

Analogue gravity with waves in nonlinear media

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Quantum Optics Group

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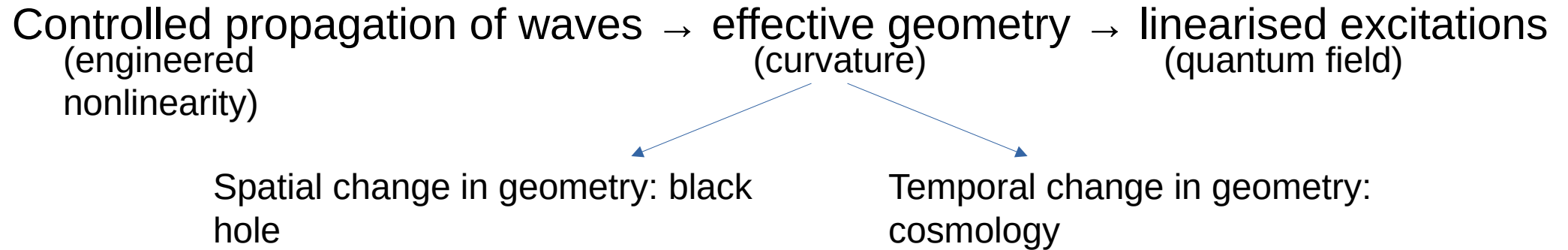
GDR Complexe 8/12/2022

The propagation of waves in nonlinear media may be controlled to engineer situations where the waves propagate as though they were on a curved spacetime, like around a black hole or in an inflating universe. This enables the experimental simulation of field theories on curved spacetime.

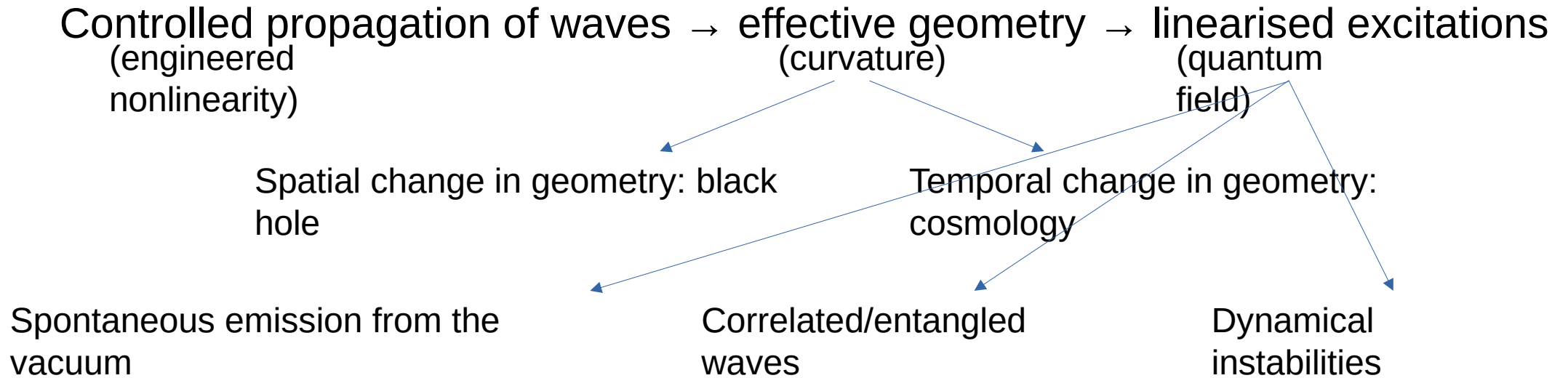
The **propagation of waves in nonlinear media** may be controlled to engineer situations where the waves propagate as though they were on an **effectively curved geometry**, like around a black hole or in an inflating universe. This enables the **experimental study of field theories** on curved geometries.

Controlled propagation of waves
(engineered nonlinearity) → effective geometry
(curvature) → linearised excitations
(quantum field)

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How to create the spacetime of a Schwarzschild black hole in the laboratory?
curvature

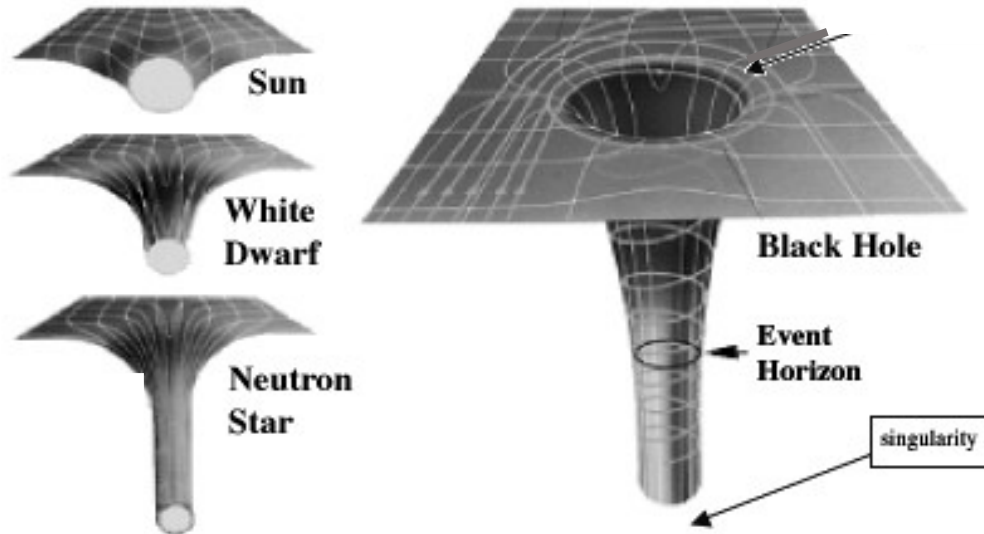
How to observe the Hawking effect in the laboratory?
quantum field

Our experiment with a quantum fluid of microcavity polaritons
engineered nonlinearity correlated waves

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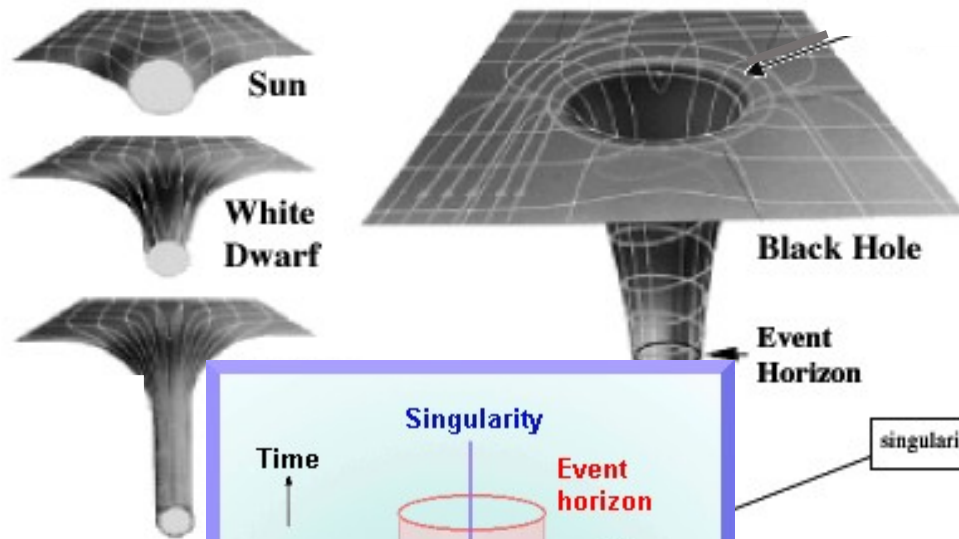
Nasa website

Penrose, 1972

General Relativity identifies gravity with curvature of spacetime

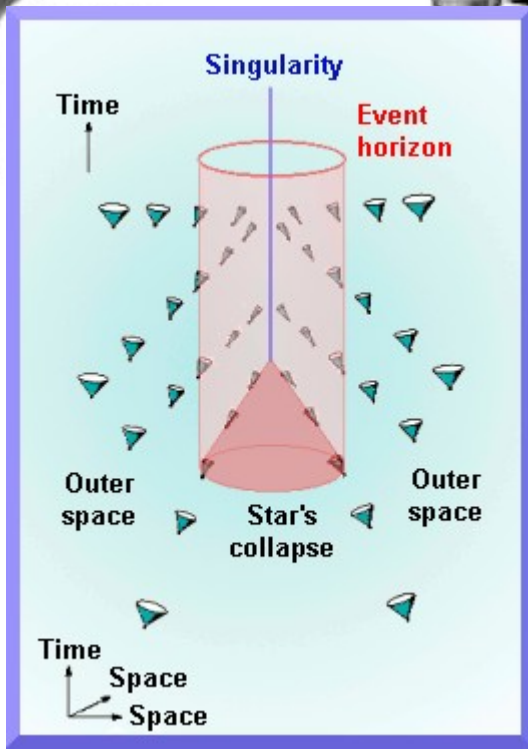
Point of no return: event horizon

General Relativity: nothing can escape a black hole



Nasa website

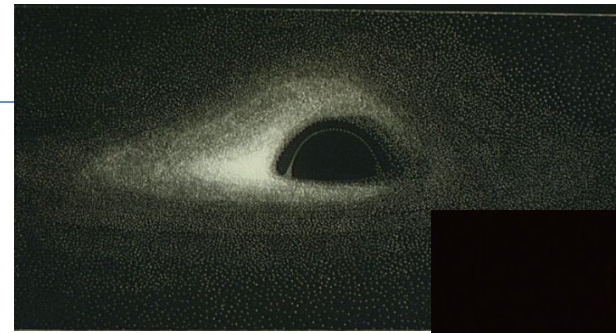
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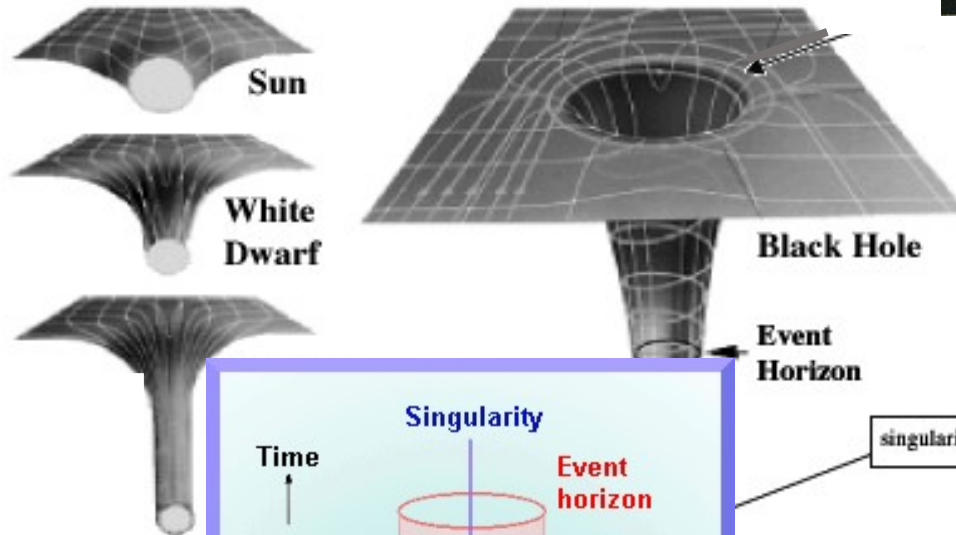
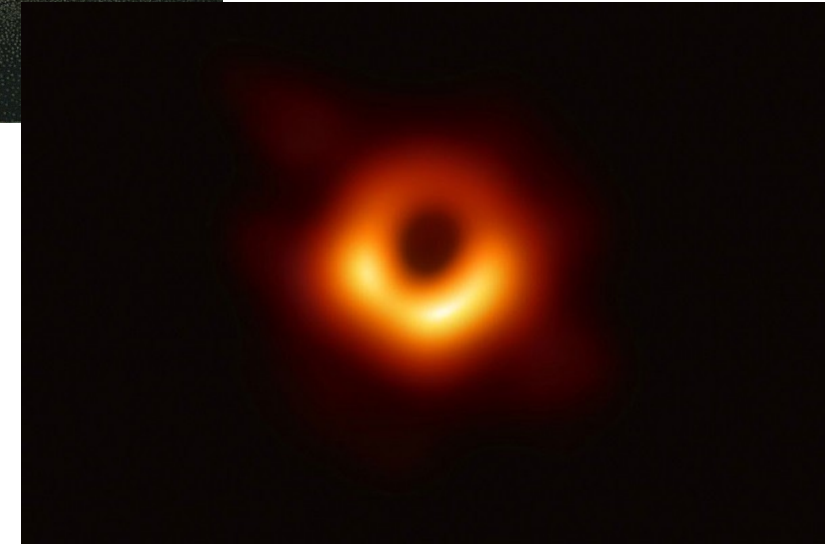
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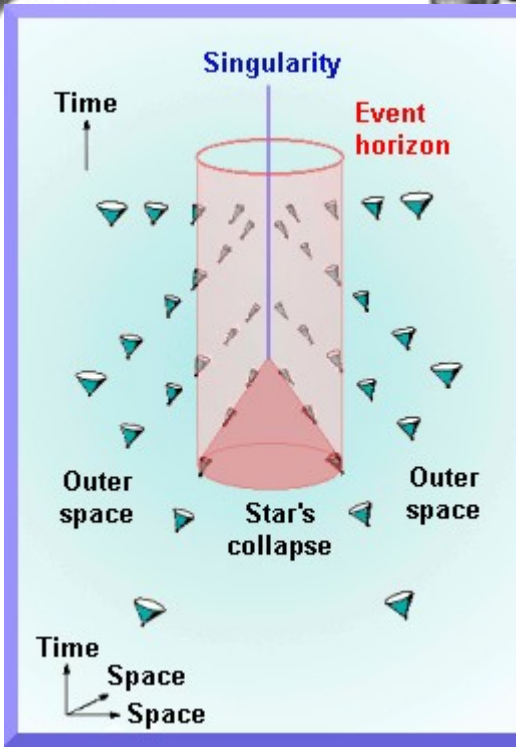
Luminet, 1979

EHT, 2019



Nasa website

Penrose, 1972



singularity

General Relativity identifies gravity with curvature of spacetime

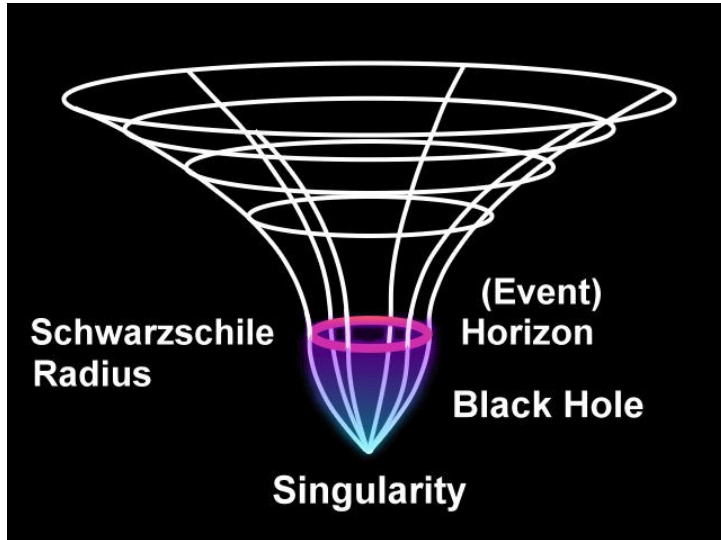
Point of no return: event horizon

General Relativity: nothing can escape a black hole

Black holes are characterised by their mass, their momentum and their charge

Non-rotating black hole : Schwarzschild

Schwarzschild black holes are characterised solely by their mass → Schwarzschild black hole = 4-sphere



<https://universe-review.ca/F05-galaxy02.htm>

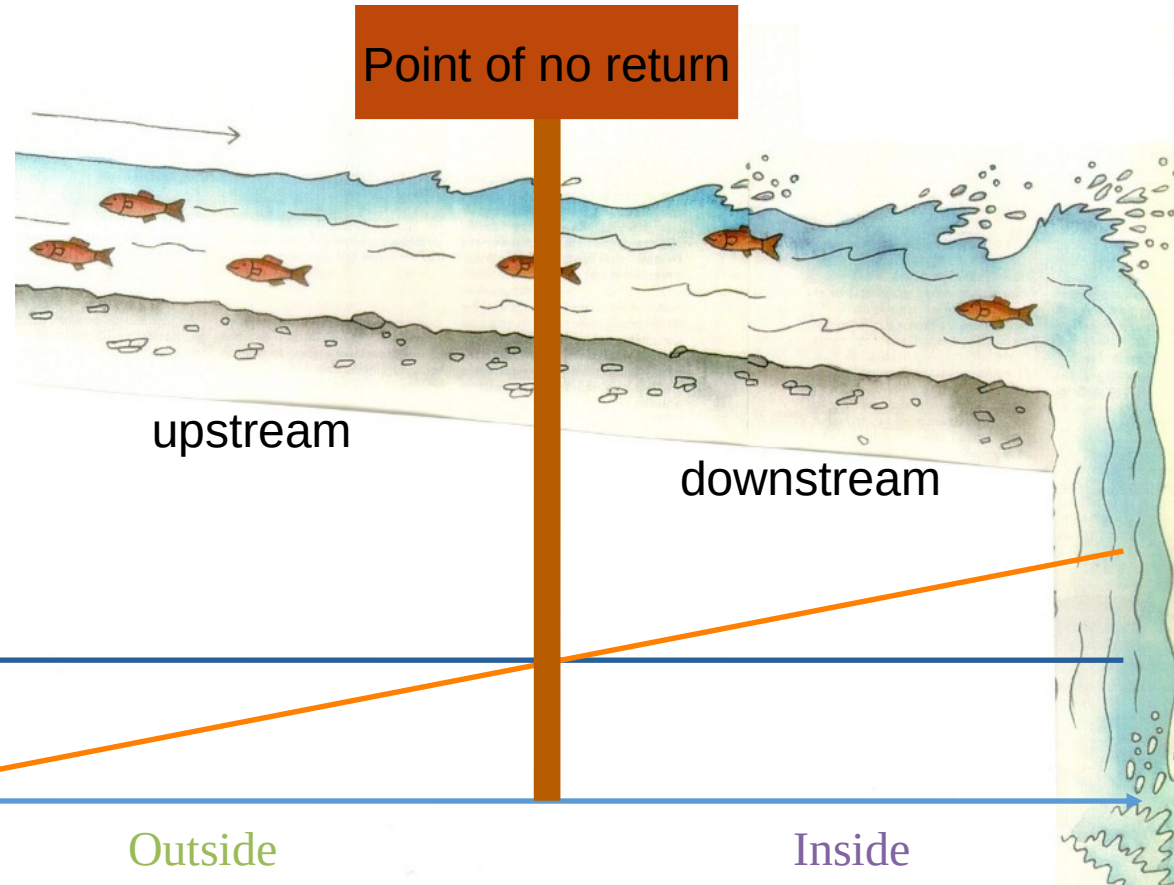
Event horizon = point of no return

Rotational spatial
symmetry → full description in
1+1D



Non-rotating black hole : Schwarzschild

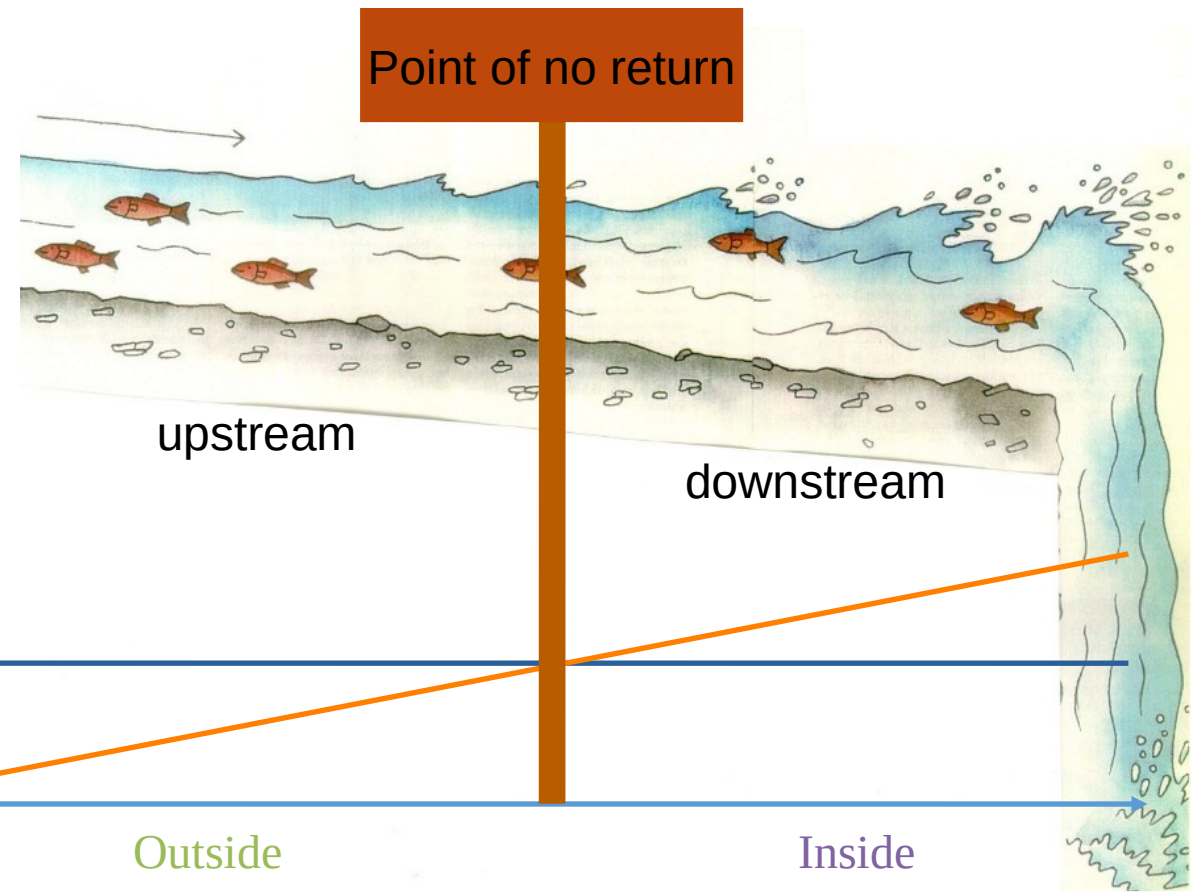
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Rotational spatial symmetry
 → full description in 1+1D



