An introduction to the localization landscape

Applications to semiconductor and cold atoms systems

Jean-Philippe Banon

Laboratoire de Physique de la Matière Condensée, CNRS, Ecole polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau, France

GdR Complexe Annual Workshop - Paris - Dec 07-09 2022

Context

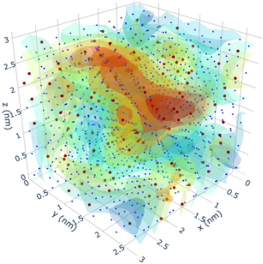
Localization of waves and the localization landscape

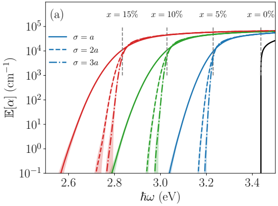
- Schrödinger equation (M. Filoche, S. Mayboroda)
- Membrane vibration (P. Sebbah)
- Gross-Pitaevskii equation (Filippo Stellin's poster)
- Tight-binding models (M. Filoche, F. Mortessagne)
- Random matrix (M. Filoche, S. Mayboroda, T. Tao)
- Dirac fermions (C. Beenakker)
- Acoustic waves (D. Colas, C. Bellis, B. Lombard, R. Cottereau)
- Electromagnetic waves? (F. Mortessagne)
- Magnetic Schrödinger (B. Poggi, Alioune Seye's poster)
- Many-body systems (V. Galitski)

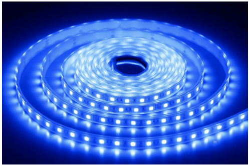
Focus on Schrödinger equation for **cold atoms** and **semiconductor** problems.

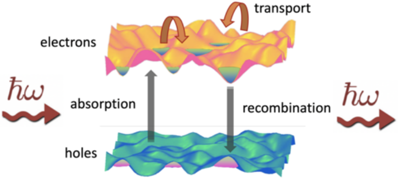
Context: Modeling radiative processes in disordered alloys

$In_xGa_{1-x}N$









Beer-Lambert law

$$I(z) = I_0 \exp(-\alpha(\omega)z)$$

Localization landscape

What is the localization landscape?

Original motivation: finding a bounding function to the eigenfunctions.

Schrödinger eigenvalue problem

$$-\frac{\hbar^2}{2m}\Delta\psi + V\psi = E\psi$$

Integral representation for ψ

$$\psi(\mathbf{r}) = \int G(\mathbf{r},\mathbf{r}')E\psi(\mathbf{r}')\,\mathrm{d}^d r'$$

Straightforward upper bound

$$|\psi(\mathbf{r})| \leq \int |G(\mathbf{r},\mathbf{r}')E\psi(\mathbf{r}')|\,\mathrm{d}^d r' \leq |E|\|\psi\|_\infty \int |G(\mathbf{r},\mathbf{r}')|\,\mathrm{d}^d r'$$

Hence

$$\frac{|\psi(\mathbf{r})|}{\|\psi\|_\infty} \leq |E|\mathcal{L}(\mathbf{r})$$

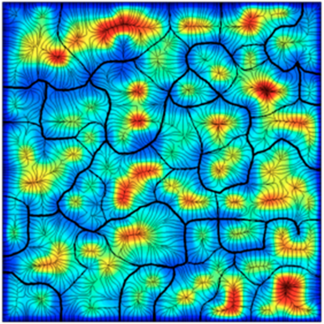
If $G \geq 0$, the landscape is easily obtained by solving

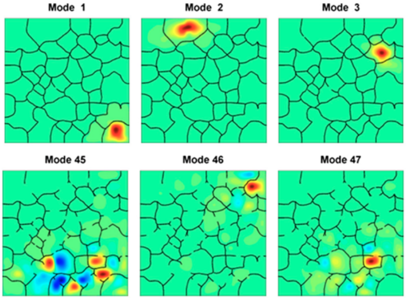
$$H\mathcal{L} = -\frac{\hbar^2}{2m}\Delta\mathcal{L} + V\mathcal{L} = 1$$

⋆ M. Filoche and S. Mayboroda, Proceedings of the National Academy of Sciences 109, 14761 (2012)

The landscape bounds the eigenstates

$$\frac{|\psi(\mathbf{r})|}{\|\psi\|_\infty} \leq |E|\mathcal{L}(\mathbf{r})$$





⋆ M. Filoche and S. Mayboroda, Proceedings of the National Academy of Sciences 109, 14761 (2012)

The spectral picture

Spectral decomposition of the localization landscape?

$$\mathcal{L}(\mathbf{r}) = \sum_k \mathcal{L}_k \psi_k(\mathbf{r})$$

By definition

$$1 = H\mathcal{L} = H \sum_k \mathcal{L}_k \psi_k = \sum_k \mathcal{L}_k E_k \psi_k$$

Project against ψ_n

$$\mathcal{L}_n E_n = \langle 1, \psi_n \rangle$$

Spectral decomposition

$$\mathcal{L}(\mathbf{r}) = \sum_k \frac{\langle 1, \psi_k \rangle}{E_k} \psi_k(\mathbf{r})$$

