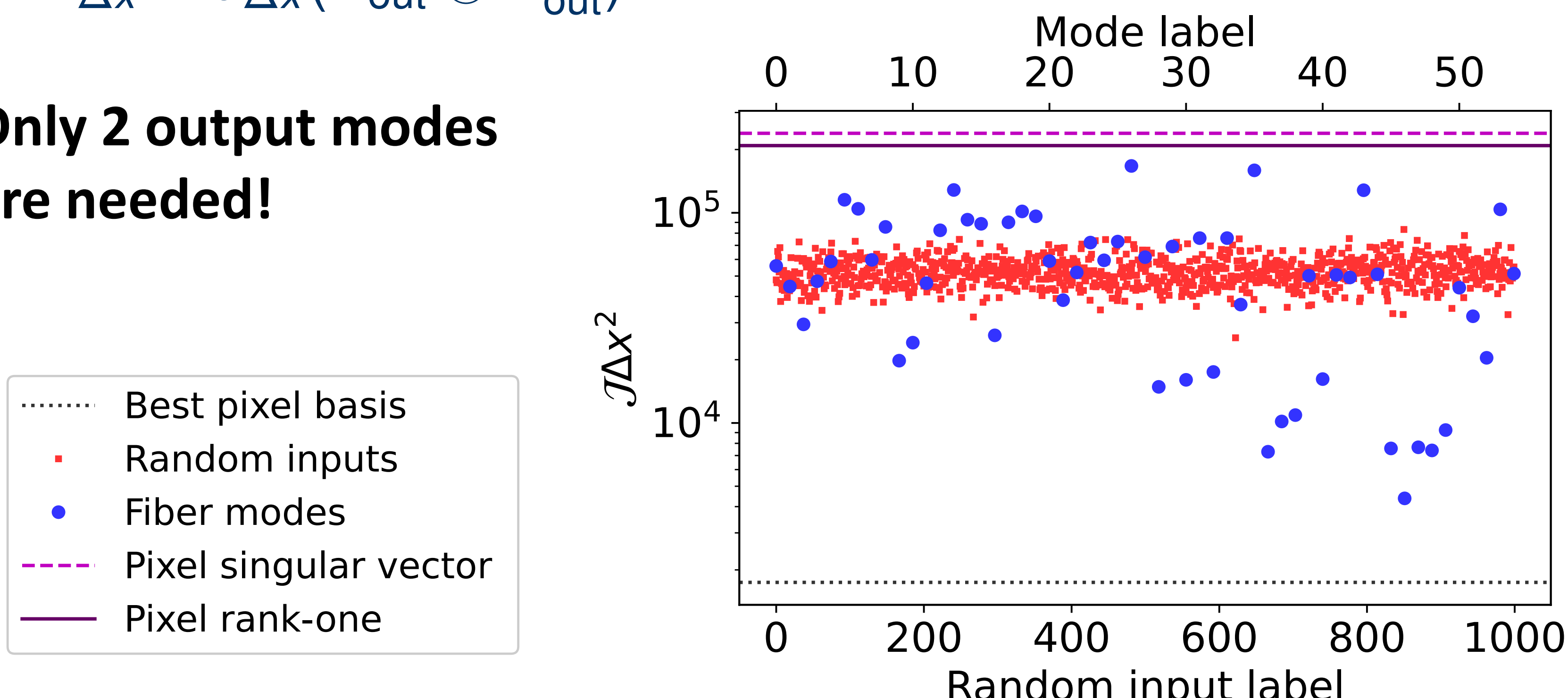
The outmodes $\mathbf{p}_q$ correspond to the columns of **P**

$$\mathcal{J} = \sum_{q=1}^{Q} \left[\sum_{n,n'} p^*_{nq} \partial_{\Delta x} (E_{\text{out},n} E^*_{\text{out},n'}) p_{n'q} \right]^2 = \sum_{q=1}^{Q} \langle \mathbf{p}_q, \mathbf{Y}_{\Delta x} \cdot \mathbf{p}_q \rangle^2$$

$$\mathbf{Y}_{\Delta x} = \partial_{\Delta x} (\mathbf{E}_{\text{out}} \otimes \mathbf{E}^*_{\text{out}})$$

Rank-2 matrix

Only 2 output modes are needed!