Schwarzschild geometry ↔ waterfall geometry



Wave equation for scalar field on Schwarzschild geometry:

$$g_{schw}^{\mu\nu} = \begin{pmatrix} -1 & -v \\ -v & (c^2 - v^2) \end{pmatrix}$$

Inverse metric tensor of Painlevé-Gullstrand metric in 1+1D

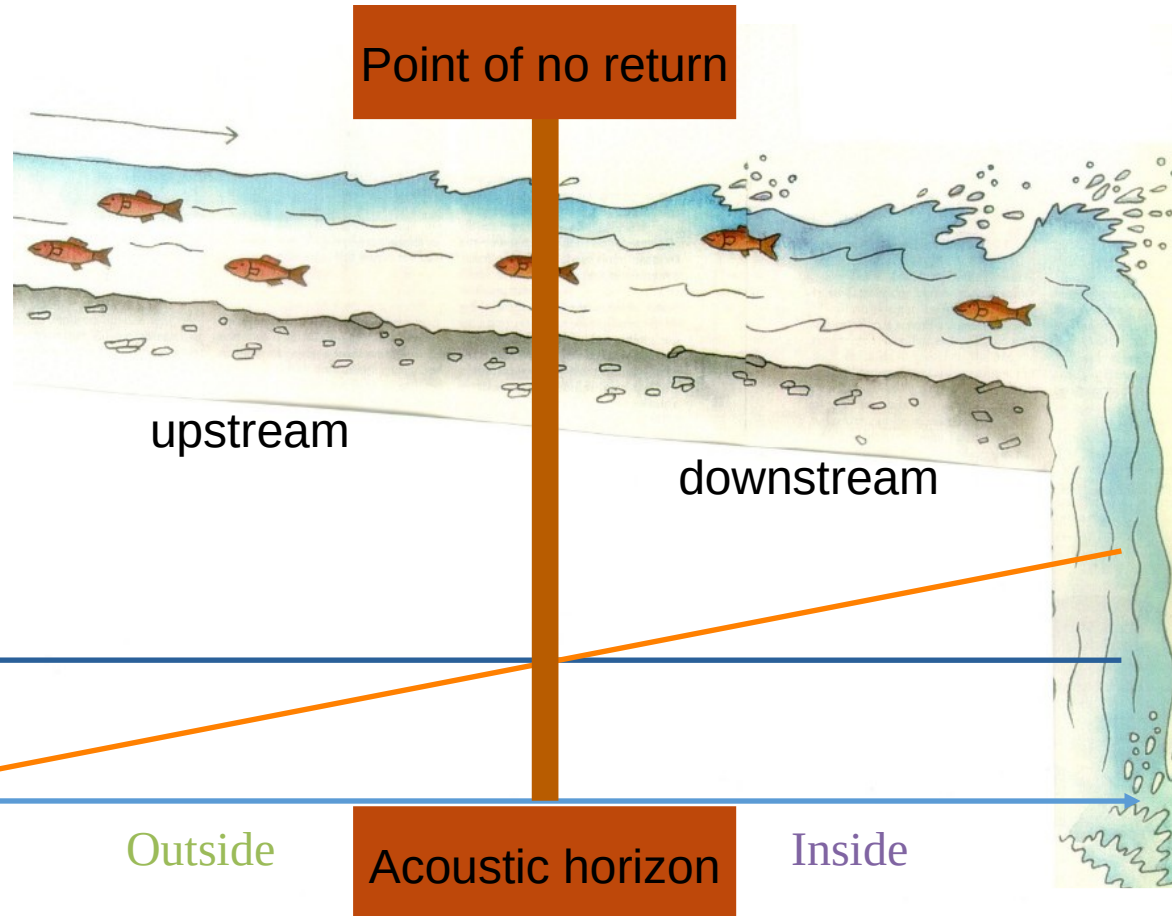
Flow velocity of fluid v

Speed of fish c

Outside

Inside

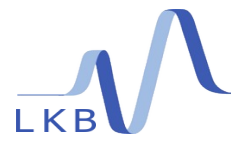
Schwarzschild geometry \leftrightarrow waterfall geometry



Wave equation for scalar field on Schwarzschild geometry:

$$g_{schw}^{\mu\nu} = \begin{pmatrix} -1 & -v \\ -v & (c^2 - v^2) \end{pmatrix}$$

$$g_{Unruh}^{\mu\nu} = \begin{pmatrix} -1 & -v \\ -v & (c^2 - v^2) \end{pmatrix}$$

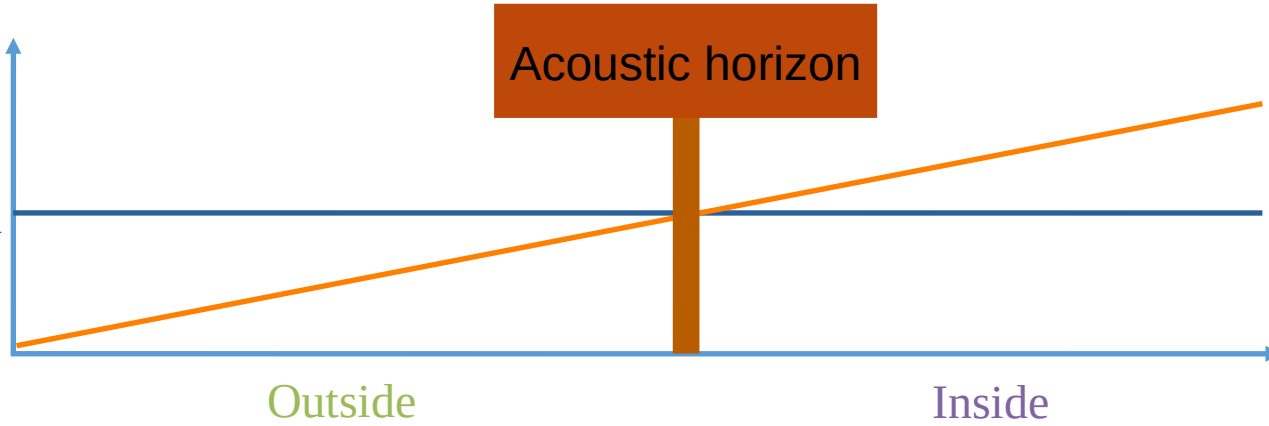


Quantised acoustic field in waterfall geometry

Schwarzschild geometry \leftrightarrow waterfall geometry

Flow velocity of fluid v

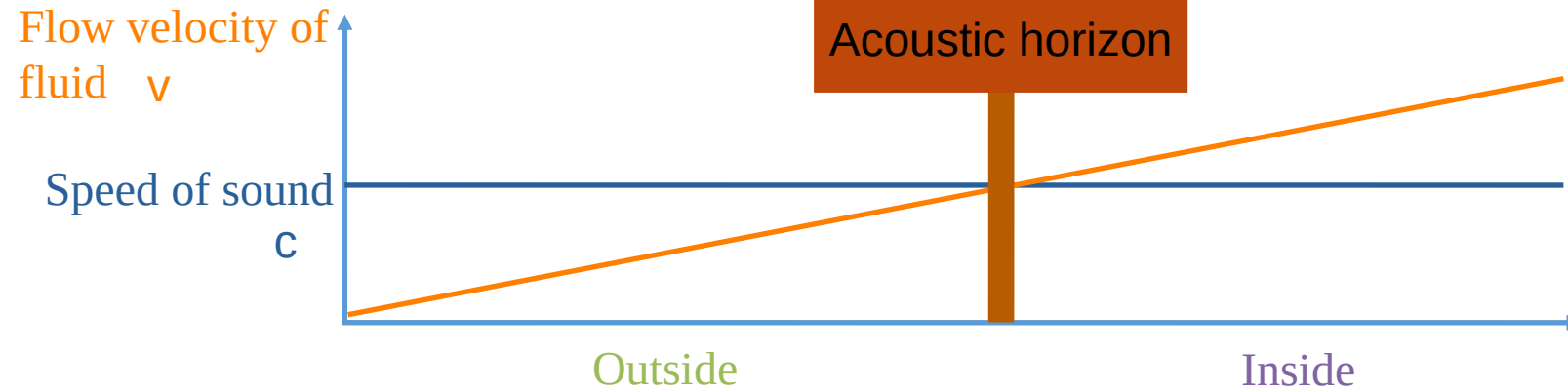
Speed of sound c



Wave equation of quantised acoustic field:

$$g_{Unruh}^{\mu\nu} = \begin{pmatrix} -1 & -v \\ -v & (c^2 - v^2) \end{pmatrix}$$

Schwarzschild geometry \leftrightarrow waterfall geometry



Quantised acoustic field:

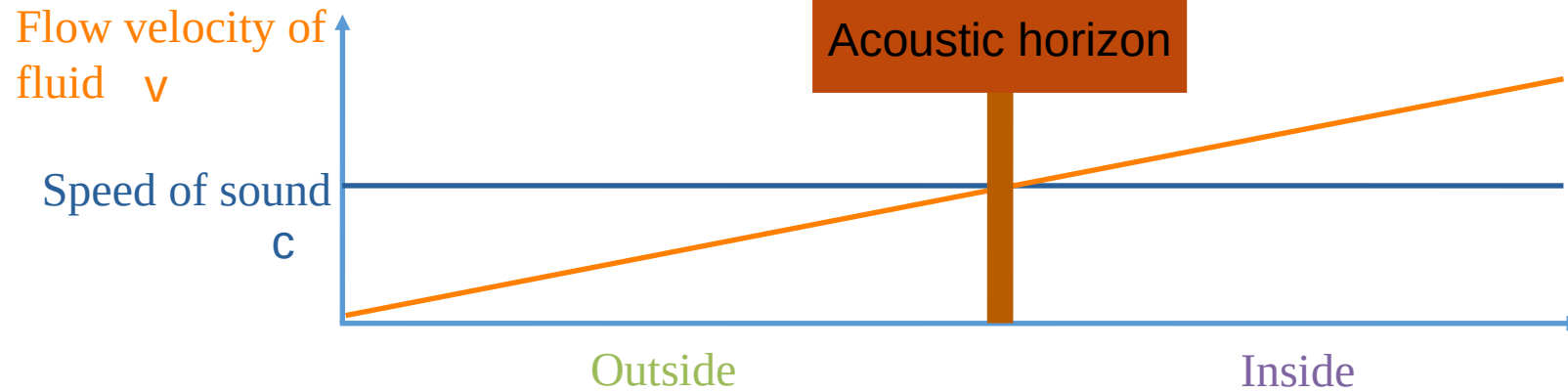
$$\text{in: } \phi = \int d\omega (a_\omega f_\omega + a_\omega^\dagger f_\omega^*) \quad a |0\rangle = 0$$

$$\text{out: } \phi = \int d\omega (\bar{a}_\omega F_\omega + \bar{a}_\omega^\dagger F_\omega^*) \quad \bar{a} |\bar{0}\rangle = 0$$

Express out modes in terms of in modes:

$$F_\omega = \int d\omega' (\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f_{\omega'}^*)$$

Schwarzschild geometry \leftrightarrow waterfall geometry



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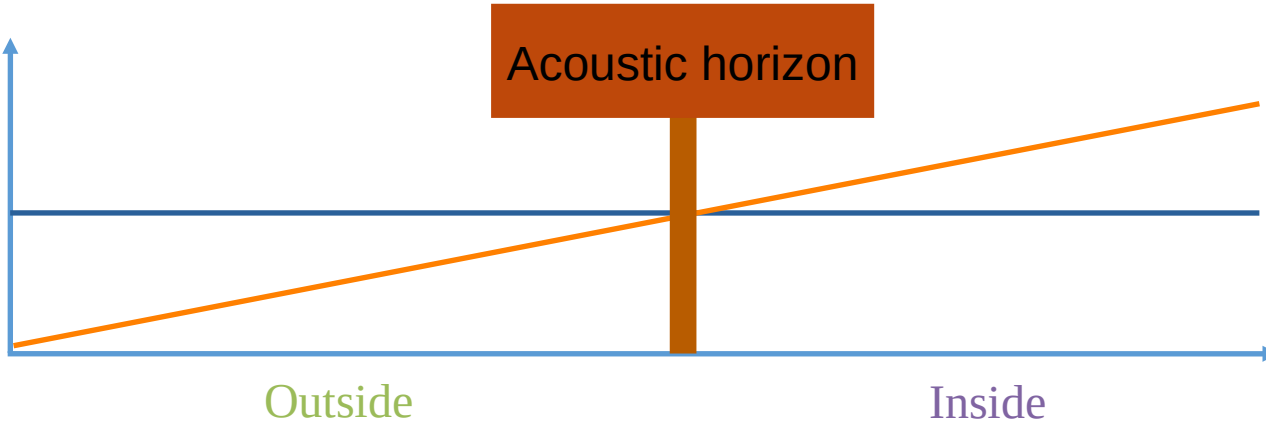
Different speed on either side of the horizon

$$\Rightarrow |\bar{0}\rangle \neq |0\rangle \Rightarrow \beta_{\omega\omega'} \neq 0$$

Schwarzschild geometry \leftrightarrow waterfall geometry

Flow velocity of fluid v

Speed of sound c



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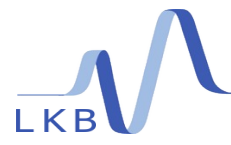
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mixing of positive and negative frequency waves
 \Rightarrow mixing of creation and annihilation operators

$$a |\bar{0}\rangle = \sum_{\omega'} \beta_{\omega\omega'} |\bar{1}\rangle > 0$$

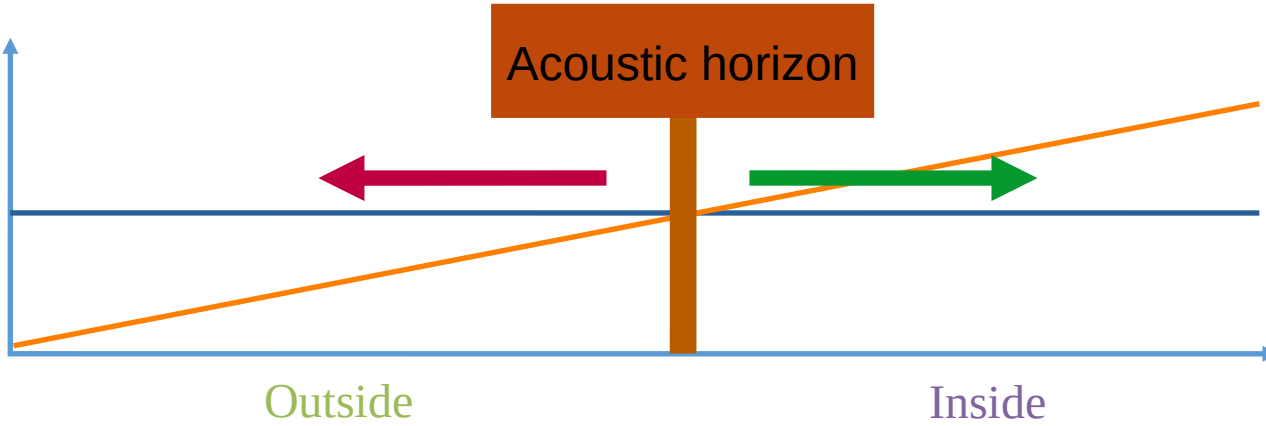


Quantised acoustic field in waterfall geometry

Schwarzschild geometry ↔ waterfall geometry

Flow velocity of fluid v

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Quantised acoustic field:

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Different speed on either side of the horizon

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mixing of positive and negative frequency waves
 \Rightarrow mixing of creation and annihilation operators

Spontaneous emission from the vacuum!

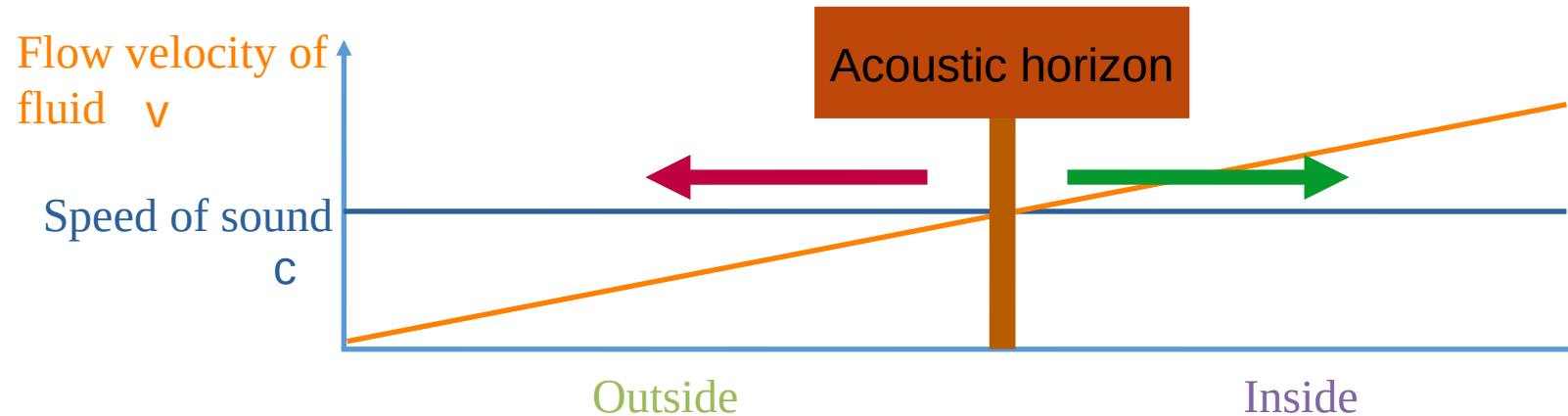
Hawking effect

How to create the spacetime of a Schwarzschild black hole in the laboratory?
curvature

How to observe the Hawking effect in the laboratory?
quantum field

Our experiment with a quantum fluid of microcavity polaritons
engineered nonlinearity correlated waves

- 1) create a transsonic fluid \rightarrow acoustic horizon where $v=c$
different speed on either side of acoustic horizon \rightarrow mixing of positive and negative frequency waves
 \rightarrow spontaneous emission of phonon pairs from the vacuum
Unruh *PRL* **46** 1351 (1981)
- 2) observe Hawking spectrum
- 3) observe correlations across the horizon



Sound waves

In BEC

Hawking correlations

Steinhauer 2019

Black hole laser?

Steinhauer 2014

In fluid of light

(microcavity polaritons)

Proof of principle by Amo and Bloch 2015

New experiments in Paris 2022

Gravity/Capillary waves

Scattering at the white hole

Rousseaux and Leonhardt 2008

Weinfurtner and Unruh 2010

Correlations across the WH horizon

Rousseaux and Parentani 2016

Correlations across the BH horizon

Rousseaux 2020

Rotating black hole - superradiance

Weinfurtner 2016

Rotating black hole - oscillation of light rings (QNMs)

Weinfurtner 2020

Light waves

Scattering at the BH/WH horizon

König and Leonhardt (Fibre) 2008

Faccio (Bulk) 2010

König (Fibre) 2012

Wang (Fibre) 2013

Murdoch (Fibre) 2015?