Consequences

"*Low energy landscape*"

- $1/E_k \to 0$
- $\langle 1, \psi_k \rangle \to 0$ (oscillating ψ_k)

$\rightarrow$ As $E_k \to \infty$, $\mathcal{L}_k \to 0$

"*Sum of local ground states*"

- If at low energy the ψ_k have almost disjoint supports Ω_k then

$\rightarrow$ Locally $\mathcal{L}|_{\Omega_k} \approx \frac{\langle \psi_k, 1 \rangle}{E_k} \psi_k$

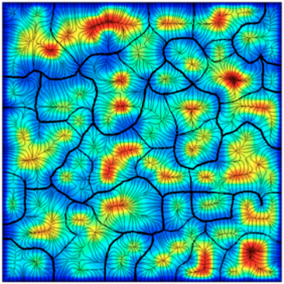
⋆ M. Piccardo *et al.* Phys. Rev. B **95**, 144205 (2017)

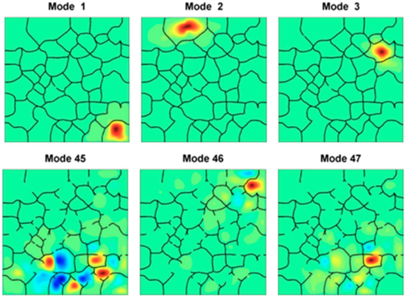
⋆ **Filippo Stellin's poster on localization for Gross-Pitaevskii**

The numerical linear algebra picture

Spectral decomposition

$$\mathcal{L}(\mathbf{r}) = \sum_k \frac{\langle 1, \psi_k \rangle}{E_k} \psi_k(\mathbf{r})$$





Ideas: construct a smart basis of functions based on the domain decomposition as good guesses for Rayleigh-Ritz iterations / solve eigenvalue problem on smaller subdomain / rank reduction.

The effective potential picture

$$H\mathcal{L} = 1$$

Change of unknown function $\psi = \mathcal{L}\varphi$

$$-\frac{\hbar^2}{2m}\Delta(\mathcal{L}\varphi) + V\mathcal{L}\varphi = E\mathcal{L}\varphi$$

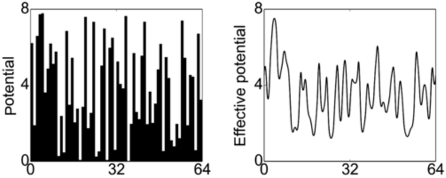
$$\vdots$$

Property for *any* ψ

$$\langle\psi|\, H\, |\psi\rangle = \frac{1}{2m}\langle\hat{\mathbf{p}}\,\psi|\,\hat{\mathbf{p}}\,\psi\rangle + \langle\psi|\, V\, |\psi\rangle$$

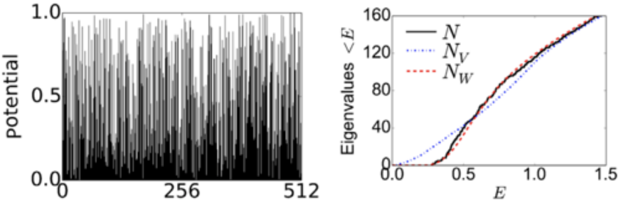
$$\langle\psi|\, H\, |\psi\rangle = \frac{1}{2m}\langle\mathcal{L}\,\hat{\mathbf{p}}\,\frac{\psi}{\mathcal{L}}|\mathcal{L}\,\hat{\mathbf{p}}\,\frac{\psi}{\mathcal{L}}\rangle + \langle\psi|\,\frac{1}{\mathcal{L}}\,|\psi\rangle$$

$$\underbrace{-\frac{\hbar^2}{2m}\frac{1}{\mathcal{L}^2}\nabla\cdot\left[\mathcal{L}^2\nabla\varphi\right]}_{\text{Eff. kinetic energy}} + \underbrace{\frac{1}{\mathcal{L}}}_{\text{Eff. potential}}\varphi = E\varphi$$



⋆ D. N. Arnold *et al.* Phys. Rev. Lett. 116, 056602 (2016)

The effective potential picture - Weyl's law



Weyl's law

$$\mathrm{IDOS}(E) \sim \iint_{\hbar^2 k^2/2m + V(\mathbf{r}) < E} \frac{\mathrm{d}^d r\, \mathrm{d}^d k}{(2\pi)^d}$$

Modified Weyl's law

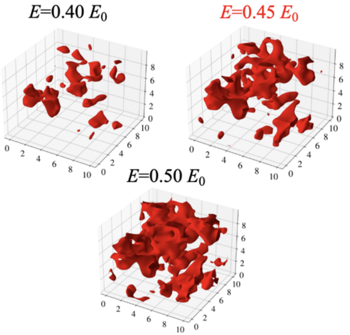
$$\mathrm{IDOS}(E) \sim \iint_{\hbar^2 k^2/2m + 1/\mathcal{L}(\mathbf{r}) < E} \frac{\mathrm{d}^d r\, \mathrm{d}^d k}{(2\pi)^d}$$

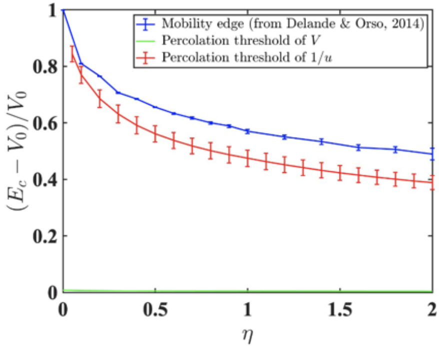
⋆ D. N. Arnold *et al.* Phys. Rev. Lett. 116, 056602 (2016)

⋆ D. N. Arnold *et al.* SIAM J. Sci. Comput., 41(1), B69–B92 (2019)

The **mobility edge** for 3D cold atoms systems can be estimated by the **percolation threshold of the effective potential**.