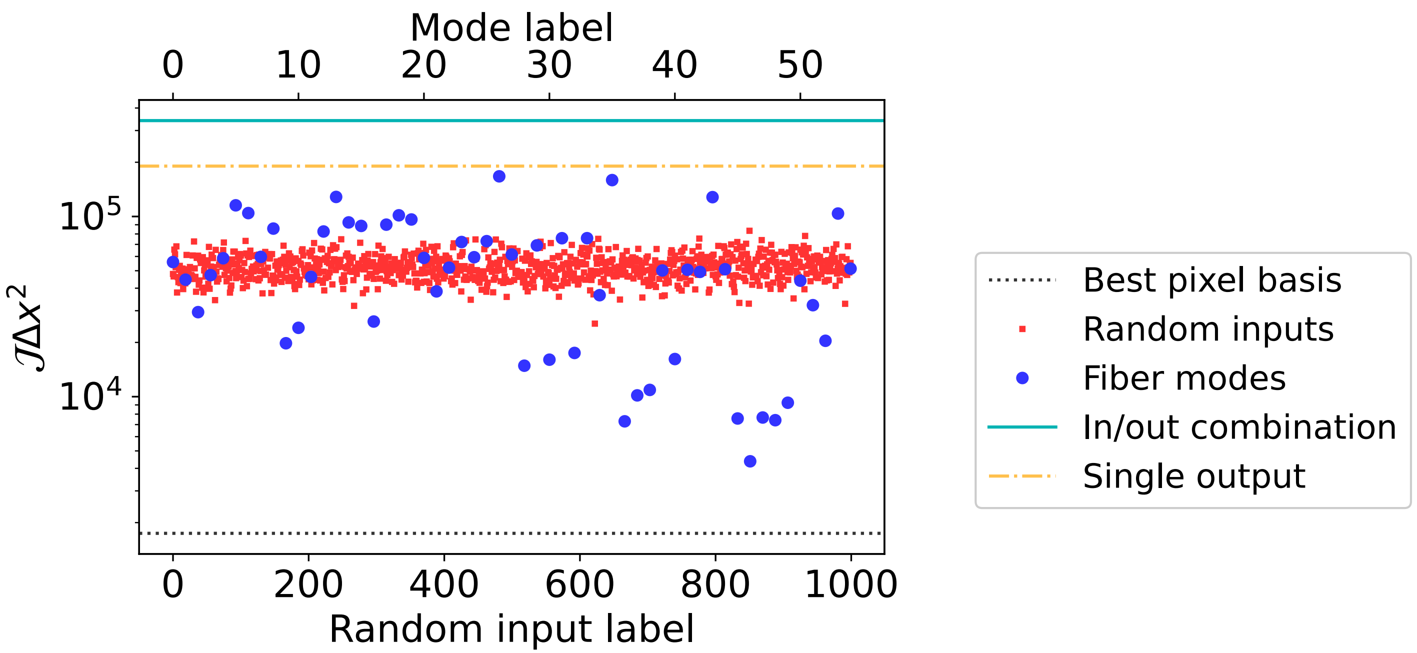


Best input-output combination

- Write in terms of input fields and **H**
- Inner products of rank 4 tensors!

$$\mathcal{J} = \sum_{q=1}^{2} \langle \mathbb{W}^{(4)}, \mathbf{p}_q^* \otimes \mathbf{p}_q \otimes \mathbf{E}_{\text{in}} \otimes \mathbf{E}_{\text{in}}^* \rangle^2 \qquad \mathbb{W}_{ijkl}^{(4)} = \partial_\theta (H_{ik}^* H_{jl})$$

- For one output mode then rank-one approximation

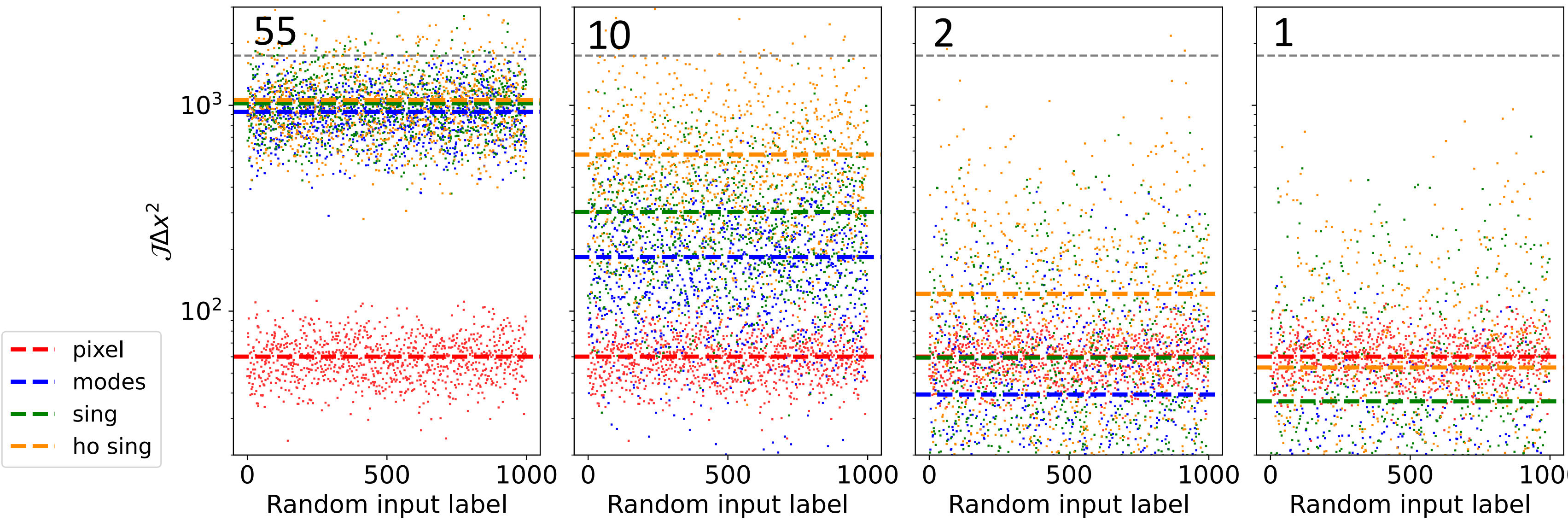


Best generic output modes

- Assume we do not know in advance the input field
- Need to choose output basis that works **best for most fields**
- Candidates:
  1. Pixels
  2. Fiber modes
  3. Singular output modes
  4. HO singular output modes

$$\mathbf{H} = \sum_i \sigma_i \mathbf{u}_i \otimes \mathbf{v}_i^*$$

$$\mathbb{W}^{(4)} = \sum_{i,j,k,l} s_{ijkl} \mathbf{u}_i \otimes \mathbf{u}_j^* \otimes \mathbf{v}_k^* \otimes \mathbf{v}_l$$



Conclusions

- Avoid disorder (imaging , telecom):
  - Generalized principal modes
  - Optimal for estimating
  - **But** information in the phase
- Crash with disorder (sensing):
  - Crashing modes
  - Given by the HOSVD
  - Good for intensity-based measurements
  - Importance of output projection

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