Bose (Fibre) 2015

Ciret (waveguide) 2016

Kanakis (Fibre) 2016

Gaafar (waveguide) 2017

König and Jacquet (Fibre) 2018

Leonhardt (Fibre) 2019

Negative frequency waves

König and Faccio 2012

König 2014, 2015

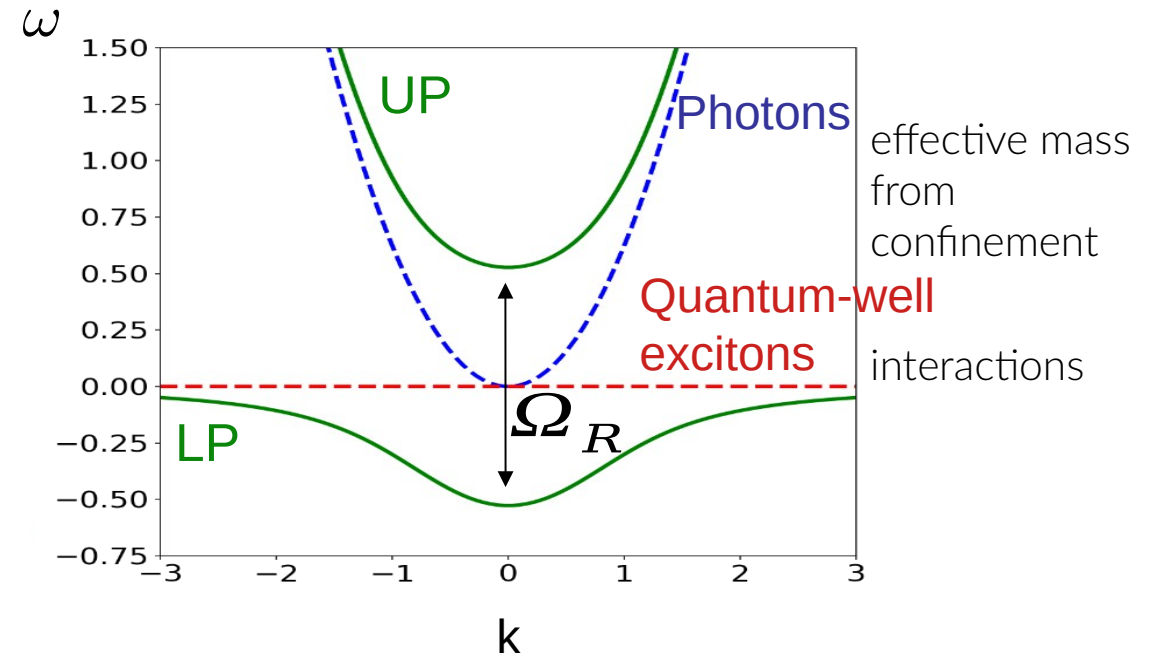
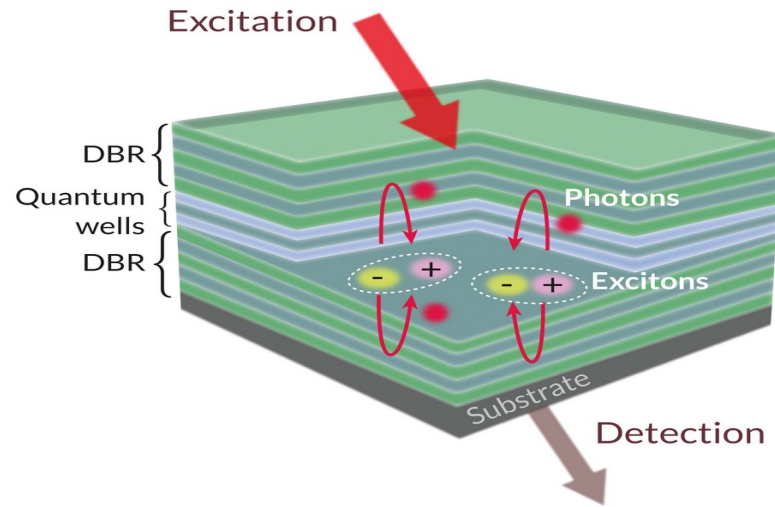
Universality of the Hawking effect, Unruh and Schützhold PRD 71 024028 (2005)?

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Our experiment with a quantum fluid of microcavity polaritons
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Polaritons= photons dressed with material excitations that live in the cavity plane



Dynamics in the cavity plane described by Gross-Pitaevskii (Nonlinear Schrödinger) equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m_{LP}^*} + gn \right) \psi - \frac{i\hbar\gamma}{2} \psi + P(r, t)$$

- g polariton-polariton interaction constant
- γ losses
- P pump

Driven-dissipative dynamics → Out-of-equilibrium system

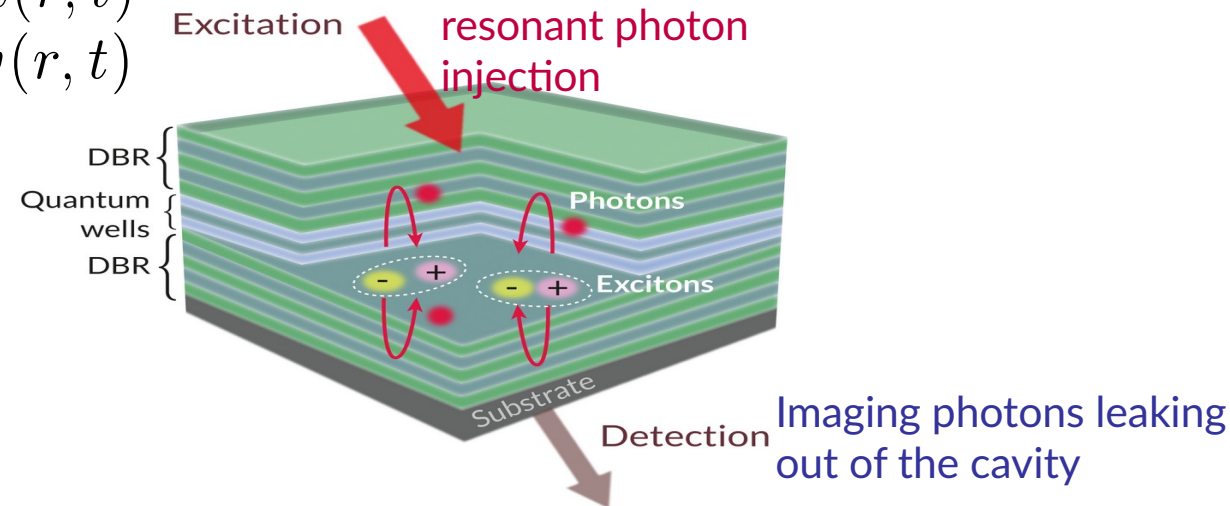
Polaritons= photons dressed with material excitations that live in the cavity plane

$$I(\mathbf{r}, t) \rightarrow n(\mathbf{r}, t)$$

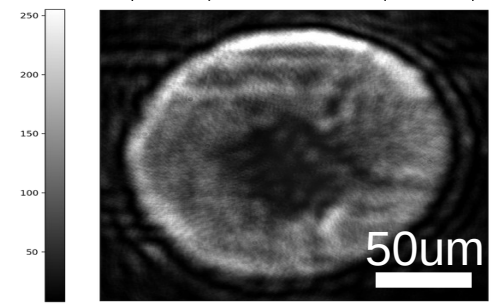
$$\phi(\mathbf{r}, t) \rightarrow v(\mathbf{r}, t)$$

Phase + intensity profile of driving field

→ Spatial Light Modulator (SLM)



$$n(\mathbf{r}, t) \rightarrow I(\mathbf{r}, t)$$



$$v(\mathbf{r}, t) \rightarrow \phi(\mathbf{r}, t)$$

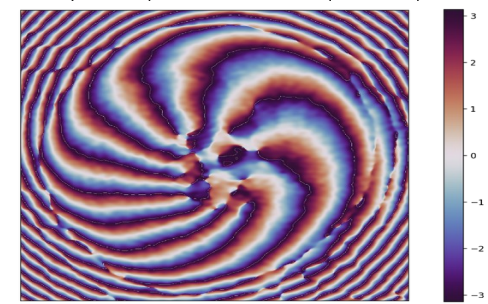
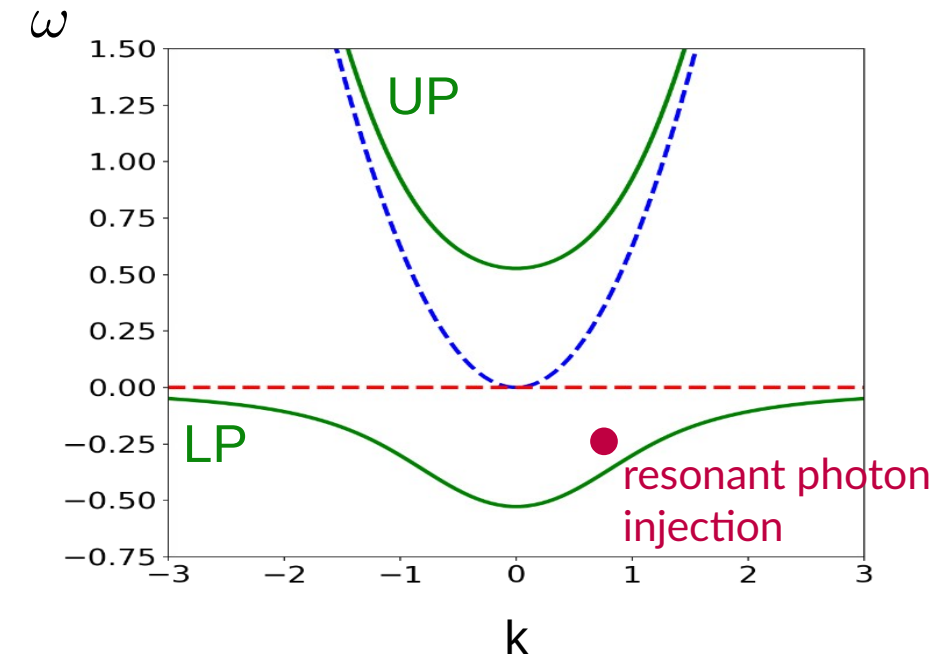


Image of the cavity plane

→ density map: $c \propto \sqrt{n}$

→ velocity map: $v \propto \nabla \phi$



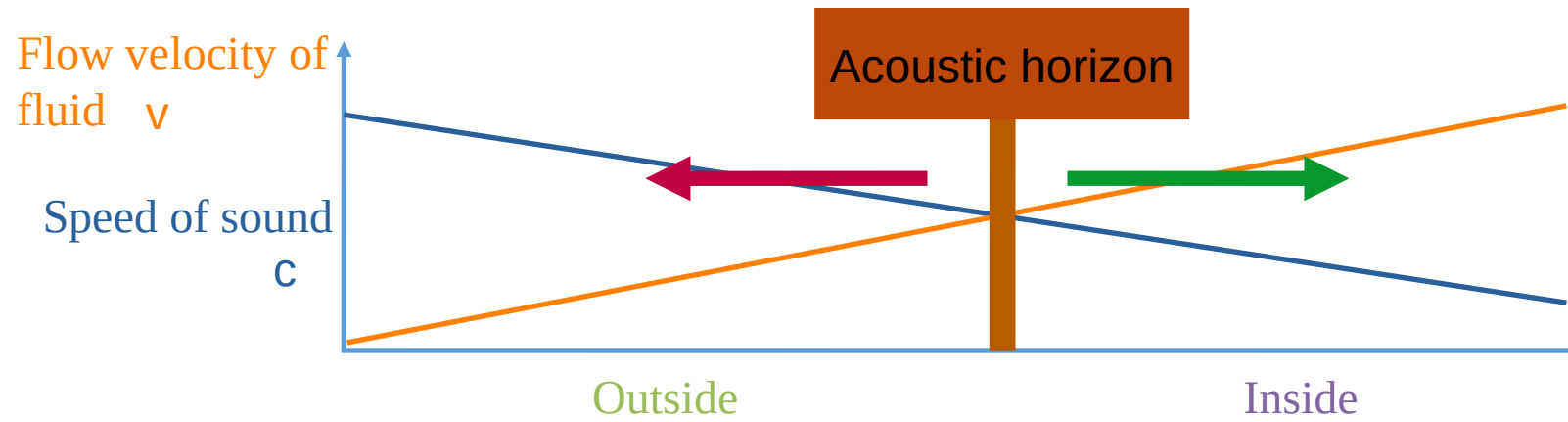
Full optical experiment

(2D planar sample, no microstructure)

- 1) create a transsonic fluid \rightarrow acoustic horizon where $v=c$
- 2) observe Hawking spectrum
- 3) observe correlations across the horizon

Unruh *PRL* **46** 1351 (1981)

Visser *Class Quant Grav* **15** 1767 (1998)



First proposal by Solnyshkov *et al.* *PRB* **84** 233405 (2011)

Numerical studies in Gerace and Carusotto *PRB* **86** 144505 (2012)

Grisins *et al.* *PRB* **94** 144518 (2016)

Jacquet *et al.* *EPJD* **76** 152 (2022)

Proof of principle experiments for acoustic horizon by Nguyen *et al.* *PRL* **114** 036402 (2015)

Jacquet *et al.* *PTRSA* **378** 201190225 (2020)

Hawking effect has not been seen in polaritons to date



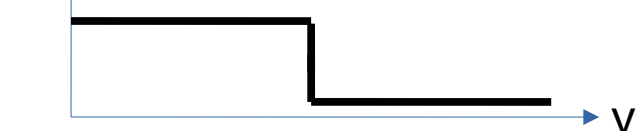
Acoustic horizon

New data by PhD student Kévin Falque

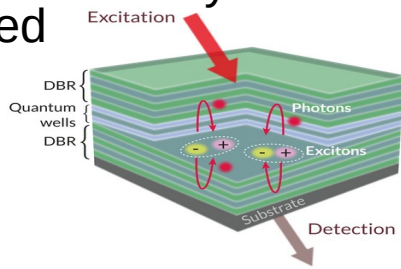
Flow velocity of fluid v

Speed of sound c

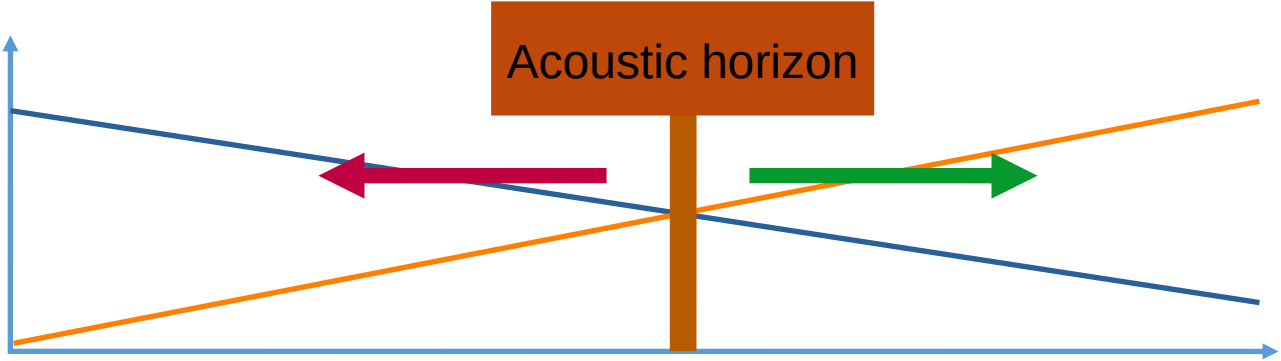
$|F_p|$



+ phase controlled with SLM

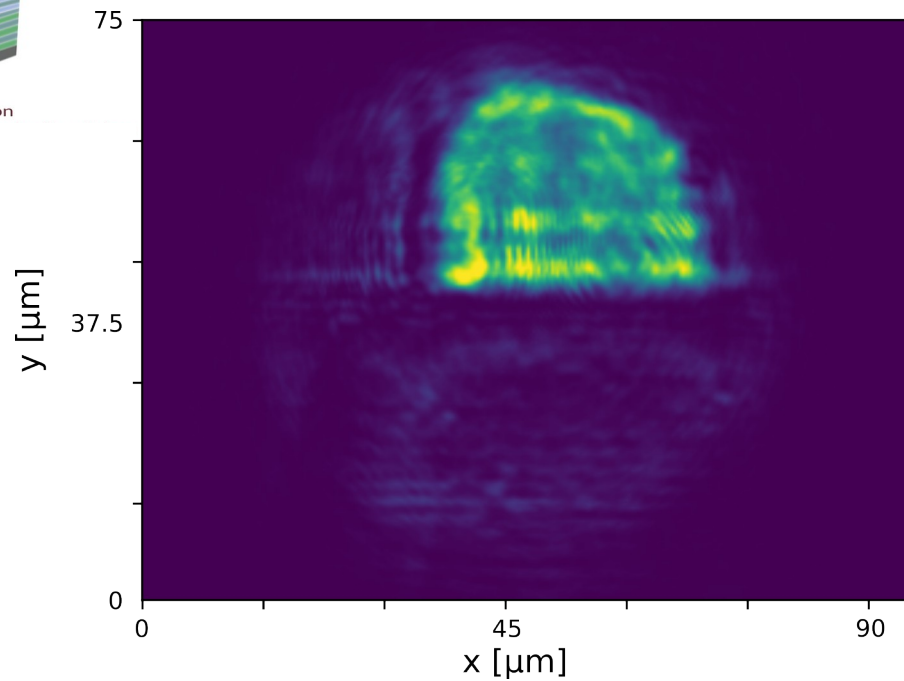


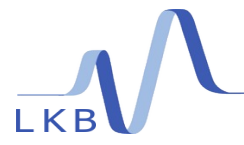
Acoustic horizon



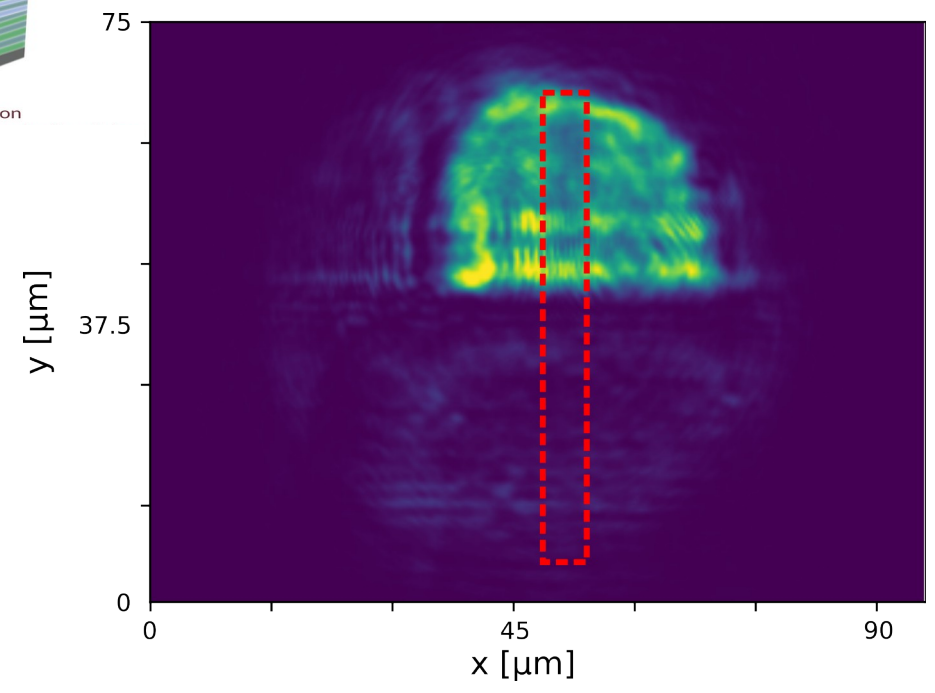
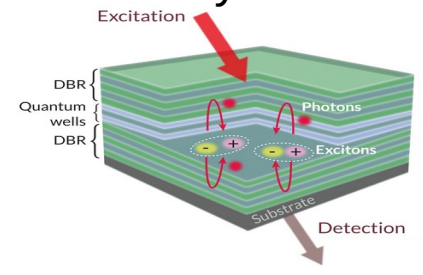
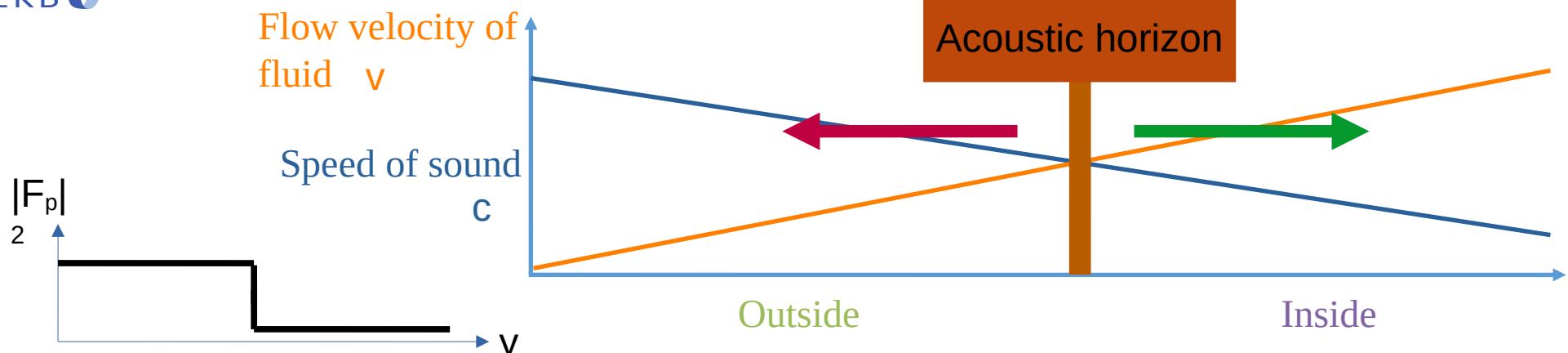
Outside