Conjecture: percolation threshold < mobility edge.





Courtesy of Pierre Pelletier.

A phase space description of the eigenstates

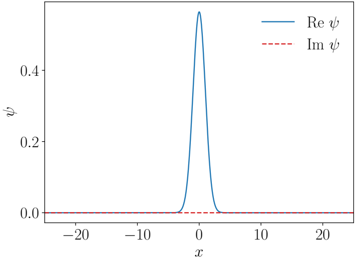
Wigner transform

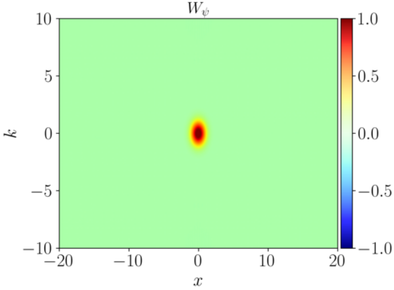
The *Wigner transform* of a function ψ is defined by

$$W_\psi(\mathbf{r},\mathbf{k}) = \int \psi^*\Big(\mathbf{r}-\frac{\mathbf{x}}{2}\Big)\psi\Big(\mathbf{r}+\frac{\mathbf{x}}{2}\Big)\ \exp\big(-i\,\mathbf{k}\cdot\mathbf{x}\big)\ \mathrm{d}^3x\,.$$

Examples

$$\psi(x) \propto \exp\left(-\frac{x^2}{2\sigma^2}\right) \qquad W_\psi(x,k) \propto \exp\left(-\frac{x^2}{\sigma^2} - \sigma^2 k^2\right)$$





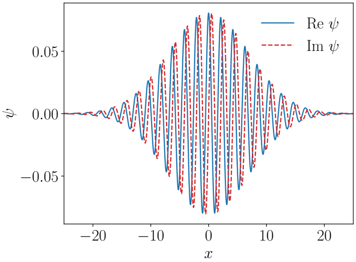
Wigner transform

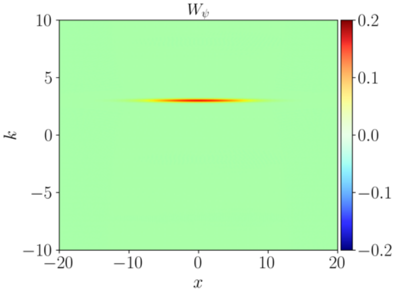
The *Wigner transform* of a function ψ is defined by

$$W_\psi(\mathbf{r},\mathbf{k}) = \int \psi^*\Big(\mathbf{r}-\frac{\mathbf{x}}{2}\Big)\psi\Big(\mathbf{r}+\frac{\mathbf{x}}{2}\Big)\,\exp\big(-i\,\mathbf{k}\cdot\mathbf{x}\big)\,\mathrm{d}^3x\,.$$

Examples

$$\psi(x) \propto \exp\left(-\frac{x^2}{2\sigma^2}\right)\exp(ik_0x) \qquad W_\psi(x,k) \propto \exp\left(-\frac{x^2}{\sigma^2} - \sigma^2(k-k_0)^2\right)$$





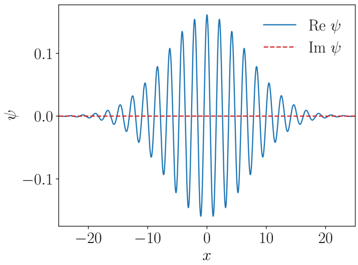
Wigner transform

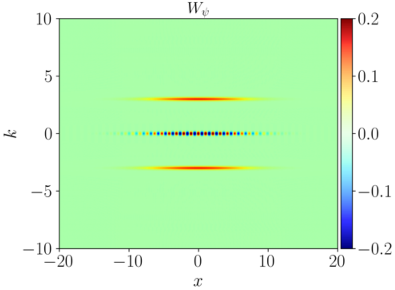
The *Wigner transform* of a function ψ is defined by

$$W_\psi(\mathbf{r},\mathbf{k}) = \int \psi^*\Big(\mathbf{r} - \frac{\mathbf{x}}{2}\Big)\psi\Big(\mathbf{r} + \frac{\mathbf{x}}{2}\Big)\ \exp\big(-i\,\mathbf{k}\cdot\mathbf{x}\big)\ \mathrm{d}^3 x\,.$$

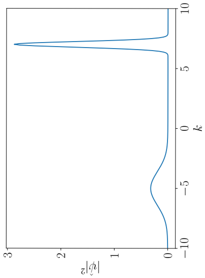
Examples

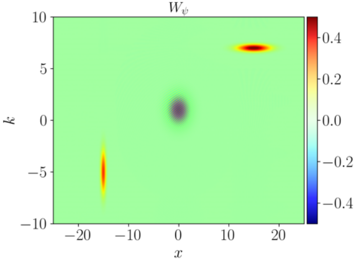
$$\psi(x) \propto \exp\left(-\frac{x^2}{2\sigma^2}\right)\cos(ik_0 x)$$

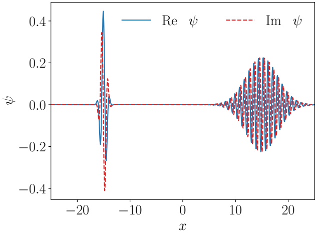




Properties of the Wigner transform







$$\int W_\psi(\mathbf{r},\mathbf{k})\,\mathrm{d}^d r = |\hat{\psi}(\mathbf{k})|^2$$

$$\int W_\psi(\mathbf{r},\mathbf{k})\,\frac{\mathrm{d}^d k}{(2\pi)^d} = |\psi(\mathbf{r})|^2$$

$$\iint W_\psi(\mathbf{r},\mathbf{k})\,\frac{\mathrm{d}^d r\,\mathrm{d}^d k}{(2\pi)^d} = 1$$

Density of states in phase space, a central object

Schrödinger eigenvalue problem

$$-\frac{\hbar^2}{2m}\Delta\psi_n + V\psi_n = E_n\psi_n$$

A list of usual densities of states

$$\mathrm{IDOS}(E) = \sum_n \Theta(E - E_n)$$

$$\mathrm{DOS}(E) = \sum_n \delta(E - E_n)$$

$$\mathrm{LDOS}(\mathbf{r}, E) = \sum_n |\psi_n(\mathbf{r})|^2 \delta(E - E_n)$$

$$A(\mathbf{k}, E) = \sum_n |\hat{\psi}_n(\mathbf{k})|^2 \delta(E - E_n)$$

Density of states in phase space, a central object

A density of states in *phase space* (φDOS)?

$$\mathcal{D}(\mathbf{r},\mathbf{k},E) = \sum_n W_{\psi_n}(\mathbf{r},\mathbf{k})\delta(E-E_n)$$

Link to the usual densities of states

$$\mathrm{IDOS}(E) = \sum_n \Theta(E-E_n) = \int_{-\infty}^{E} \iint \mathcal{D}(\mathbf{r},\mathbf{k},\varepsilon)\,\frac{\mathrm{d}^d r\,\mathrm{d}^d k}{(2\pi)^d}\,\mathrm{d}\varepsilon$$

$$\mathrm{DOS}(E) = \sum_n \delta(E-E_n) = \iint \mathcal{D}(\mathbf{r},\mathbf{k},E)\,\frac{\mathrm{d}^d r\,\mathrm{d}^d k}{(2\pi)^d}$$

$$\mathrm{LDOS}(\mathbf{r},E) = \sum_n |\psi_n(\mathbf{r})|^2\delta(E-E_n) = \int \mathcal{D}(\mathbf{r},\mathbf{k},E)\,\frac{\mathrm{d}^d k}{(2\pi)^d}$$

$$A(\mathbf{k},E) = \sum_n |\hat{\psi}_n(\mathbf{k})|^2\delta(E-E_n) = \int \mathcal{D}(\mathbf{r},\mathbf{k},E)\,\mathrm{d}^d r$$

IDOS and integrated φDOS

Interpretation of Weyl's law for the IDOS in phase-space.

Definition of the φDOS

$$\mathcal{D}(\mathbf{r},\mathbf{k},E) = \sum_n W_{\psi_n}(\mathbf{r},\mathbf{k})\delta(E-E_n)$$

Property

$$\iint \underbrace{\int_{-\infty}^{E} \mathcal{D}(\mathbf{r},\mathbf{k},\varepsilon)\,\mathrm{d}\varepsilon}_{\sum_{E_n<E} W_{\psi_n}(\mathbf{r},\mathbf{k})} \frac{\mathrm{d}^d r\,\mathrm{d}^d k}{(2\pi)^d} = \mathrm{IDOS}(E) \sim \underbrace{\iint_{\frac{\hbar^2 k^2}{2m}+V(\mathbf{r})<E} \frac{\mathrm{d}^d r\,\mathrm{d}^d k}{(2\pi)^d}}_{\text{Weyl's law}}$$

Conjecture / *plateau function approximation*

$$\int_{-\infty}^{E} \mathcal{D}(\mathbf{r},\mathbf{k},\varepsilon)\,\mathrm{d}\varepsilon \sim \Theta\left(E - \frac{\hbar^2 k^2}{2m} - V(\mathbf{r})\right)$$

IDOS and integrated φDOS

Interpretation of landscape-based Weyl's law for the IDOS in phase-space.