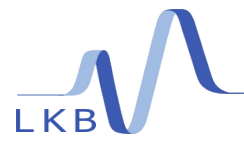
Inside



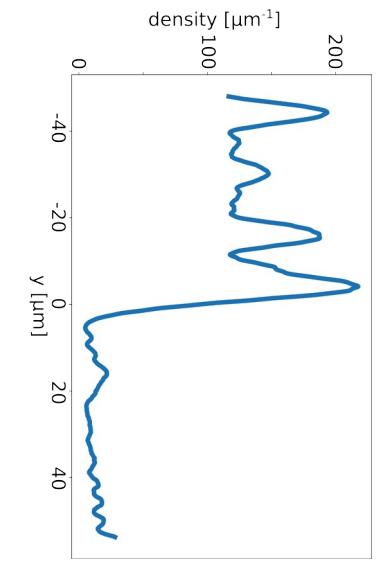
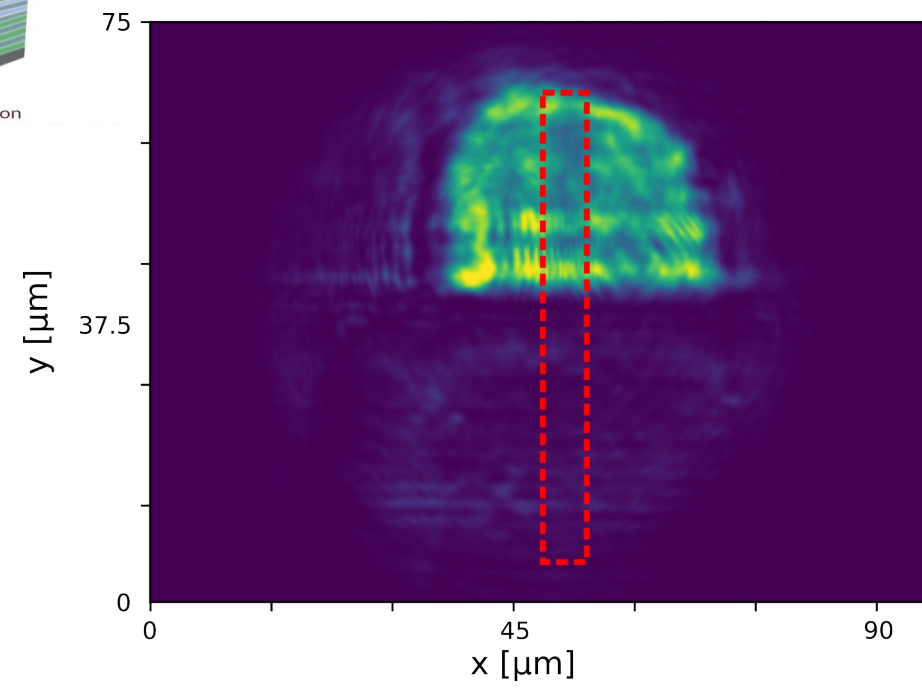
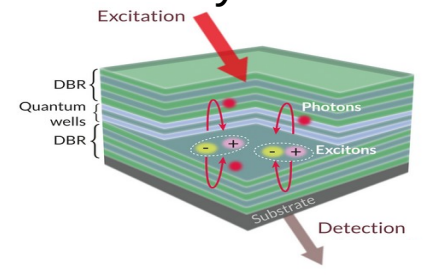
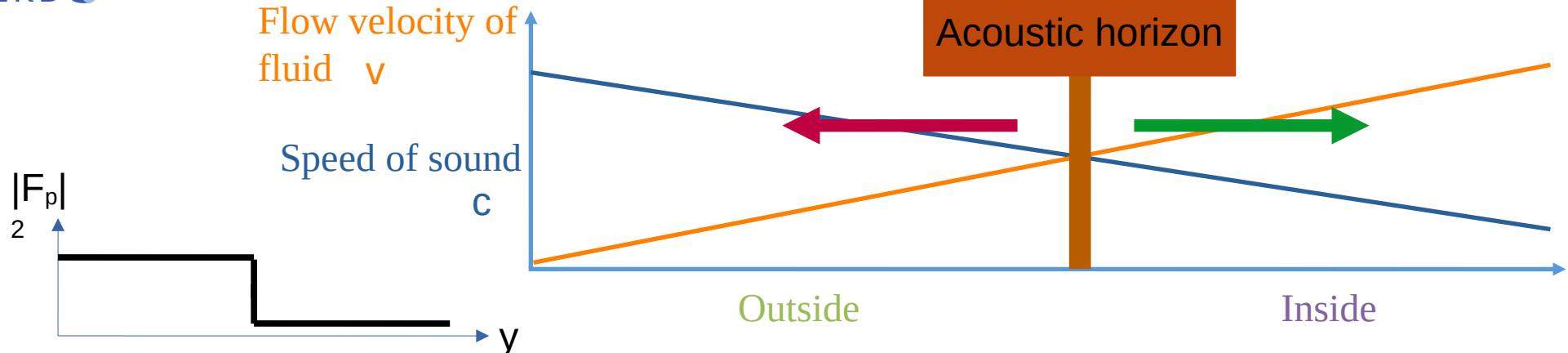


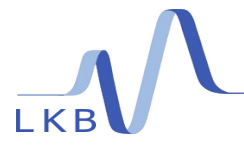
Acoustic horizon





Acoustic horizon





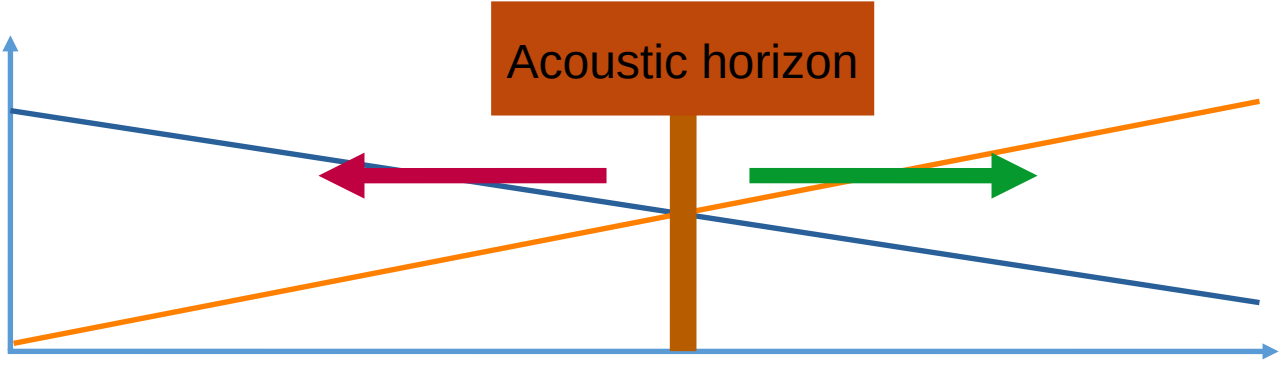
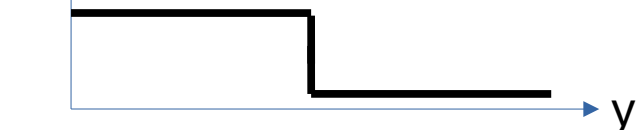
Acoustic horizon

Flow velocity of fluid v

Speed of sound c

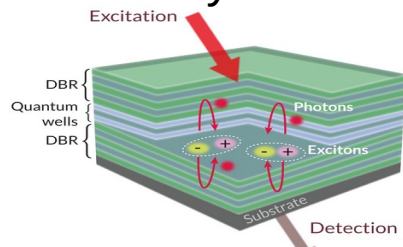
Acoustic horizon

$|F_p|$

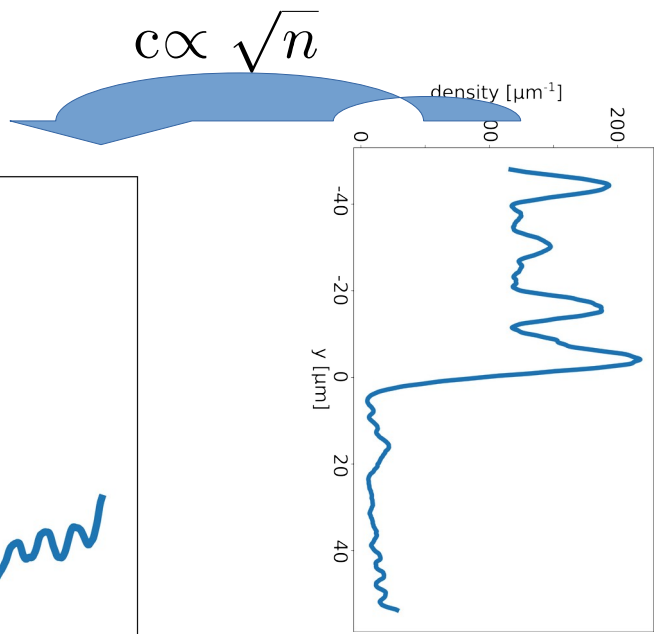
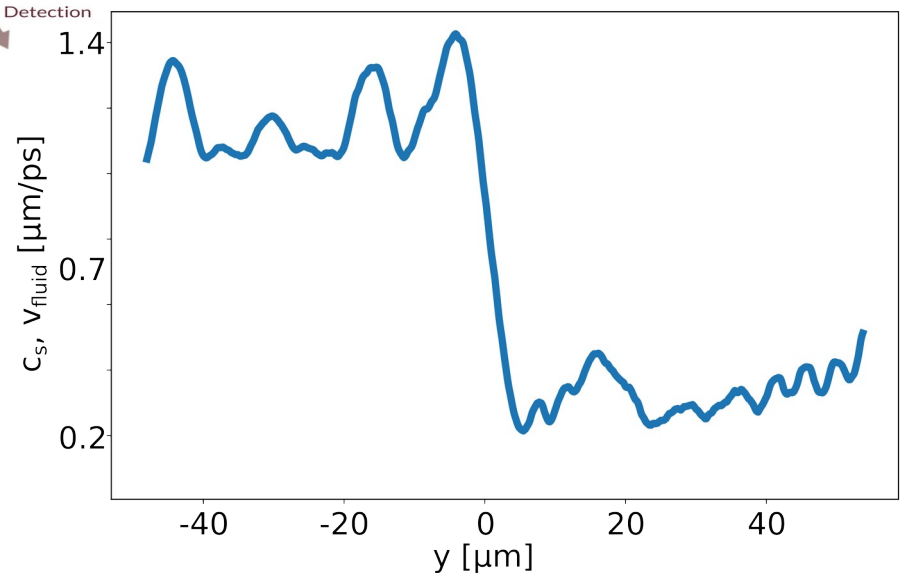


Outside

Inside



$$c \propto \sqrt{n}$$





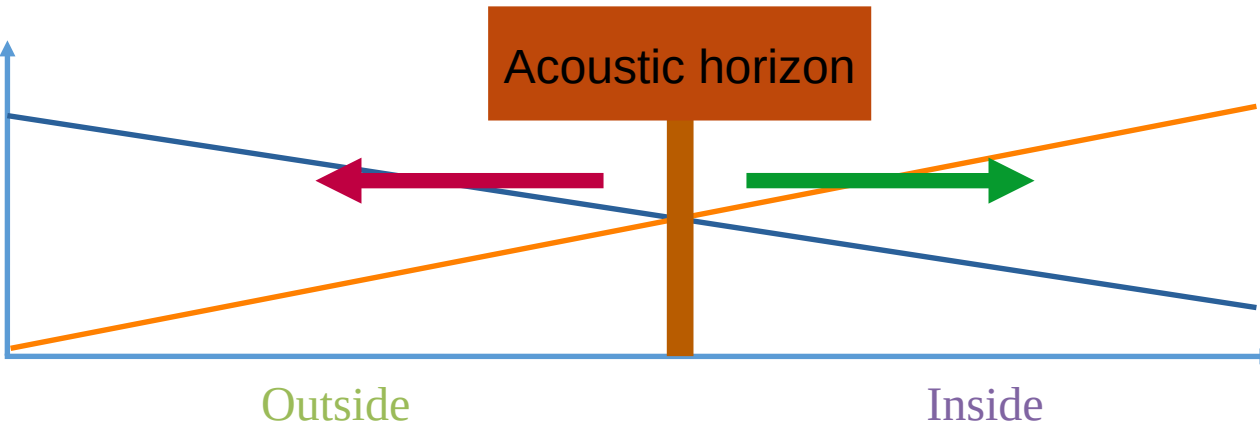
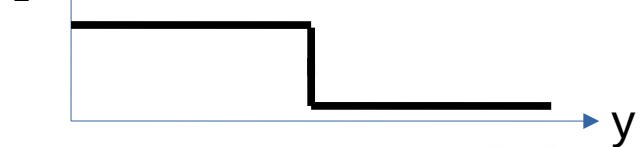
Acoustic horizon

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Flow velocity of fluid v

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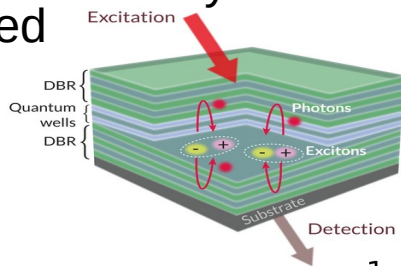


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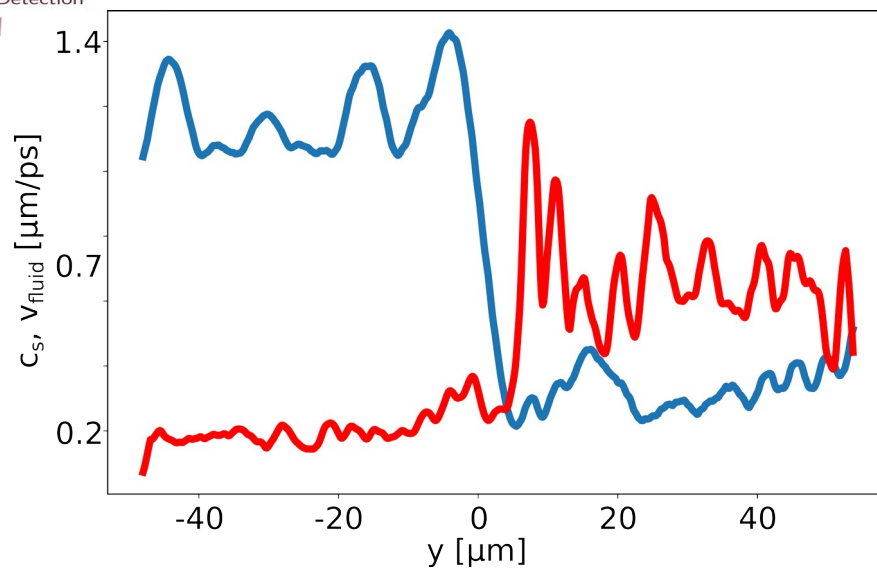
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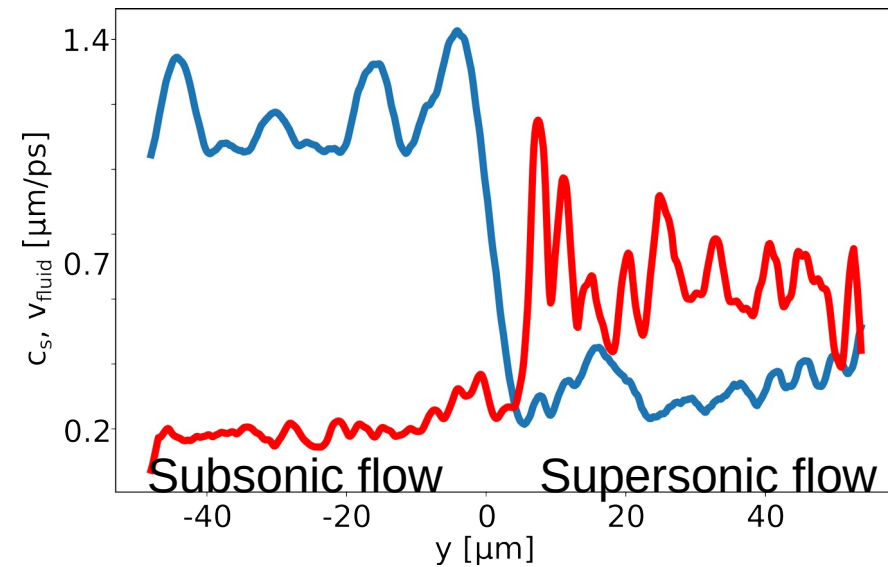
$$v \propto \nabla \phi$$

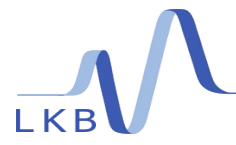


$$c \propto \sqrt{n}$$



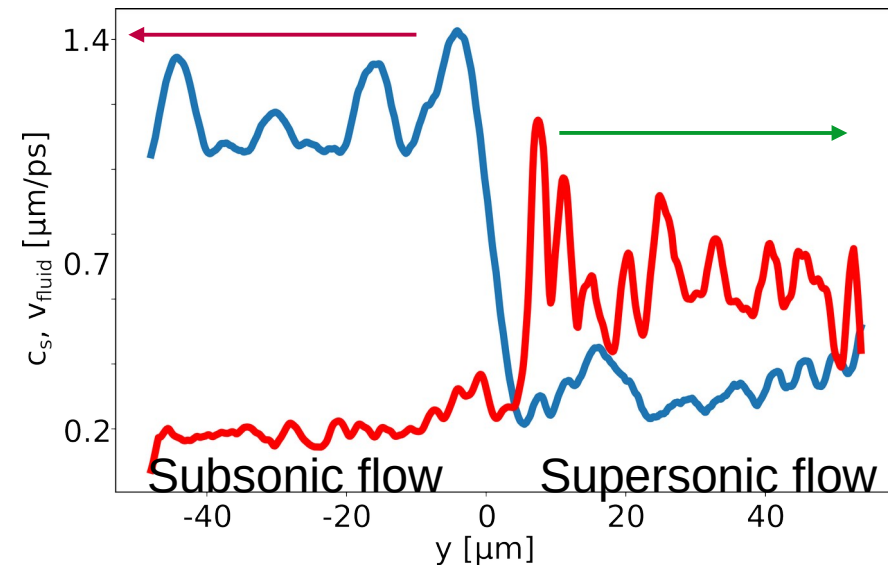
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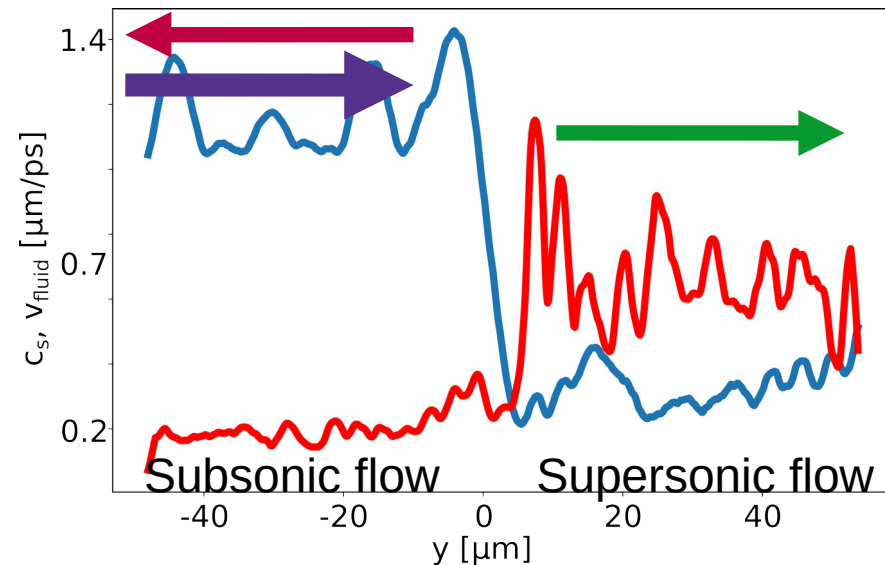
Hawking spectrum

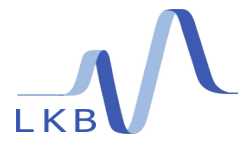
Hawking effect at the horizon: emission of acoustic waves on either side of the horizon



Hawking effect at the horizon: emission of acoustic waves on either side of the horizon

Stimulate emission with **coherent probe at input** → create acoustic wave that impinges on horizon and scatters
→ reflection = Hawking radiation
transmission = partner