Definition of the φDOS

$$\mathcal{D}(\mathbf{r},\mathbf{k},E) = \sum_n W_{\psi_n}(\mathbf{r},\mathbf{k})\delta(E-E_n)$$

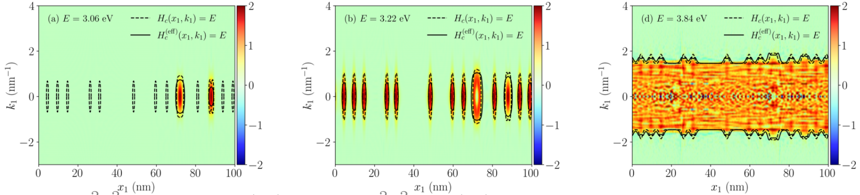
Property

$$\iint \underbrace{\int_{-\infty}^{E} \mathcal{D}(\mathbf{r},\mathbf{k},\varepsilon)\,\mathrm{d}\varepsilon}_{\sum_{E_n<E} W_{\psi_n}(\mathbf{r},\mathbf{k})} \frac{\mathrm{d}^d r\,\mathrm{d}^d k}{(2\pi)^d} = \mathrm{IDOS}(E) \approx \iint_{\frac{\hbar^2 k^2}{2m}+V^{(\mathrm{eff})}(\mathbf{r})<E} \frac{\mathrm{d}^d r\,\mathrm{d}^d k}{(2\pi)^d}$$

Conjecture / *plateau function approximation*

$$\int_{-\infty}^{E} \mathcal{D}(\mathbf{r},\mathbf{k},\varepsilon)\,\mathrm{d}\varepsilon \approx \Theta\left(E - \frac{\hbar^2 k^2}{2m} - V^{(\mathrm{eff})}(\mathbf{r})\right)$$

The plateau function approx. in a disordered potential



$H(\mathbf{r}, \mathbf{k}) = \frac{\hbar^2 k^2}{2m} + V(\mathbf{r})$, & $H^{(\mathrm{eff})}(\mathbf{r}, \mathbf{k}) = \frac{\hbar^2 k^2}{2m} + V^{(\mathrm{eff})}(\mathbf{r})$

Since we have

$$\int_{-\infty}^{E} \mathcal{D}(\mathbf{r}, \mathbf{k}, \varepsilon)\, \mathrm{d}\varepsilon \approx \Theta\left(E - \frac{\hbar^2 k^2}{2m} - V^{(\mathrm{eff})}(\mathbf{r})\right)$$

then by differentiating with respect to E we get

$$\mathcal{D}(\mathbf{r}, \mathbf{k}, \varepsilon) \approx \delta\left(E - \frac{\hbar^2 k^2}{2m} - V^{(\mathrm{eff})}(\mathbf{r})\right)$$

The φDOS is approximately a Dirac layer on the hypersurface $E = \frac{\hbar^2 k^2}{2m} + V^{(\mathrm{eff})}(\mathbf{r})$.

⋆ J.-P. Banon, P. Pelletier, C. Weisbuch, S. Mayboroda, M. Filoche, Phys. Rev. B **105**, 125422 (2022).

Application 1: local density of states

From the property

$$\mathrm{LDOS}(\mathbf{r},E) = \sum_n |\psi_n(\mathbf{r})|^2 \delta(E - E_n) = \int \mathcal{D}(\mathbf{r},\mathbf{k},E)\, \frac{\mathrm{d}^d k}{(2\pi)^d}$$

and the Dirac layer approximation

$$\mathcal{D}(\mathbf{r},\mathbf{k},\varepsilon) \approx \delta\left(E - \frac{\hbar^2 k^2}{2m} - V^{(\mathrm{eff})}(\mathbf{r})\right)$$

we get after integration over $\mathbf{k}$

$$\mathrm{LDOS}(\mathbf{r},E) = \frac{d v_d}{2(2\pi)^d}\left[\frac{2m}{\hbar^2}\right]^{d/2}\left(E - V^{(\mathrm{eff})}(\mathbf{r})\right)_+^{d/2-1}$$

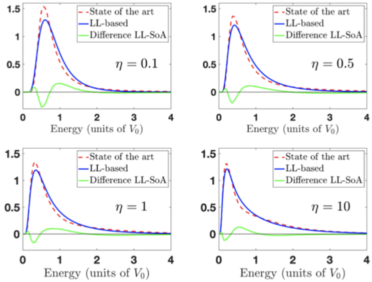
We recover the formula suggested in

⋆ M. Filoche, M. Piccardo, Y.-R. Wu, C.-K. Li, C. Weisbuch, and S. Mayboroda, Phys. Rev. B **95**, 144204 (2017).

Application 2: spectral function in cold atom systems

Spectral function

$$\begin{aligned}A(\mathbf{k},E) &= \mathbb{E}\left[\sum_n |\langle \mathbf{k}\,|\psi_n\rangle|^2 \delta(E-E_n)\right]\\ &= \mathbb{E}\left[\int \mathcal{D}(\mathbf{r},\mathbf{k},E)\,\frac{\mathrm{d}^d r}{|\Omega|}\right]\\ &\approx \mathbb{E}\left[\int \delta\left(E-\frac{\hbar^2k^2}{2m}-V^{(\mathrm{eff})}(\mathbf{r})\right)\,\frac{\mathrm{d}^d r}{|\Omega|}\right]\\ &= P_{V^{(\mathrm{eff})}}\left(E-\frac{\hbar^2k^2}{2m}\right)\end{aligned}$$



$P_{V^{(\mathrm{eff})}}$: probability density for $V^{(\mathrm{eff})}$.

Reminder: $\eta = mV_0\ell_c^2/\hbar^2$.

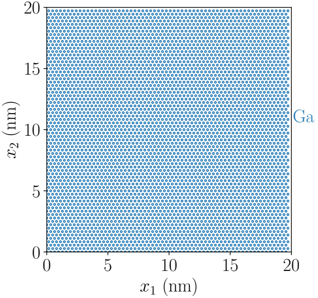
⋆ P. Pelletier, D. Delande, V. Josse, A. Aspect, S. Mayboroda, D. N. Arnold, and M. Filoche, *Spectral functions and localization-landscape theory in speckle potentials*, Phys. Rev. A **105**, 023314 (2022)

Absorption and emission of light in disordered semiconductors

Strategy

Fermi Golden rule $\underset{\text{exact}}{\longrightarrow}$ Phase space formulation $\underset{\text{approx.}}{\longrightarrow}$ Weyl-landscape law

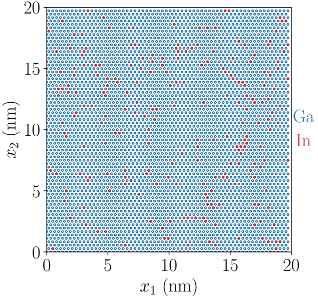
From ordered GaN...



Eigenstates in the conduction band of a periodic potential (Bloch waves)

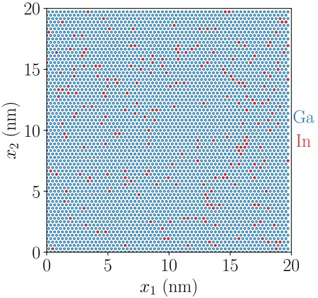
$$\psi_\mu^{(c)}(\mathbf{r}) = \underbrace{u_c(\mathbf{r})}_{\text{cell function}} \underbrace{\exp(i\,\mathbf{k}_\mu \cdot \mathbf{r})}_{\text{plane wave}}$$

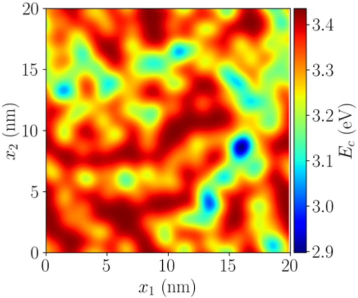
From ordered GaN... to disordered InGaN



Eigenstates in the conduction band of a periodic potential (Bloch waves)

$$\psi_\mu^{(c)}(\mathbf{r}) = \underbrace{u_c(\mathbf{r})}_{\text{cell function}} \underbrace{\exp(i\mathbf{k}_\mu \cdot \mathbf{r})}_{\text{plane wave}}$$





Eigenstates in the conduction band in the *effective mass approximation*

$$\psi_\mu^{(c)}(\mathbf{r}) = \underbrace{u_c(\mathbf{r})}_{\text{cell function}} \underbrace{\chi_\mu^{(c)}(\mathbf{r})}_{\text{envelope}} \quad \text{and} \quad \psi_\nu^{(v)}(\mathbf{r}) = u_v(\mathbf{r})\, \chi_\nu^{(v)}(\mathbf{r})$$

Schrödinger equation $$-\frac{\hbar^2}{2}\nabla\cdot\left[\frac{\nabla\chi_\mu^{(c)}}{m_c}\right] + V_c\,\chi_\mu^{(c)} = E_\mu^{(c)}\,\chi_\mu^{(c)}$$

Associated landscape $-\frac{\hbar^2}{2}\nabla\cdot\left[\frac{\nabla\mathcal{L}_c}{m_c}\right] + V_c\mathcal{L}_c = 1$ and effective potential $V_c^{(\text{eff})} = 1/\mathcal{L}_c$

Absorption coefficient

In the dipole approximation it can be shown that

$$\alpha(\omega) \propto \sum_{\mu,\nu} \underbrace{\left| \langle \chi_\mu^{(c)} | \chi_\nu^{(v)} \rangle \right|^2}_{\text{coupling}} \underbrace{\delta\left(E_\mu^{(c)} - E_\nu^{(v)} - \hbar\omega\right)}_{\text{energy conservation}}$$

$\Longrightarrow$ Solving two eigenvalue problems... numerically costly.