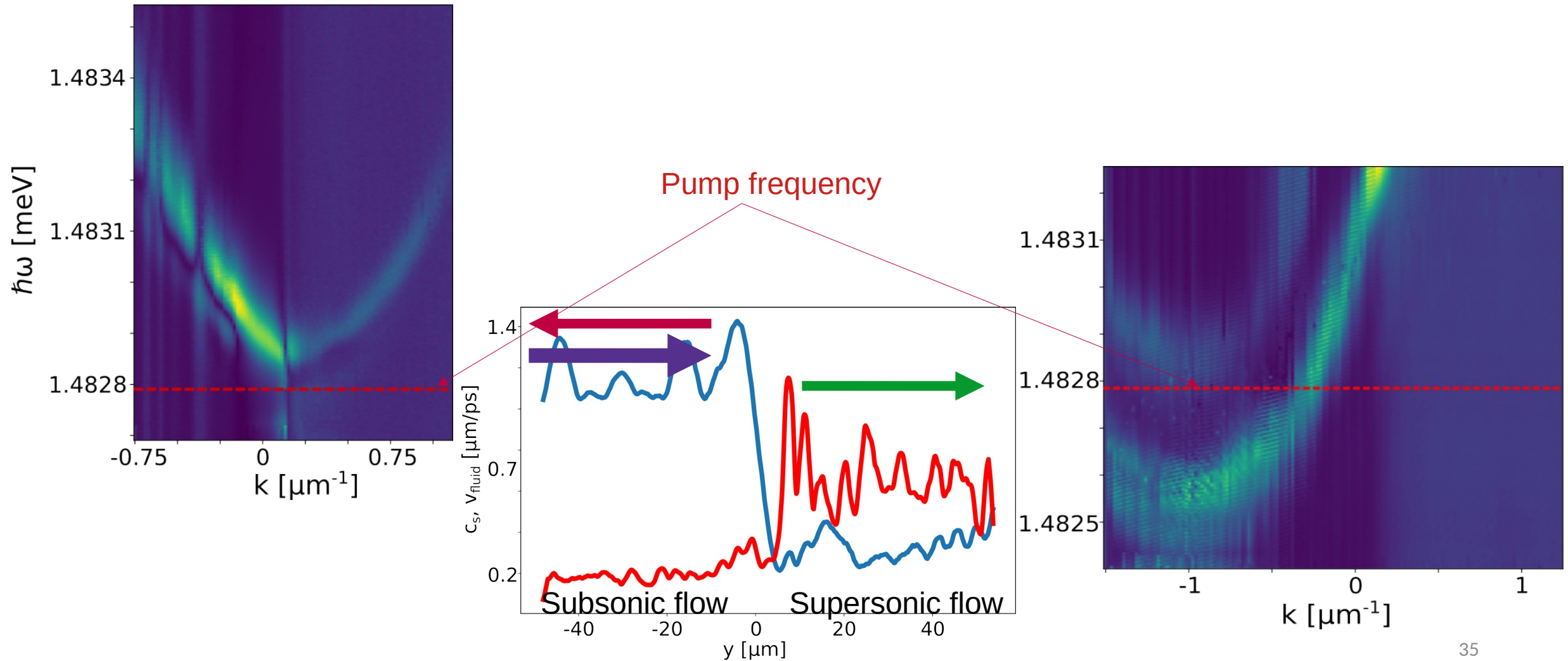


Hawking spectrum

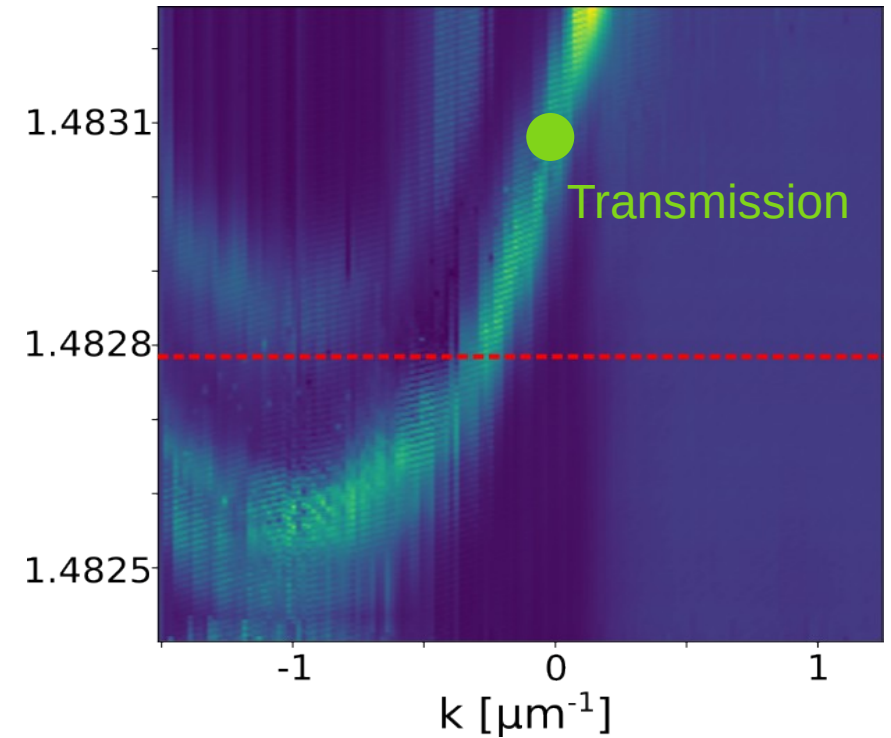
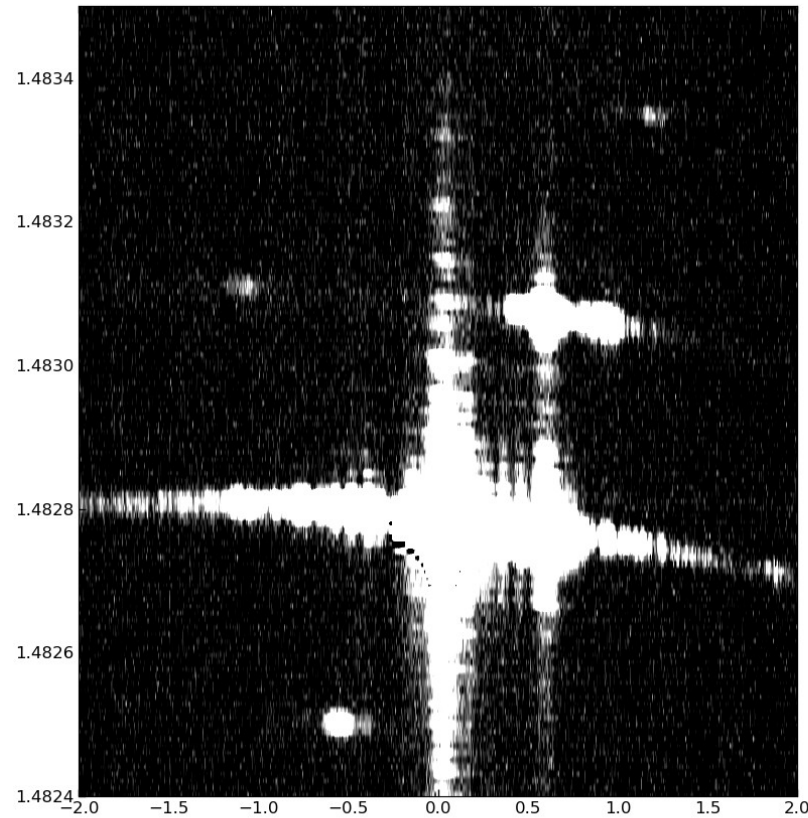
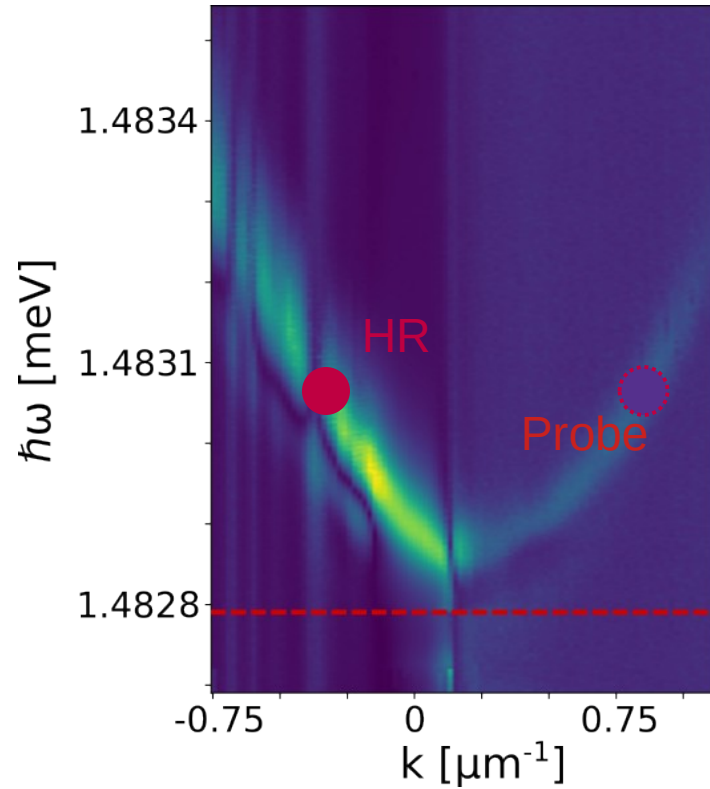
Sound waves on either side of the horizon: Bogoliubov dispersion relation

measured with coherent probe spectroscopy

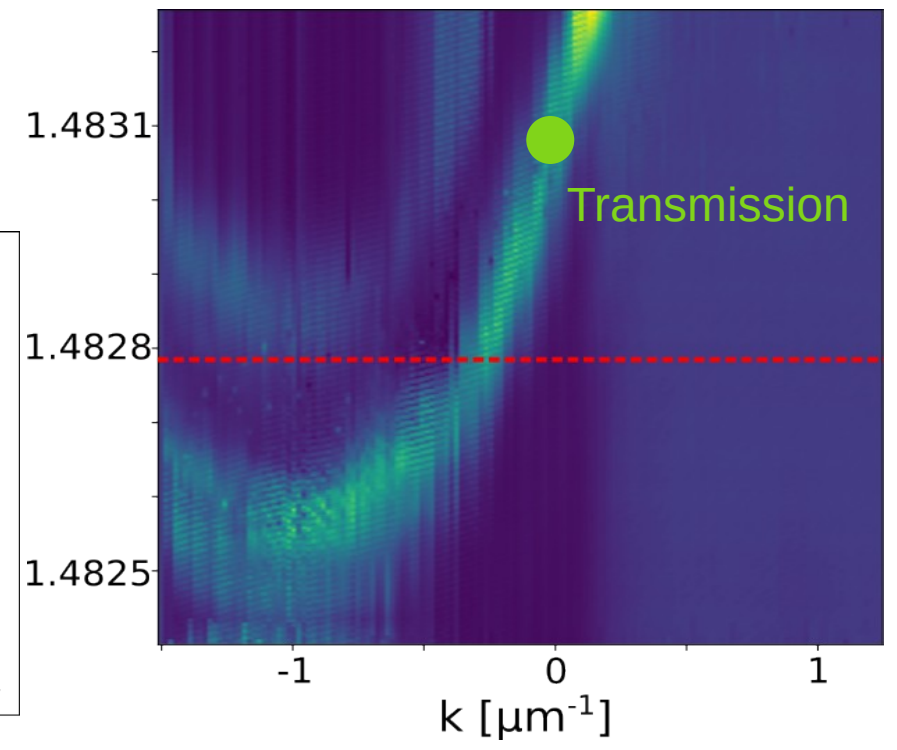
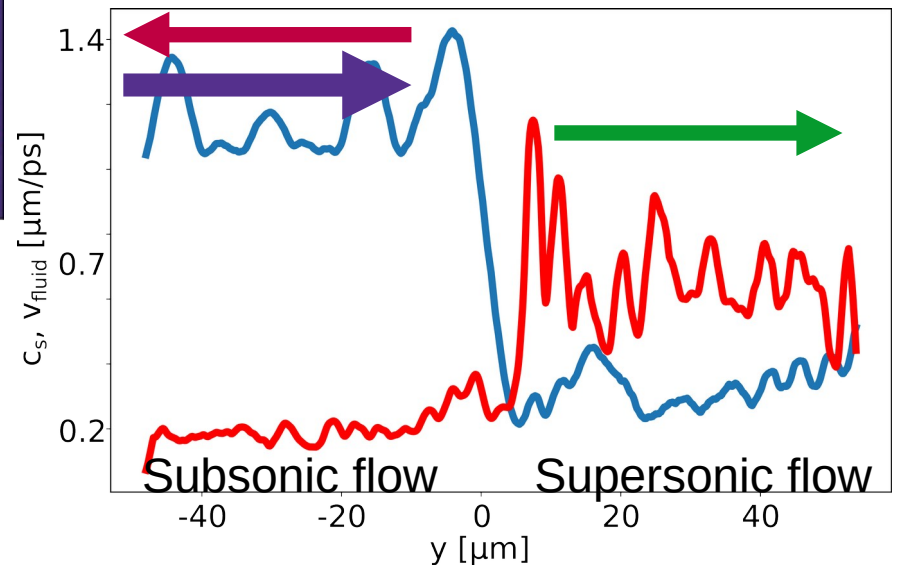
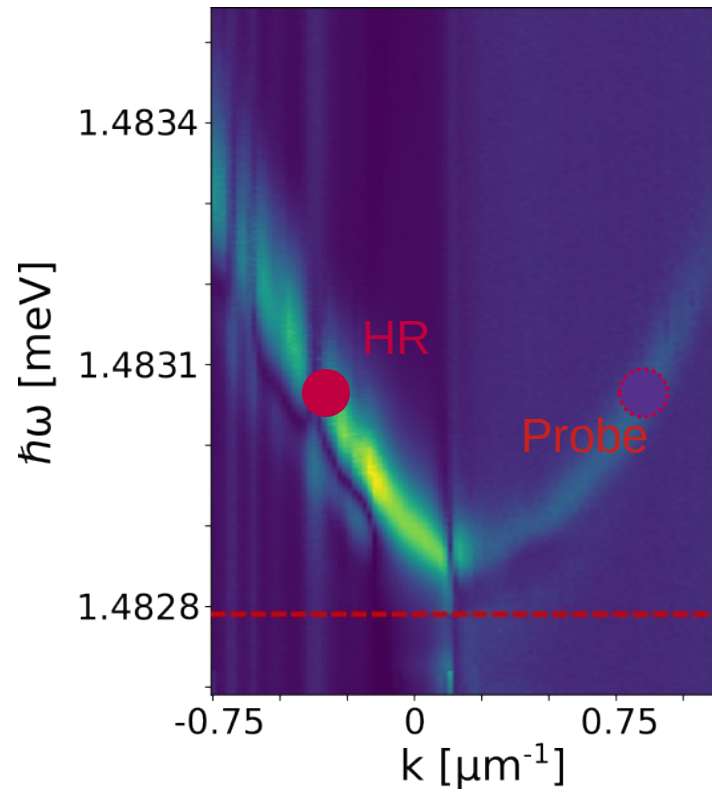
F Claude, M Jacquet *et al* PRL **129** 103601 (2022)



Stimulate emission with **coherent probe at input** → create acoustic wave that impinges on horizon and scatters
 → reflection = Hawking radiation
 transmission = partner



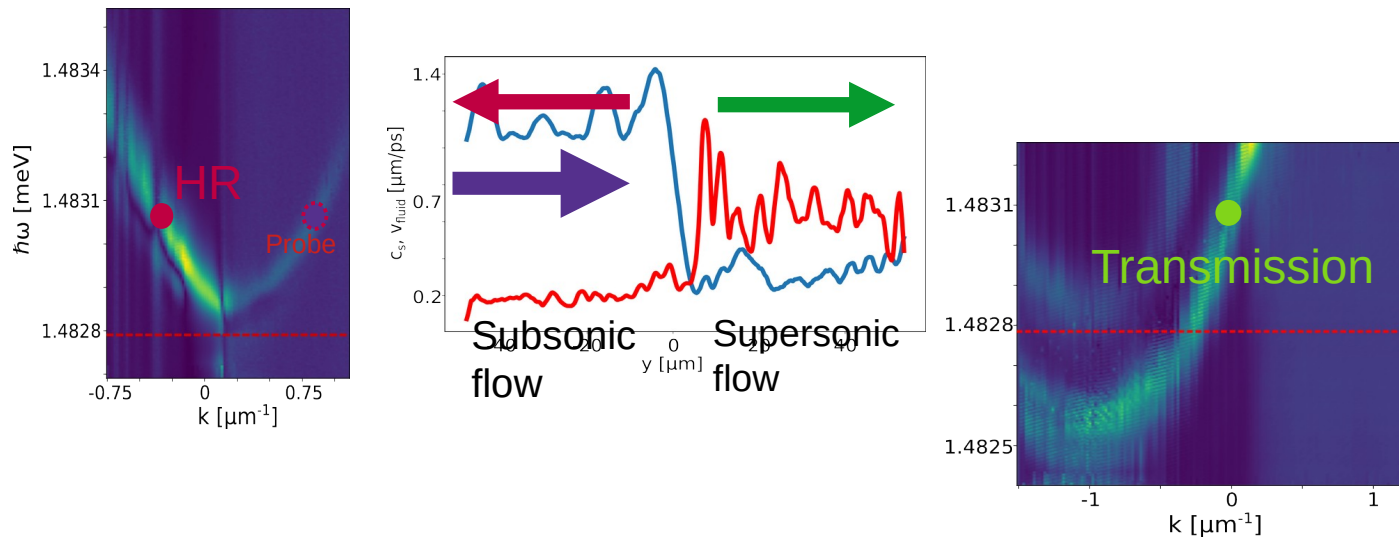
Scattering of probe at acoustic horizon = observation of Hawking effect



The **propagation of waves in nonlinear media** may be controlled to engineer situations where the waves propagate as though they were on an **effectively curved geometry**, like around a black hole or in an inflating universe. This enables the **experimental study of field theories** on curved geometries.

A transsonic fluid creates an acoustic horizon

Our experiment with a quantum fluid of microcavity polaritons: Hawking effect



New data by PhD student Kévin Falque

- Next:
- observe full spectrum of Hawking radiation, including negative frequencies
 - measure correlations across the horizon
 - simulate rotating geometry?

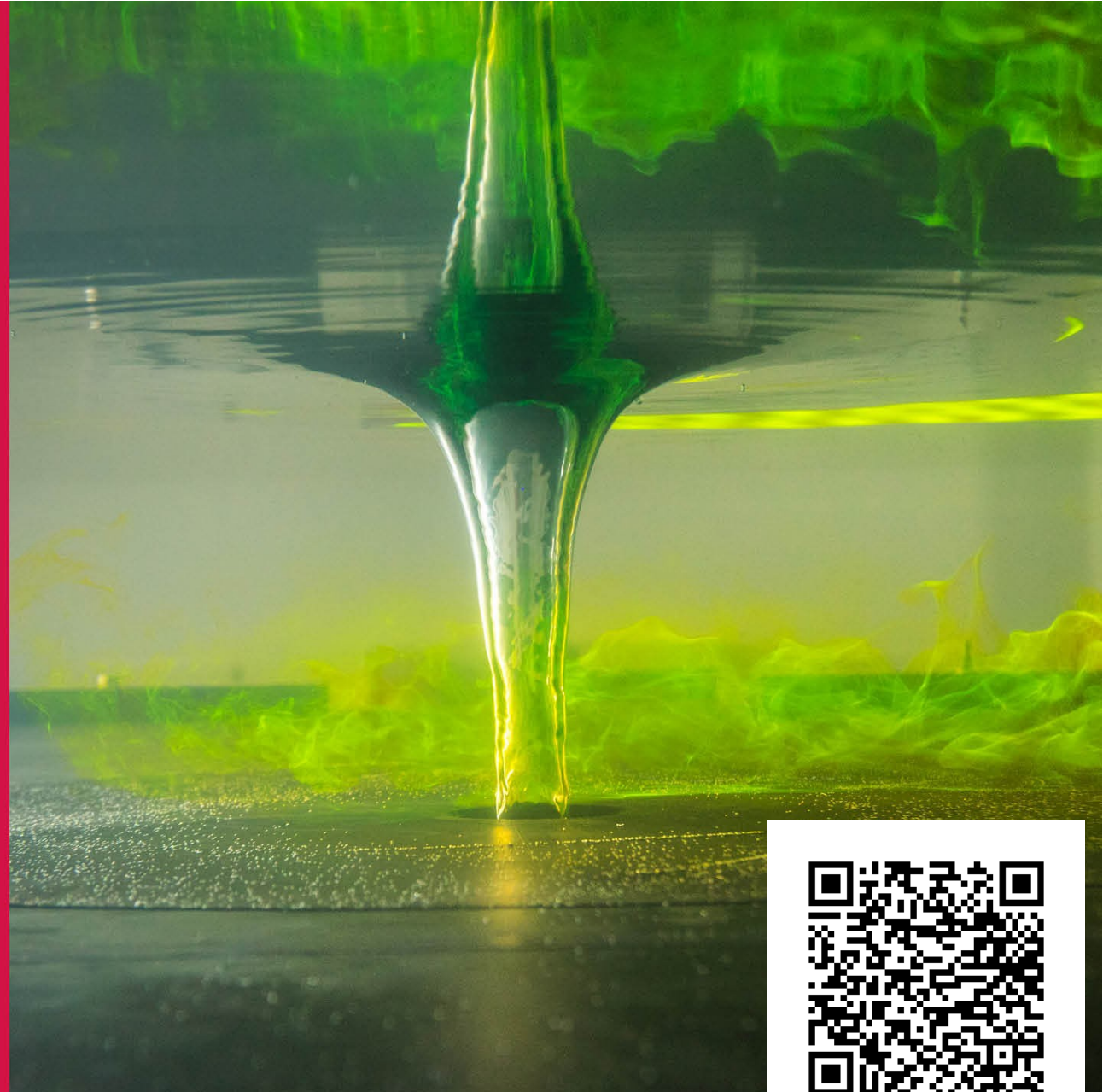
The next generation of analogue gravity experiments

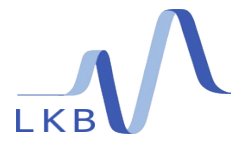
9 – 10 December 2019

Organised by Dr Maxime
Jacquet, Dr Silke Weinfurter
and Dr Friedrich König.