Absorption coefficient

Absorption coefficient

$$\alpha(\omega) \propto \sum_{\mu,\nu} \left| \langle \chi_\mu^{(c)} | \chi_\nu^{(v)} \rangle \right|^2 \delta\left(E_\mu^{(c)} - E_\nu^{(v)} - \hbar\omega\right)$$

Moyal formula

$$\left| \langle \chi_\mu^{(c)} | \chi_\nu^{(v)} \rangle \right|^2 = \iint \underbrace{W_{\chi_\mu^{(c)}}(\mathbf{r},\mathbf{k}) W_{\chi_\nu^{(v)}}(\mathbf{r},\mathbf{k})}_{\text{Wigner transforms}} \frac{\mathrm{d}^3 r\, \mathrm{d}^3 k}{(2\pi)^3}$$

Decoupling energies

$$\delta(E_\mu^{(c)} - E_\nu^{(v)} - \hbar\omega) = \int \delta(E_\mu^{(c)} - \hbar\omega - \varepsilon)\delta(E_\nu^{(v)} - \varepsilon)\, \mathrm{d}\varepsilon$$

We obtain

$$\alpha(\omega) \propto \iiint \underbrace{\sum_\mu W_{\chi_\mu^{(c)}}(\mathbf{r},\mathbf{k})\, \delta(E_\mu^{(c)} - \hbar\omega - \varepsilon)}_{\mathcal{D}^{(c)}(\mathbf{r},\mathbf{k},\varepsilon+\hbar\omega)} \underbrace{\sum_\nu W_{\chi_\nu^{(v)}}(\mathbf{r},\mathbf{k})\, \delta(E_\nu^{(v)} - \varepsilon)}_{\mathcal{D}^{(v)}(\mathbf{r},\mathbf{k},\varepsilon)}\, \mathrm{d}\varepsilon\, \frac{\mathrm{d}^3 r\, \mathrm{d}^3 k}{(2\pi)^3}$$

φDOS for the conduction and valence bands!

Formulation in phase space

Absorption coefficient

$$\alpha(\omega) \propto \sum_{\mu,\nu} \underbrace{\left| \langle \chi_\mu^{(c)} | \chi_\nu^{(v)} \rangle \right|^2}_{\text{coupling}} \underbrace{\delta\left(E_\mu^{(c)} - E_\nu^{(v)} - \hbar\omega\right)}_{\text{energy conservation}}$$

is equivalent to

$$\alpha(\omega) \propto \iiint \mathcal{D}^{(c)}(\mathbf{r}, \mathbf{k}, \varepsilon + \hbar\omega)\, \mathcal{D}^{(v)}(\mathbf{r}, \mathbf{k}, \varepsilon)\; \mathrm{d}\varepsilon\, \frac{\mathrm{d}^3 r\, \mathrm{d}^3 k}{(2\pi)^3}$$

Densities of states in phase space

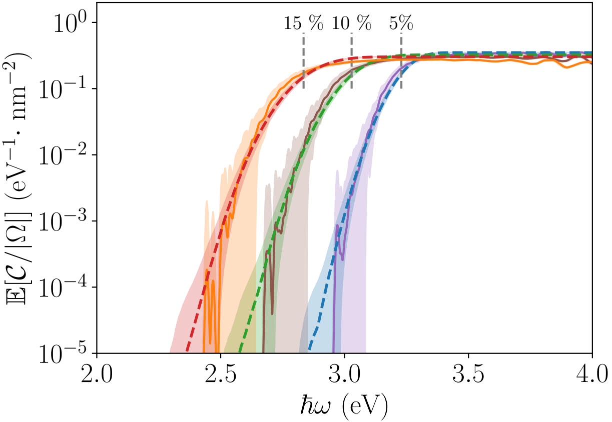
$$\mathcal{D}^{(c)}(\mathbf{r}, \mathbf{k}, E) \approx \delta\left(E - \frac{\hbar^2 |\,\mathbf{k}\,|^2}{2m_c(\mathbf{r})} - V_c^{(\text{eff})}(\mathbf{r})\right)$$

$$\mathcal{D}^{(v)}(\mathbf{r}, \mathbf{k}, E) \approx \delta\left(E - \frac{\hbar^2 |\,\mathbf{k}\,|^2}{2m_v(\mathbf{r})} - V_v^{(\text{eff})}(\mathbf{r})\right)$$

which yields the closed form expression for α in (3D)

$$\alpha(\omega) \propto \int m_r^{3/2}(\mathbf{r}) \sqrt{\left(\hbar\omega - E_g^{(\text{eff})}(\mathbf{r})\right)_+}\, \mathrm{d}r$$

Simulated absorption curves



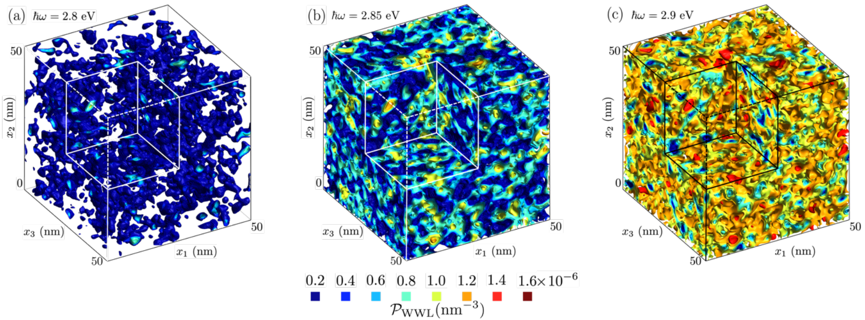
Normalized absorption coefficient spectra for 2D alloys of $\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{N}$ averaged over 100 realizations. Domain size 50 nm × 50 nm.

Computational speed-up ≈ 300.

⋆ J.-P. Banon, P. Pelletier, C. Weisbuch, S. Mayboroda, M. Filoche, Phys. Rev. B **105**, 125422 (2022).

Absorbed power density

$\hbar\omega = 2.80$ eV $\hbar\omega = 2.85$ eV $\hbar\omega = 2.9$ eV



Domain size: 50 nm × 50 nm × 50 nm. Step size $\Delta x = 3$ Å.

⋆ J.-P. Banon, P. Pelletier, C. Weisbuch, S. Mayboroda, M. Filoche, Phys. Rev. B **105**, 125422 (2022).

Acknowledgements

Generally all the members of the Simons collaboration on localization of waves, and more specifically for the work on light absorption

PMC

- Pr. Marcel Filoche
- Pierre Pelletier
- Abel Thayil
- Alioune Seye
- Pr. Jacques Peretti
- Dr Mylène Sauty

University of Minnesota

- Pr. Svitlana Mayboroda
- Pr. Douglas Arnold
- Dr Tyson Loudon

UC Santa Barbara

- Pr. Claude Weisbuch
- Pr. James Speck

University of Cambridge

- Pr. Richard Friend
- Dr Yun Liu

Fundings: Work supported by the Simons foundation grant 601944.

Thank you for your attention.

A few references to know more about ...

... the original paper on the localization landscape

⋆ M. Filoche and S. Mayboroda, Proceedings of the National Academy of Sciences 109, 14761 (2012)

... the effective potential and the modified Weyl law for the IDOS

⋆ D. N. Arnold, G. David, D. Jerison, S. Mayboroda, and M. Filoche, Phys. Rev. Lett. 116, 056602 (2016)

⋆ D. N. Arnold, G. David, M. Filoche, D. Jerison, and S. Mayboroda, SIAM Journal on Scientific Computing 41, B69 (2019)

... absorption in disordered semiconductors

⋆ J.-P. Banon, P. Pelletier, C. Weisbuch, S. Mayboroda, M. Filoche, Phys. Rev. B **105**, 125422 (2022).

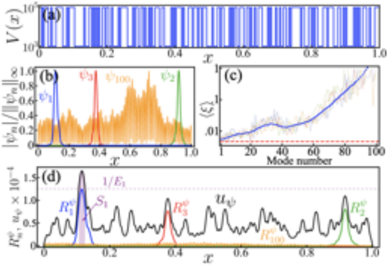
... spectral functions in cold atoms systems

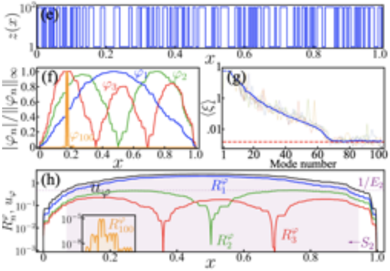
⋆ P. Pelletier, D. Delande, V. Josse, A. Aspect, S. Mayboroda, D. N. Arnold, and M. Filoche, Phys. Rev. A **105**, 023314 (2022)

Warning! Low energy $\neq$ localized

The original localization landscape for the acoustic wave equation is not informative.

Schrödinger equation	Acoustic wave equation
$$-\frac{\hbar^2}{2m}\Delta\psi_n + V\psi_n = E_n\psi_n$$	$$-\frac{1}{\rho}\nabla\cdot(\kappa\nabla\phi_n) = \omega_n^2\phi_n$$
Landscape equation	(Naive) landscape equation
$$-\frac{\hbar^2}{2m}\Delta u + Vu = 1$$	$$-\frac{1}{\rho}\nabla\cdot(\kappa\nabla u) = 1$$





⋆ David Colas, Cédric Bellis, Bruno Lombard, and Régis Cottereau, arXiv:2204.11632v1 (2022)

Acoustic landscapes

Webster transformation $\psi = \sqrt{z}\phi$ we go from

$$-\frac{1}{\rho}\nabla \cdot (\kappa \nabla \phi_n) = \omega_n^2 \phi_n$$

to

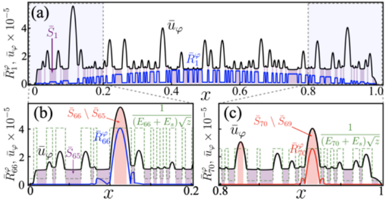
$$-\frac{1}{\sqrt{z}}\nabla \cdot \left(z \nabla \frac{\psi_n}{\sqrt{z}} \right) = E_n \psi_n$$

and an appropriate landscape is

$$-\frac{1}{\sqrt{z}}\nabla \cdot \left(z \nabla \frac{u_\psi}{\sqrt{z}} \right) + E_s u_\psi = 1$$

or equivalently in the "original" frame

$$-\nabla \cdot (z \nabla u_\phi) + E_s z u_\phi = \sqrt{z}$$



⋆ David Colas, Cédric Bellis, Bruno Lombard, and Régis Cottereau, arXiv:2204.11632v1 (2022)