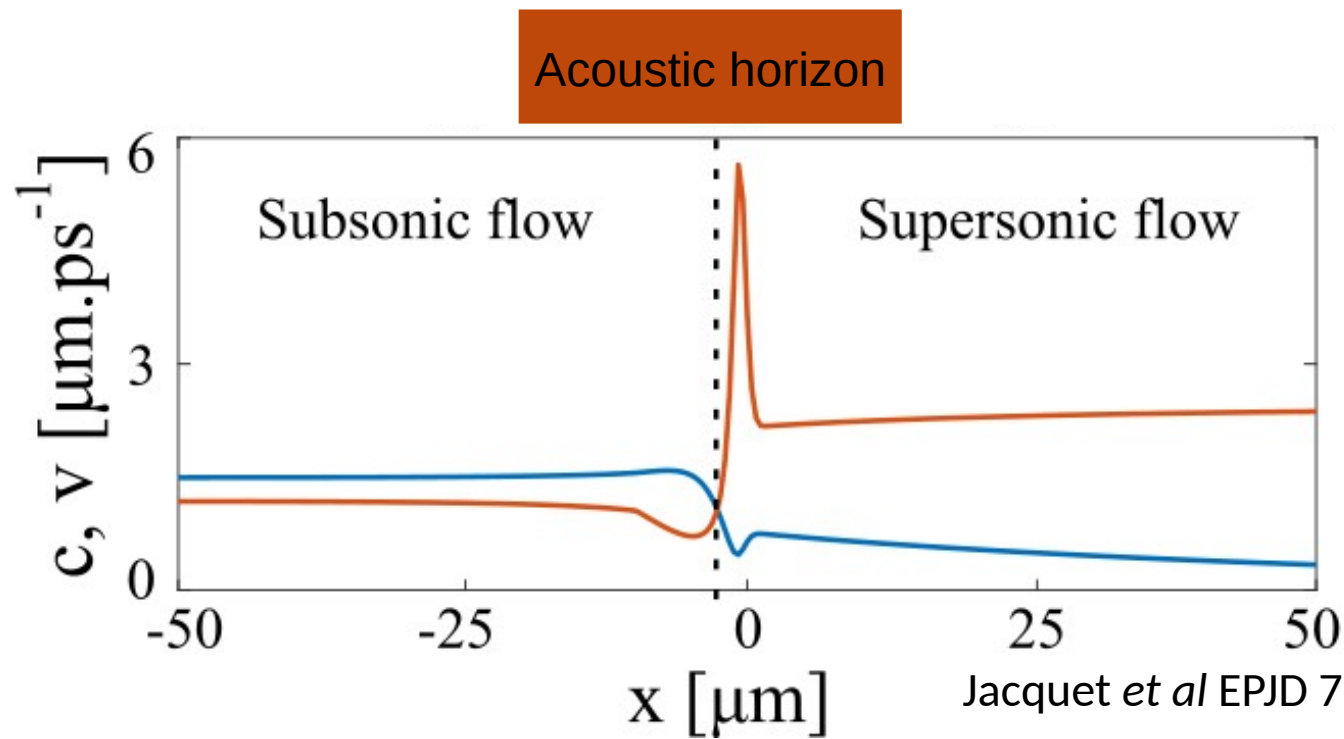
THE
ROYAL
SOCIETY

Image: © Alex Wilkinson Media.

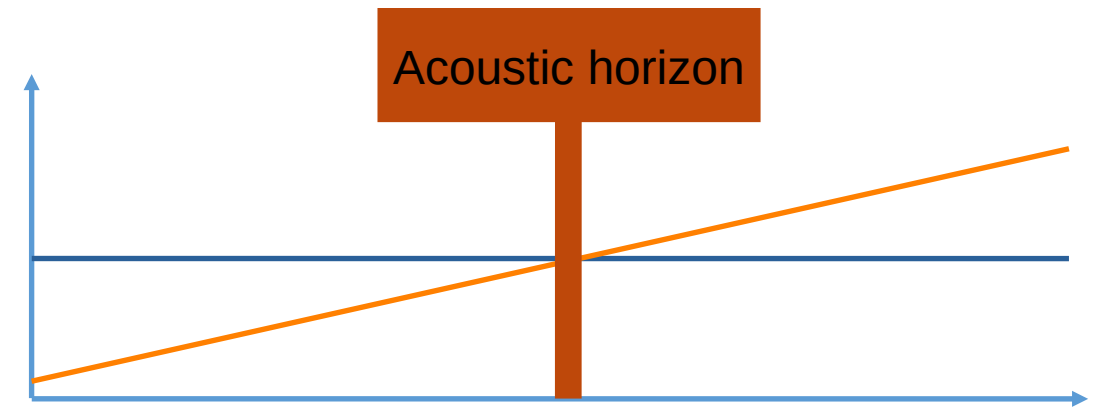




Modelling a BH spacetime with a fluid of microcavity polaritons

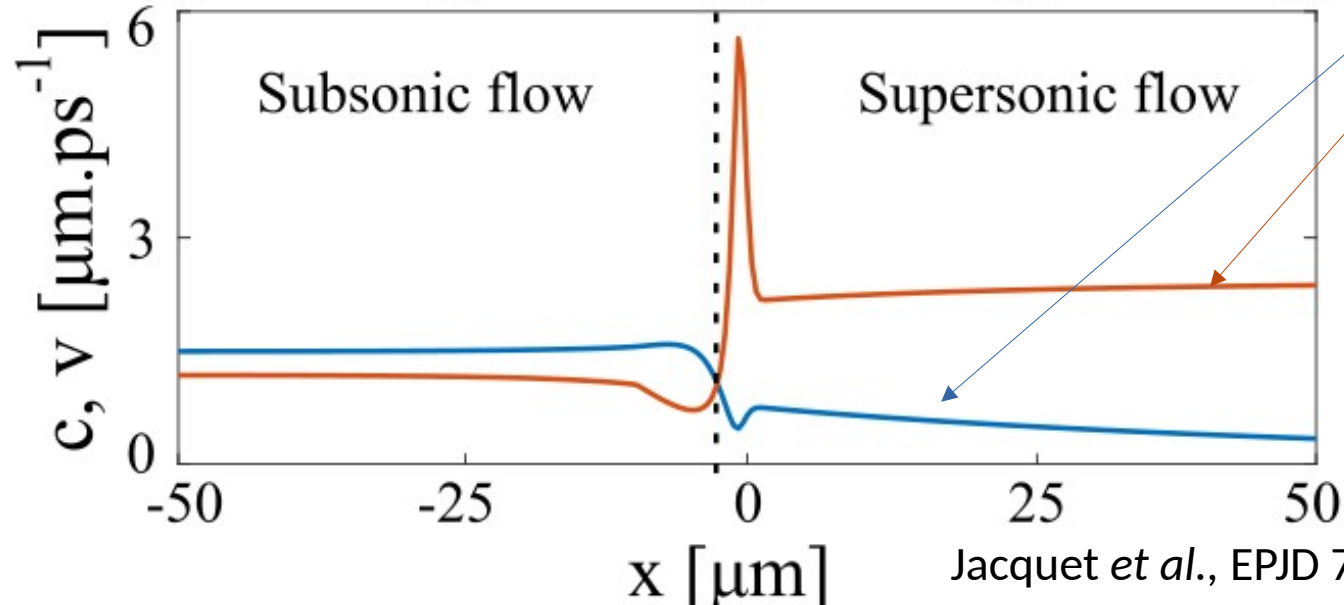
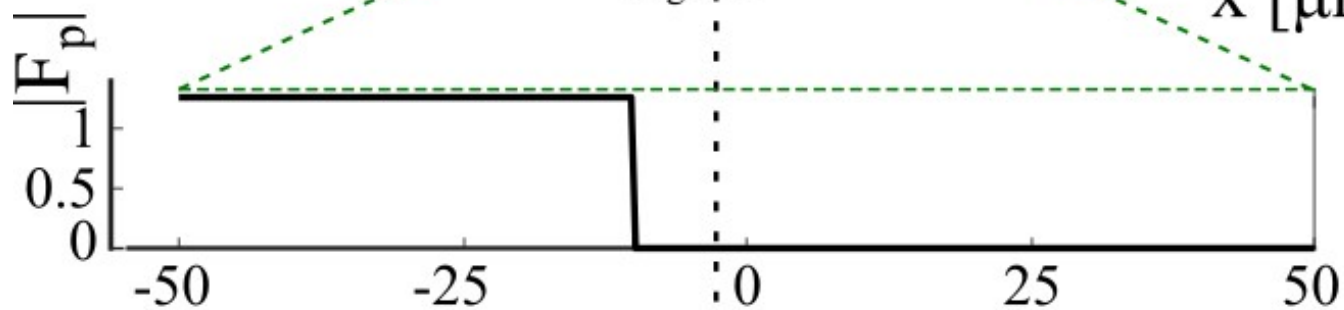
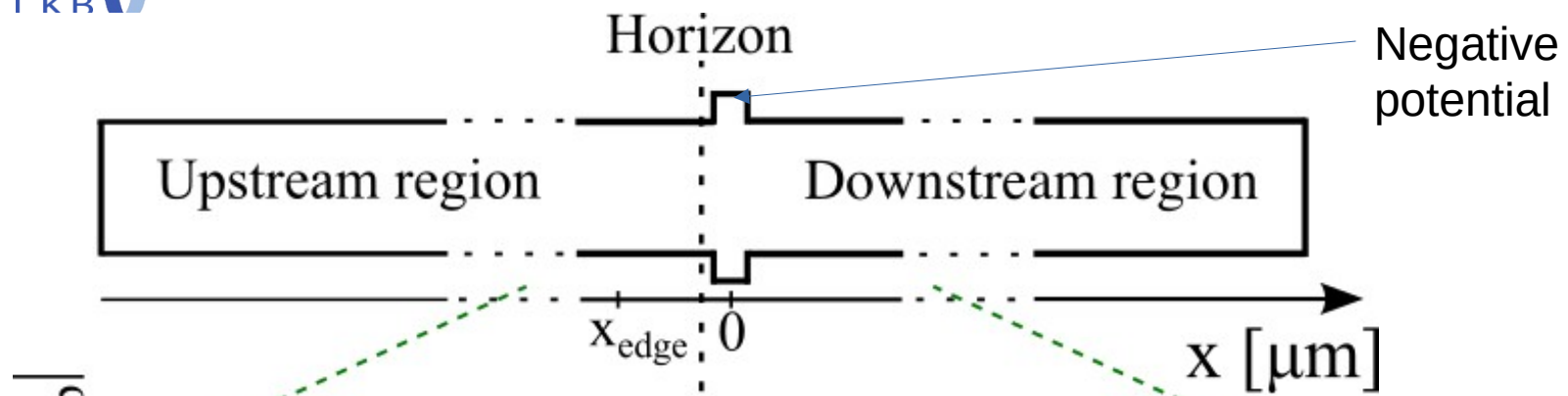


Jacquet et al EPJD 76 152 (2022)

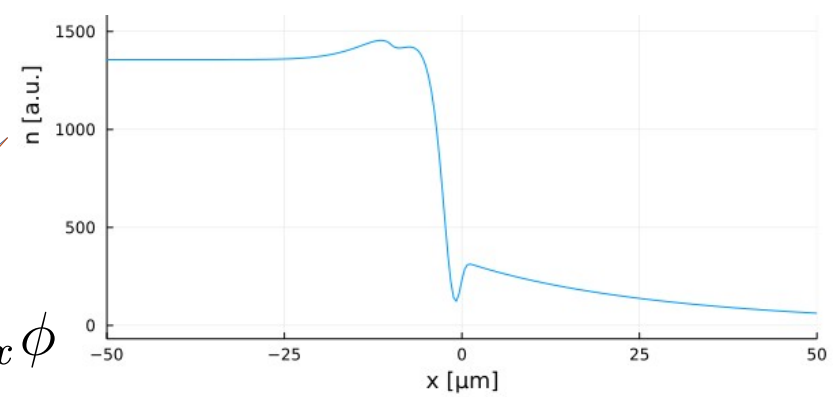




Acoustic horizon in polaritons

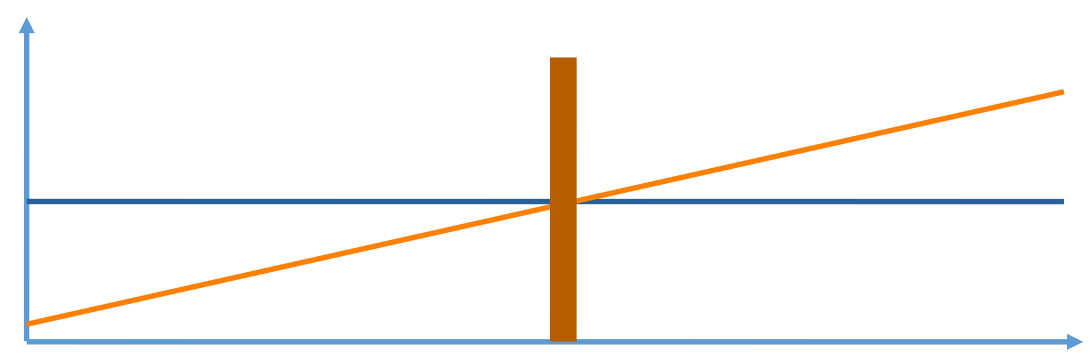


Simulate sample of Nguyen HS *et al.*,
PRL 114 036402 (2015)

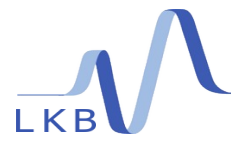


$$c \propto \sqrt{n}$$

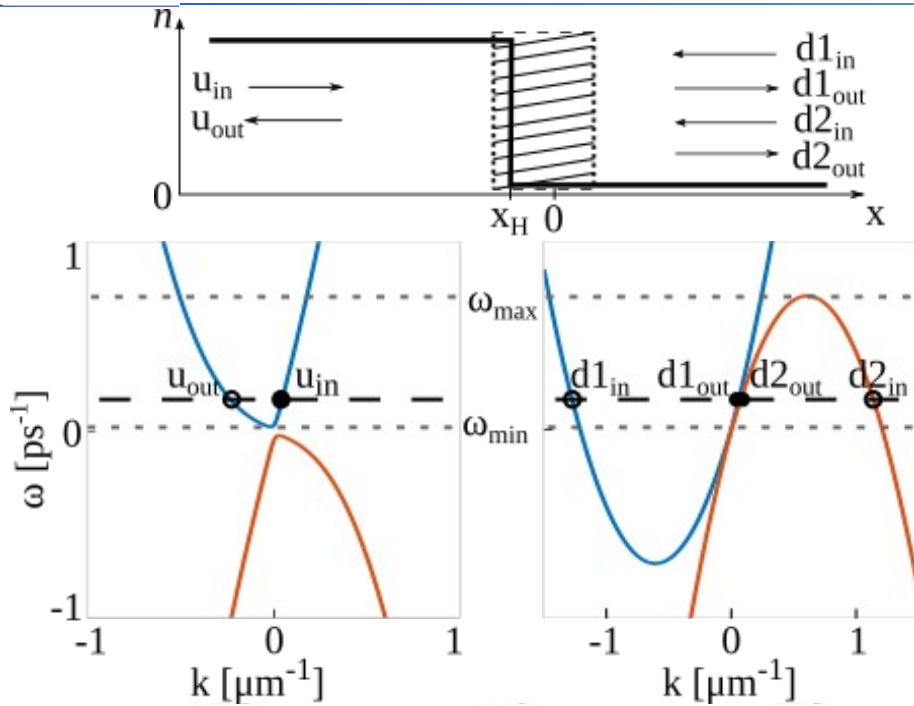
$$v \propto \partial_x \phi$$



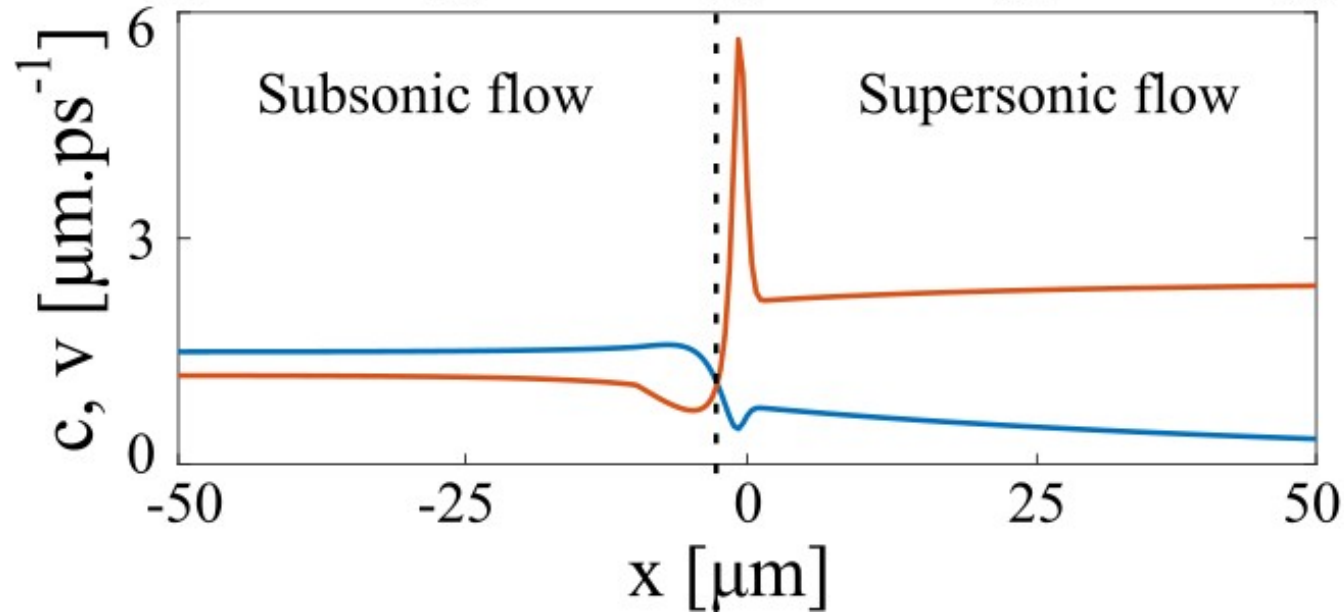
Jacquet *et al.*, EPJD 76 152 (2022)



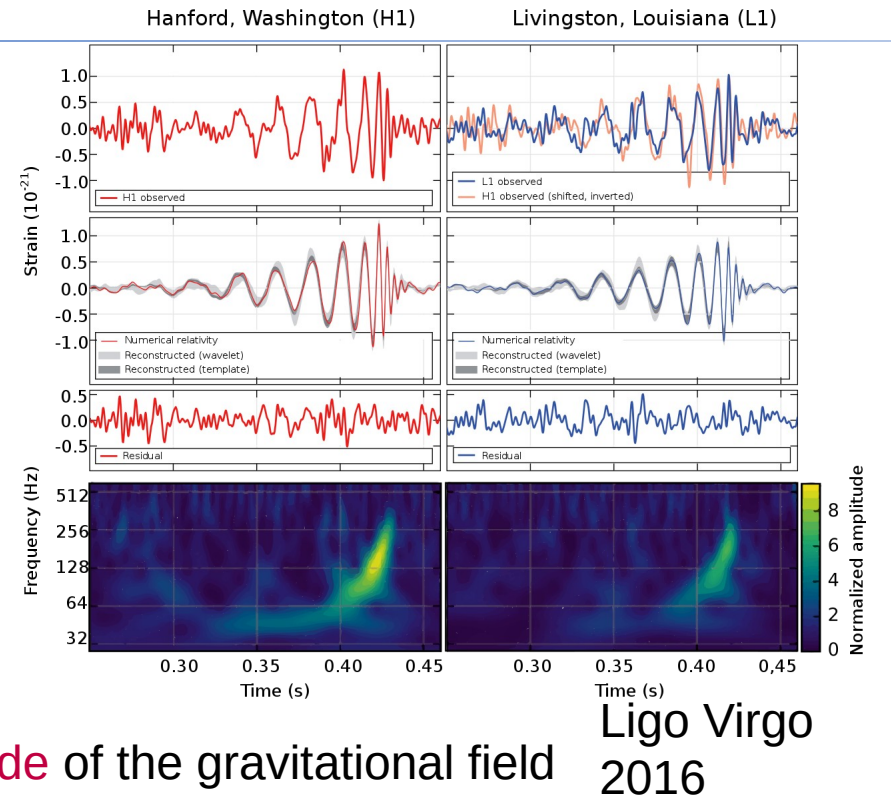
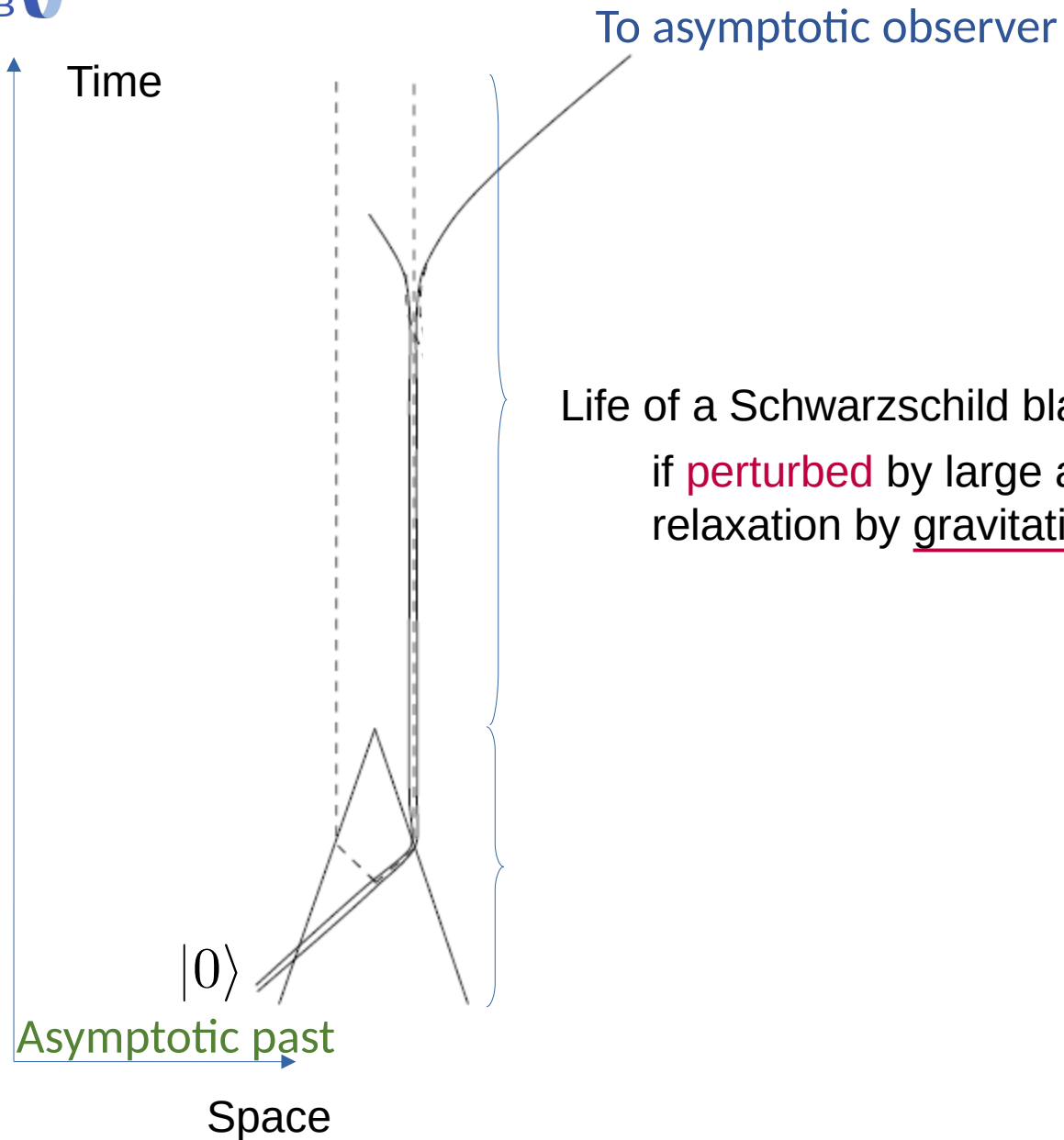
Acoustic horizon in polaritons: the modes



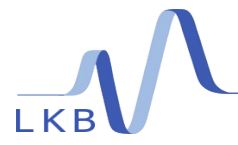
$\partial\omega/\partial k$ Group velocity of modes \rightarrow propagation w.r.t horizon



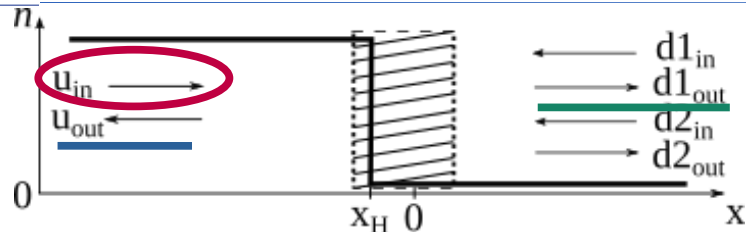
LKB Perturbation of black hole



Quasi-normal mode of the gravitational field

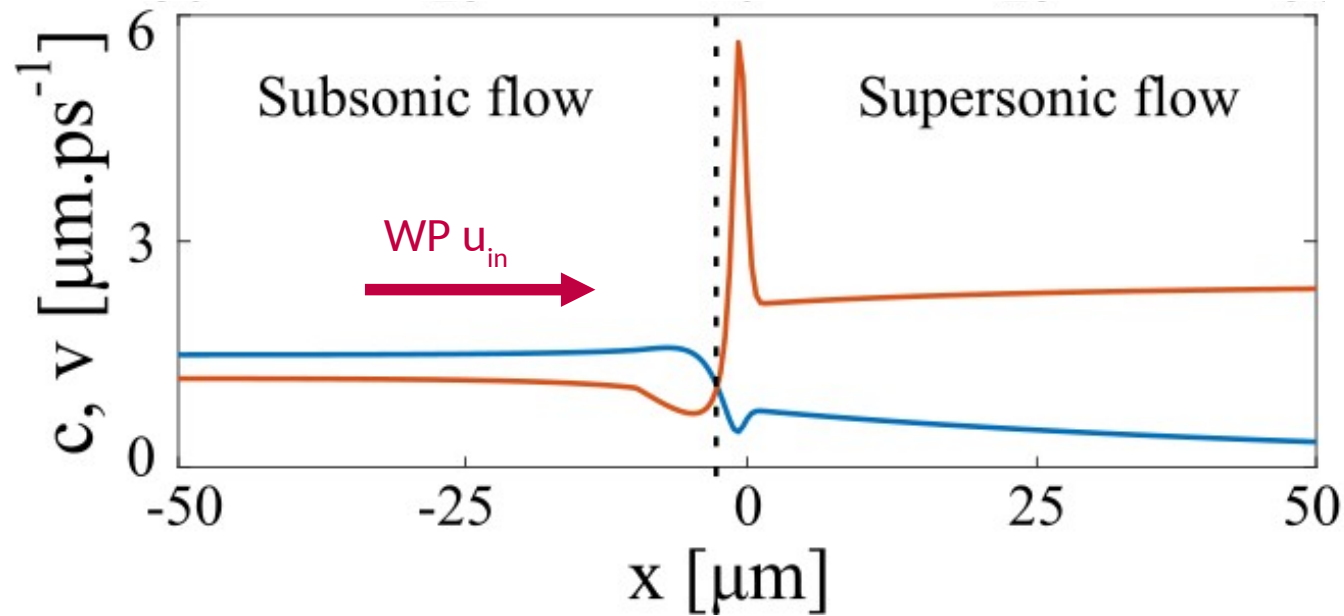
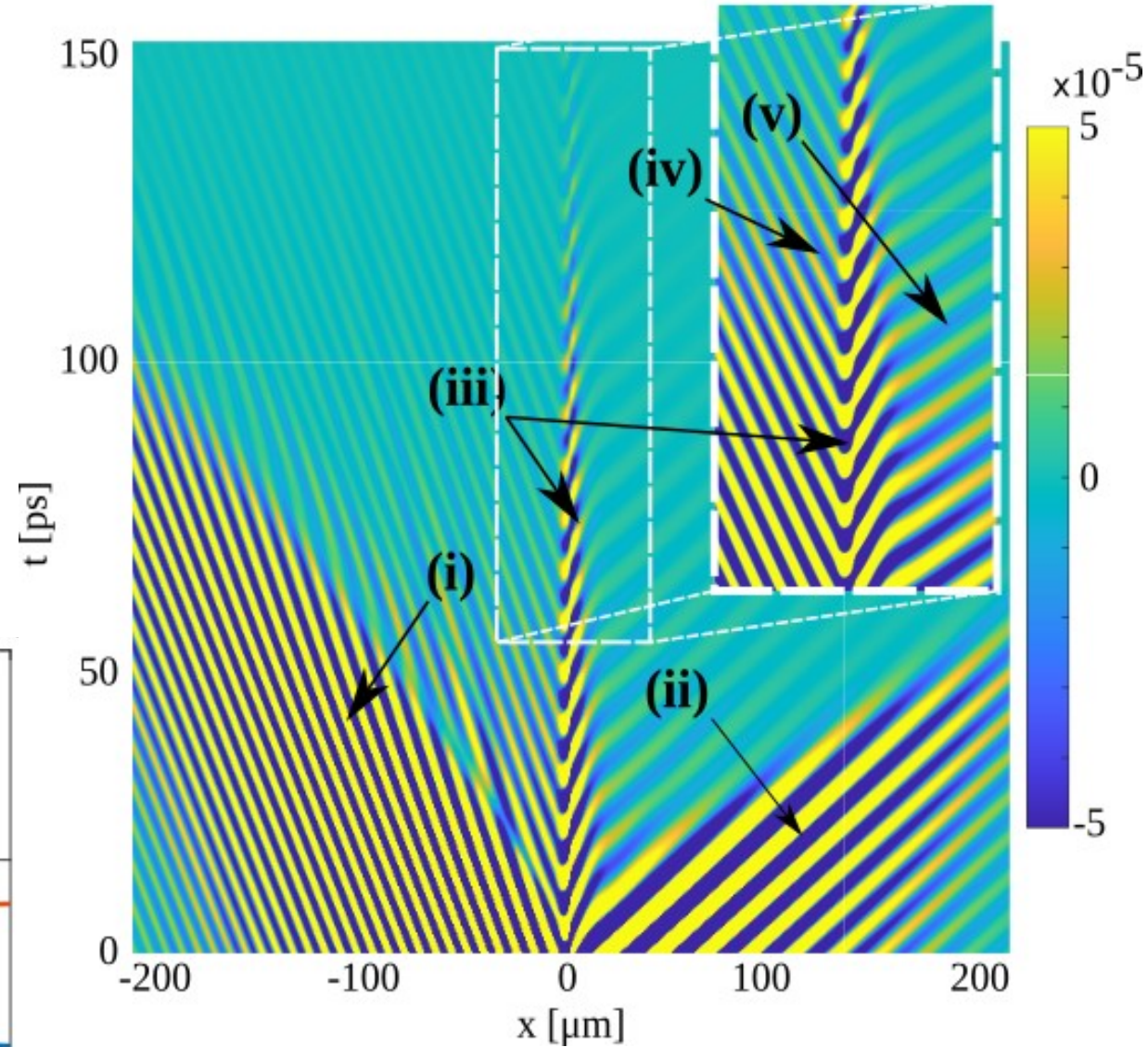


Perturb horizon with classical wavepacket



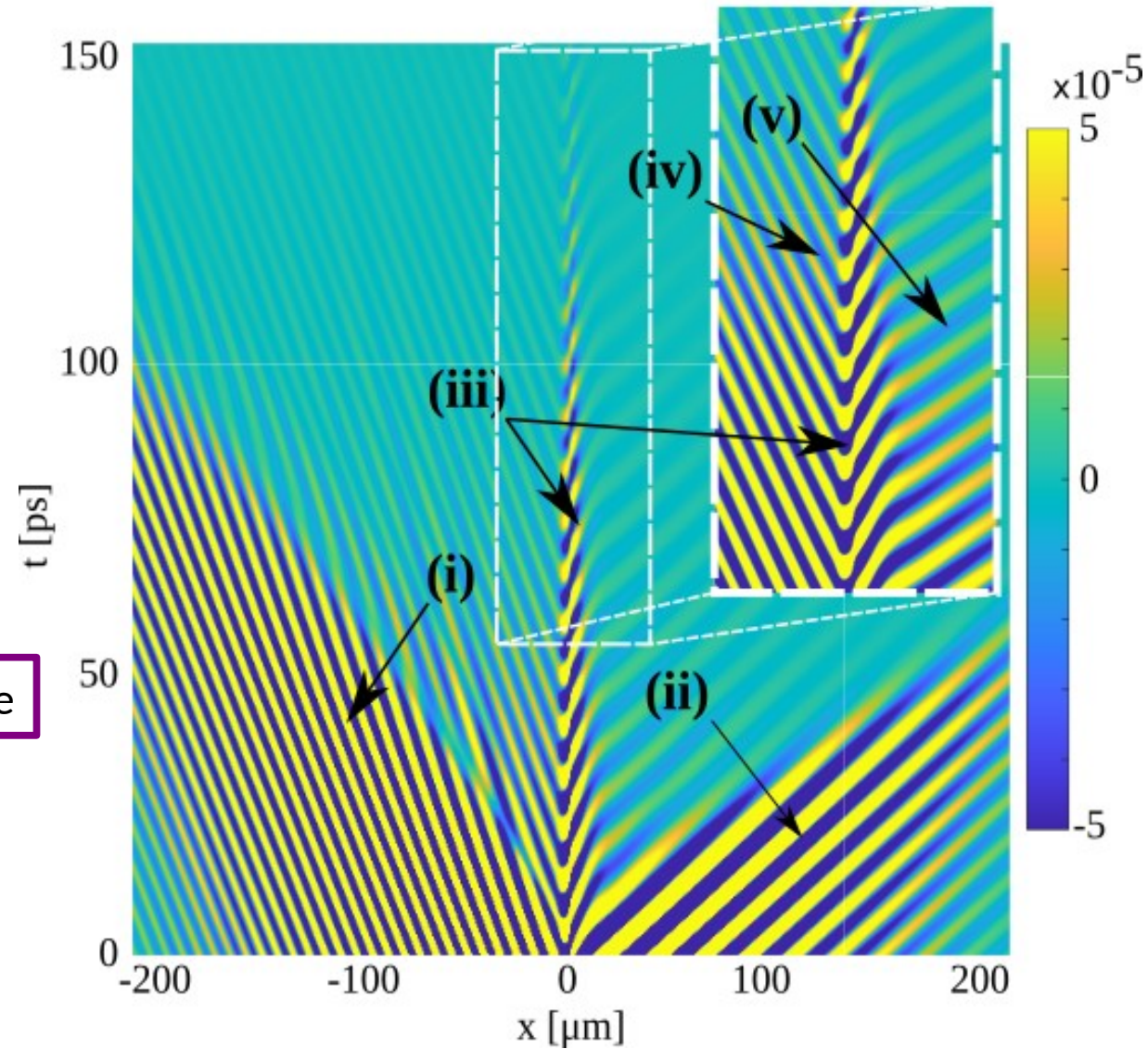
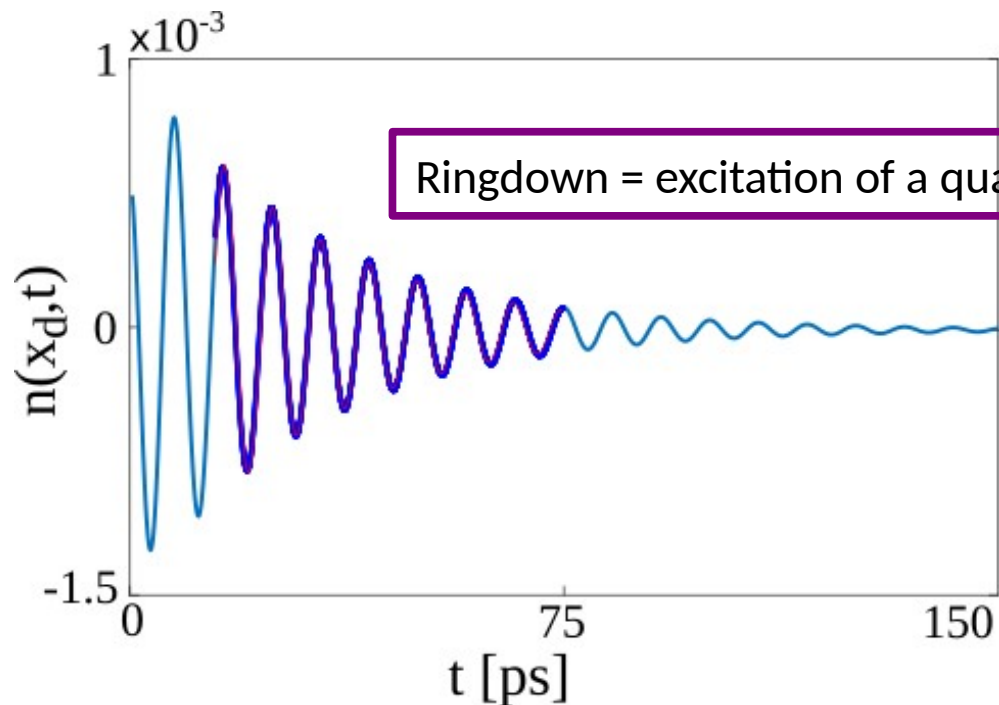
Send wavepacket u_{in} toward horizon:

- (i) reflection
- (ii) transmission
- (iii) density @horizon oscillates and dampens
- (iv) density @horizon couples with mode propagating outward
- (v) density @horizon couples with mode propagating inward



Send wavepacket u_{in} toward horizon:

- (i) reflection
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$\text{Re}(\omega)$ Frequency of oscillation
 $\text{Im}(\omega)$ Decay rate $< \gamma$ (bare polariton lifetime)

Numerical simulation: Truncated Wigner Approximation
(1 billion realisations)

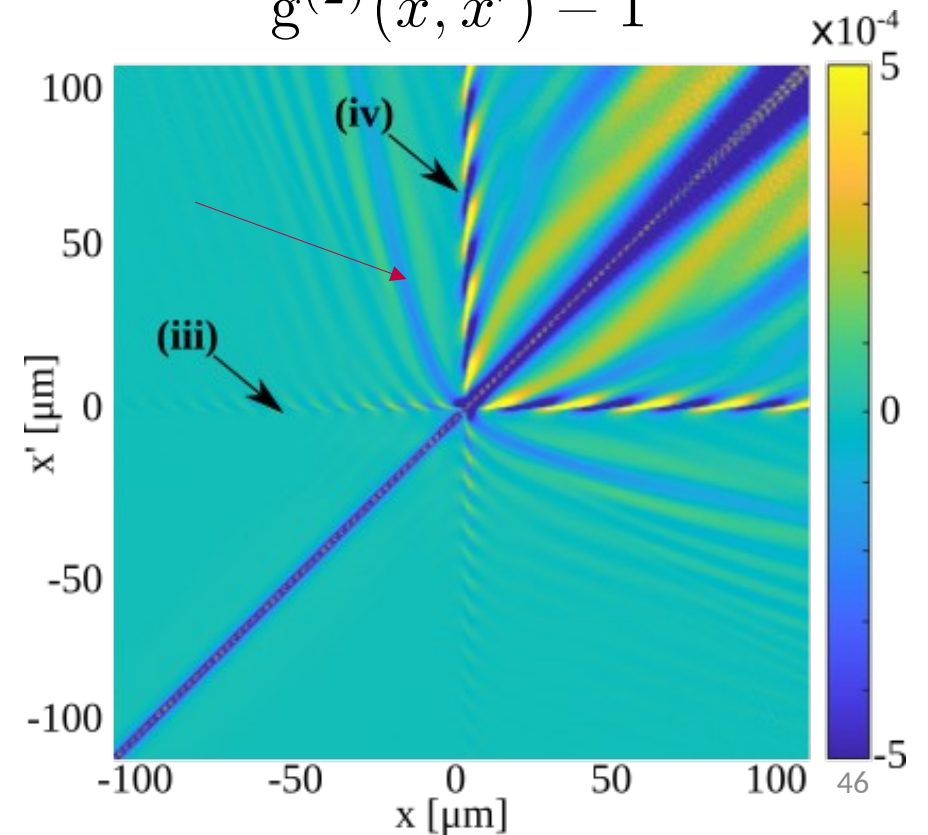
Measure equal time correlations

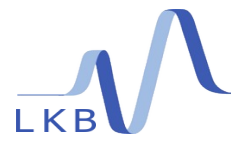
$$g^{(2)}(x, x') = \frac{\langle \Psi(x)^\dagger \Psi(x')^\dagger \Psi(x) \Psi(x') \rangle}{\langle \Psi(x)^\dagger \Psi(x) \rangle \langle \Psi(x')^\dagger \Psi(x') \rangle}$$

$$g^{(2)}(x, x') - 1$$

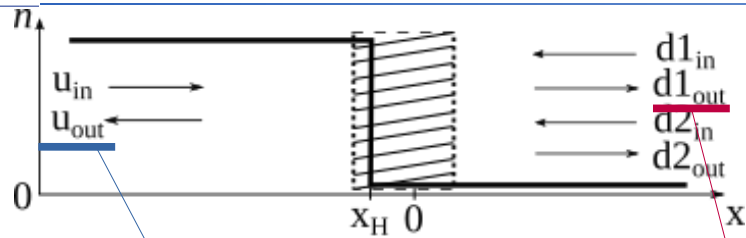
Hawking correlations

(iii) horizon - outside
(iv) horizon - inside



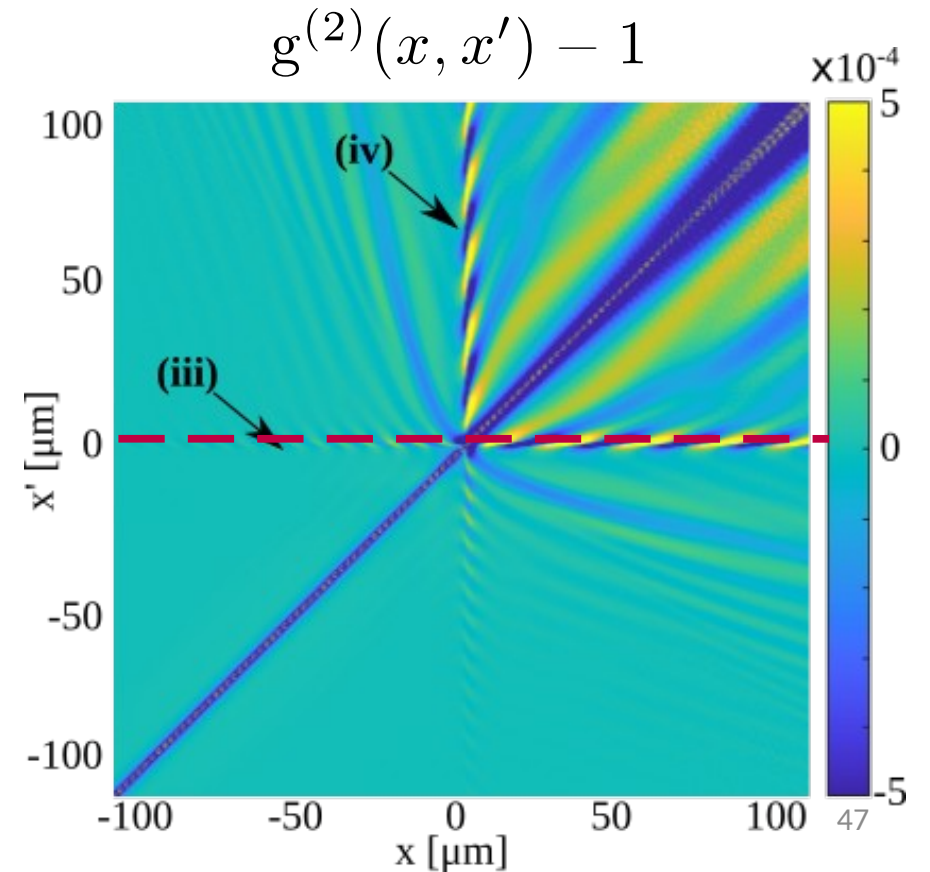
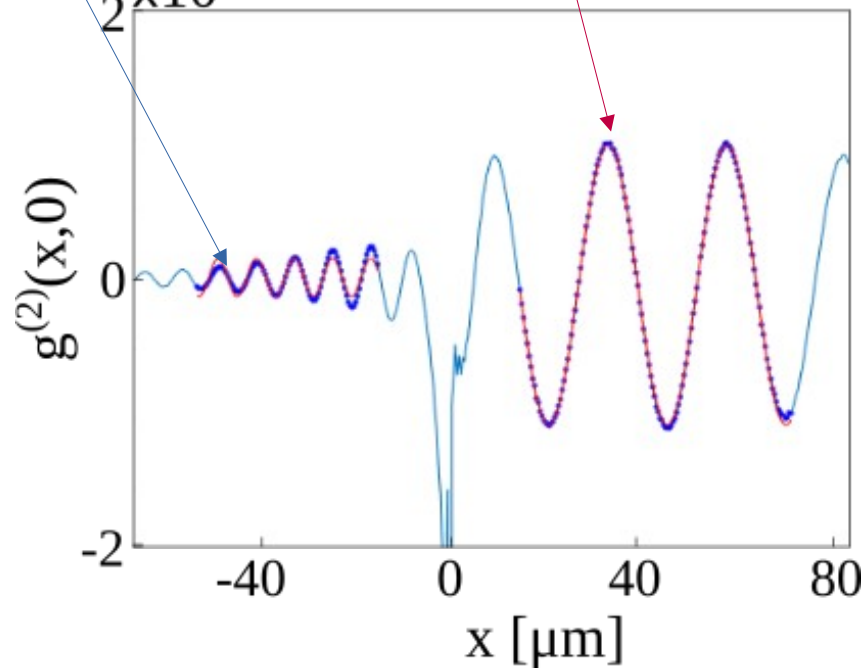


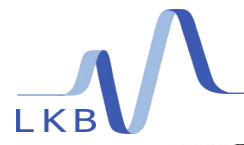
Scattering of vacuum fluctuations: horizon correlations



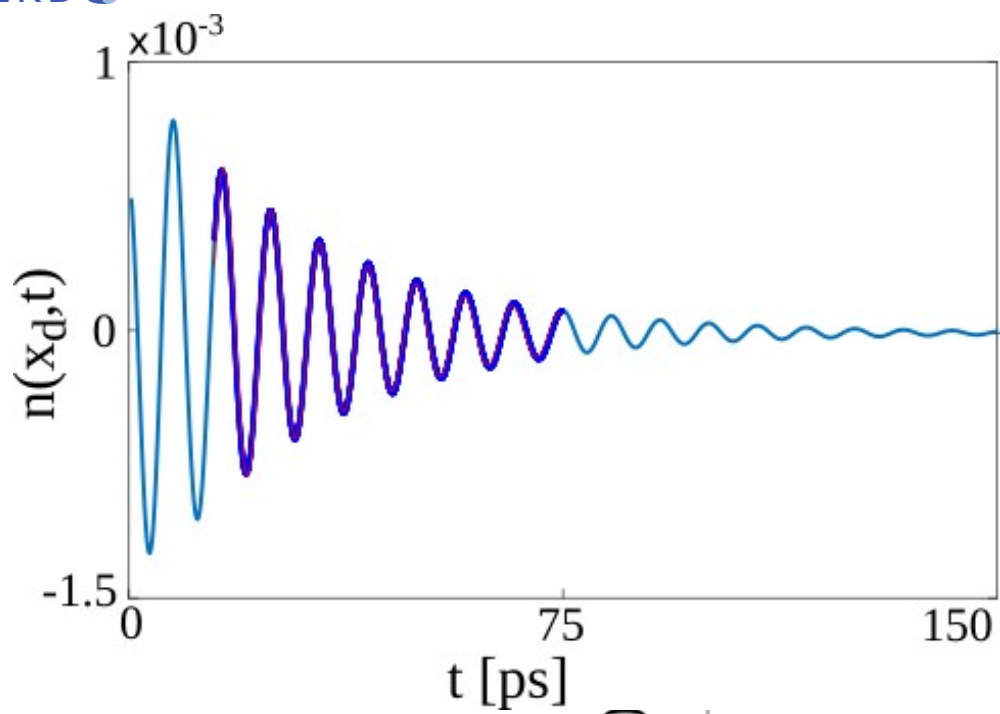
- (iii) horizon - outside
- (iv) horizon - inside

Spatial frequency?
 2×10^{-3}



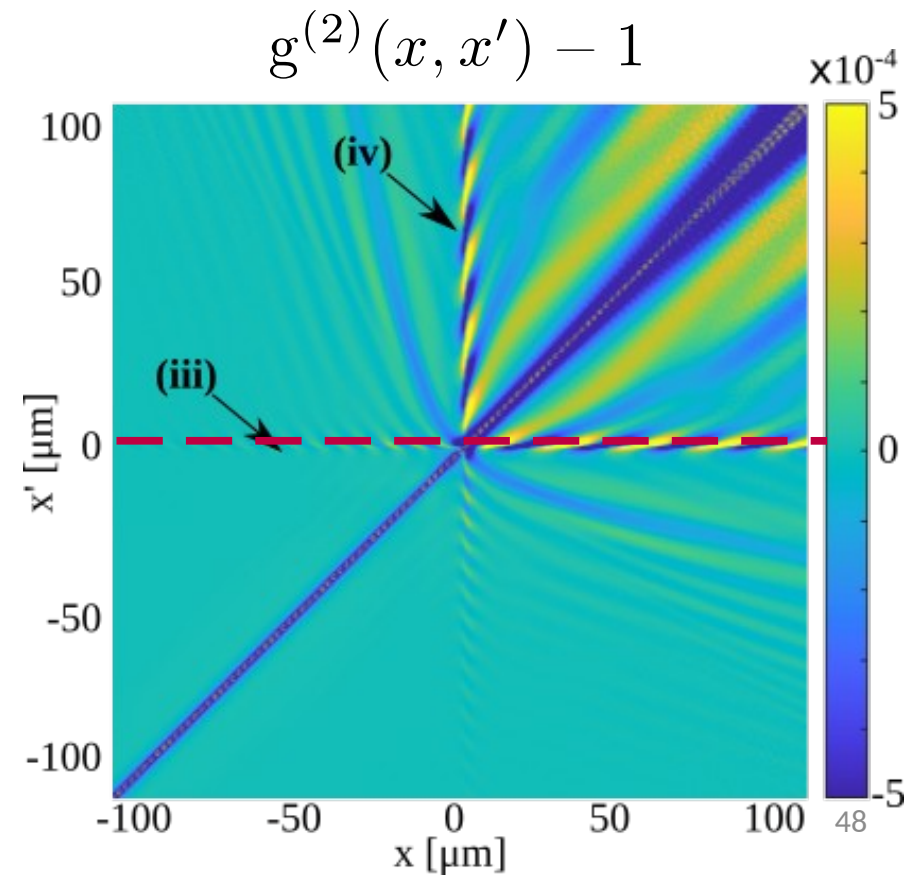
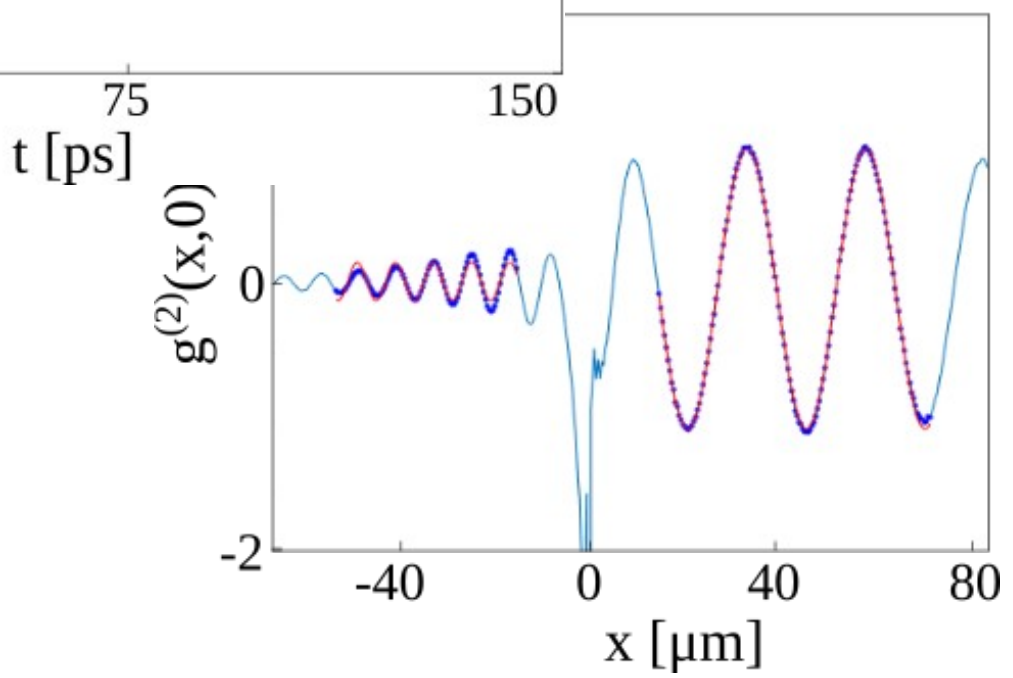


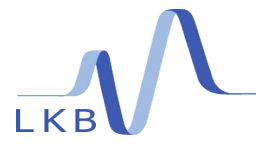
Scattering of vacuum fluctuations: excitation of a quasi normal mode



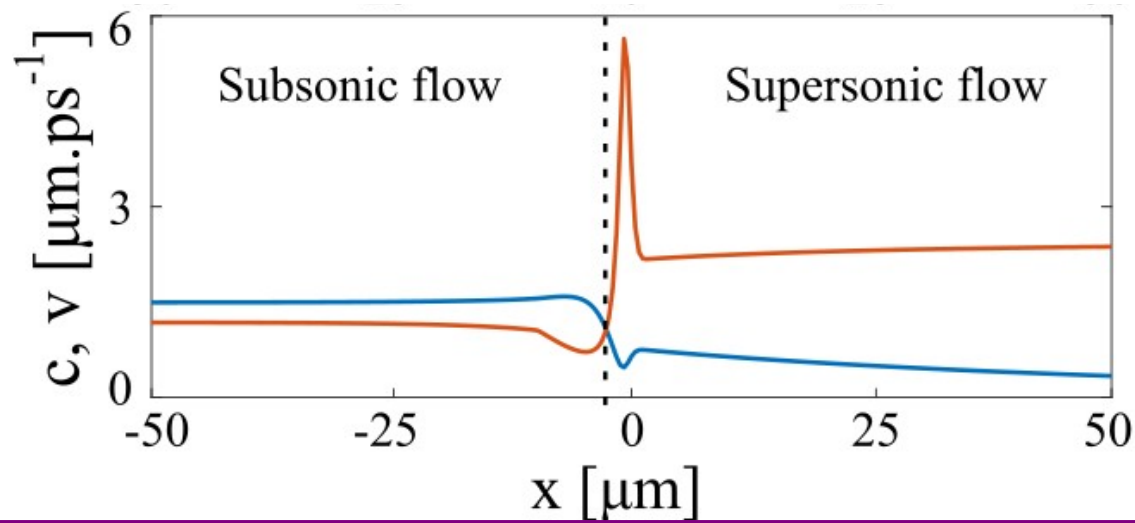
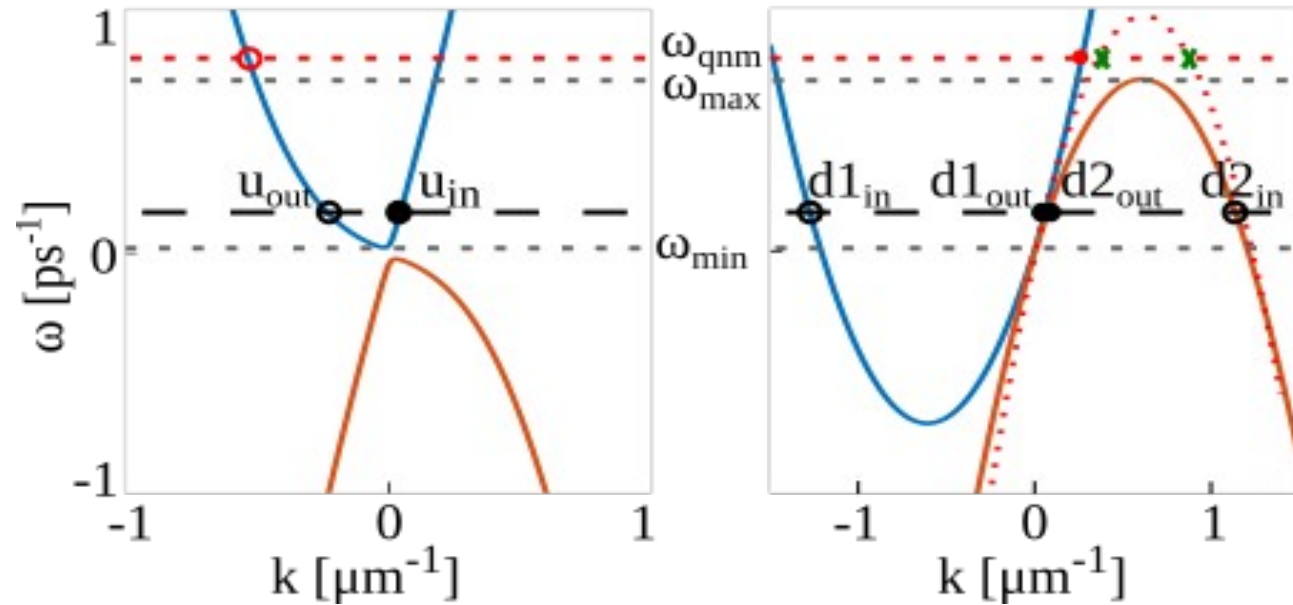
$\text{Re}(\omega)$ Frequency of oscillation
 $\text{Im}(\omega)$ Decay rate $< \gamma$

(iii) horizon - outside
(iv) horizon - inside
 → excitation of a quasi-normal mode

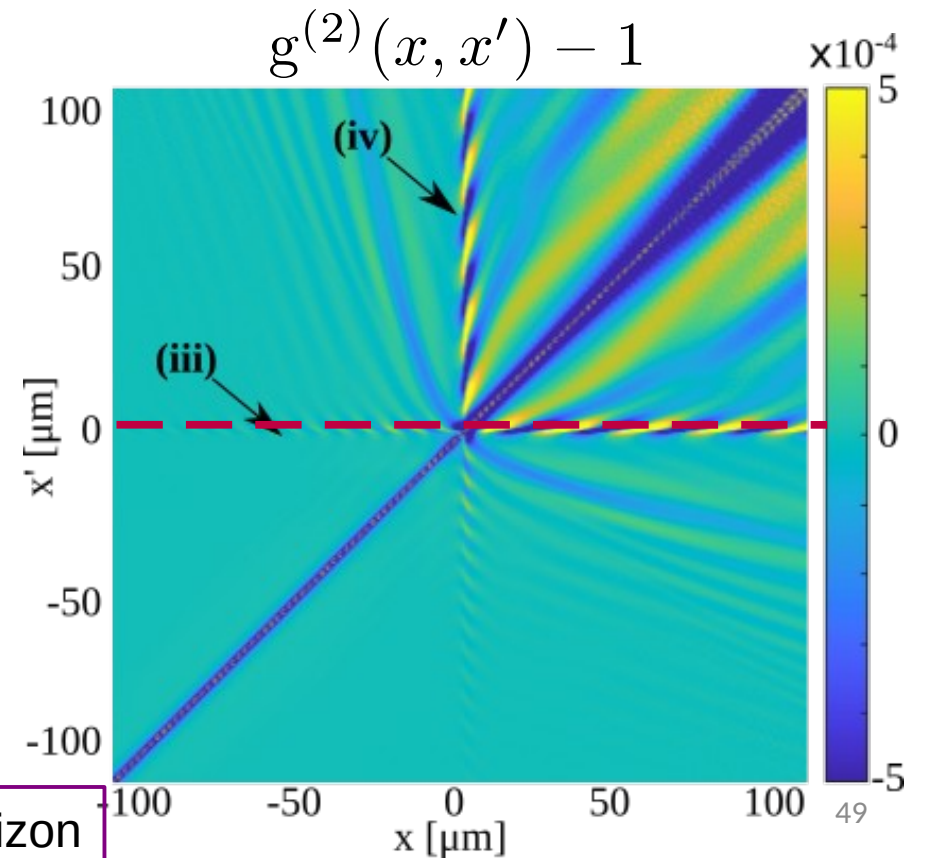




Scattering of vacuum fluctuations: effective potential



(iii) horizon - outside
(iv) horizon - inside



Velocity spike creates negative potential for negative modes near horizon



Spectral modulation

