

# Superradiant phase transition in free-space atomic clouds

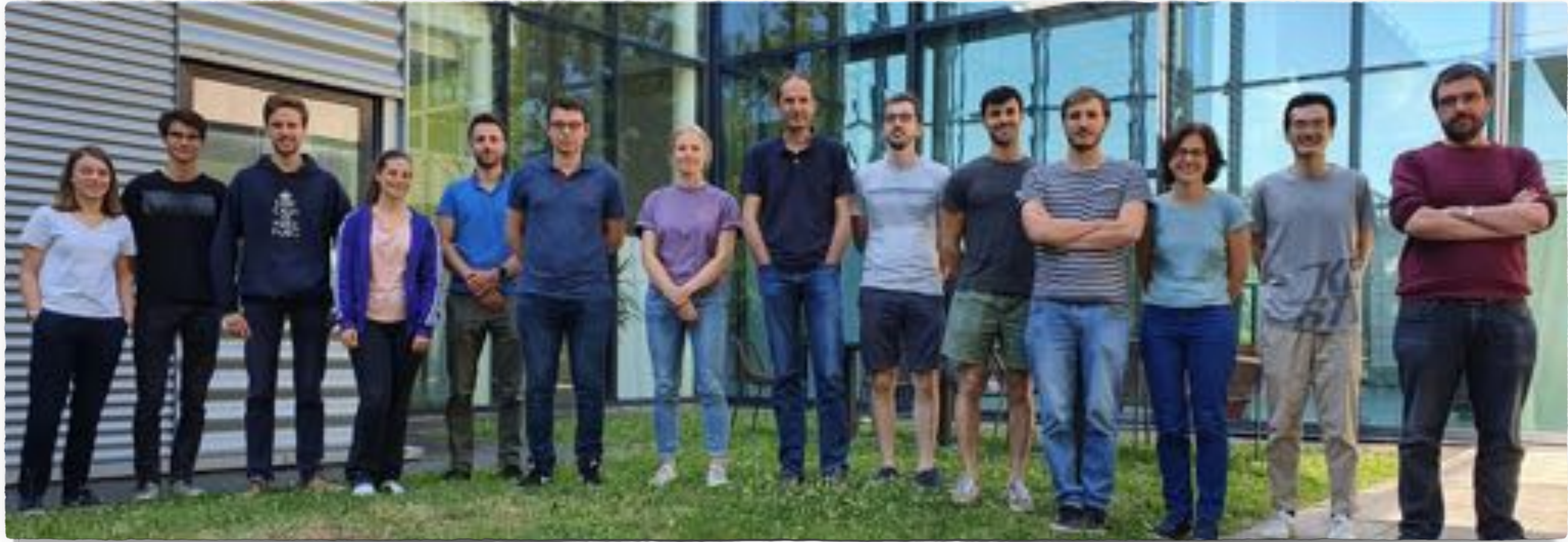
*Igor Ferrier-Barbut*

Laboratoire Charles Fabry, Institut d'Optique





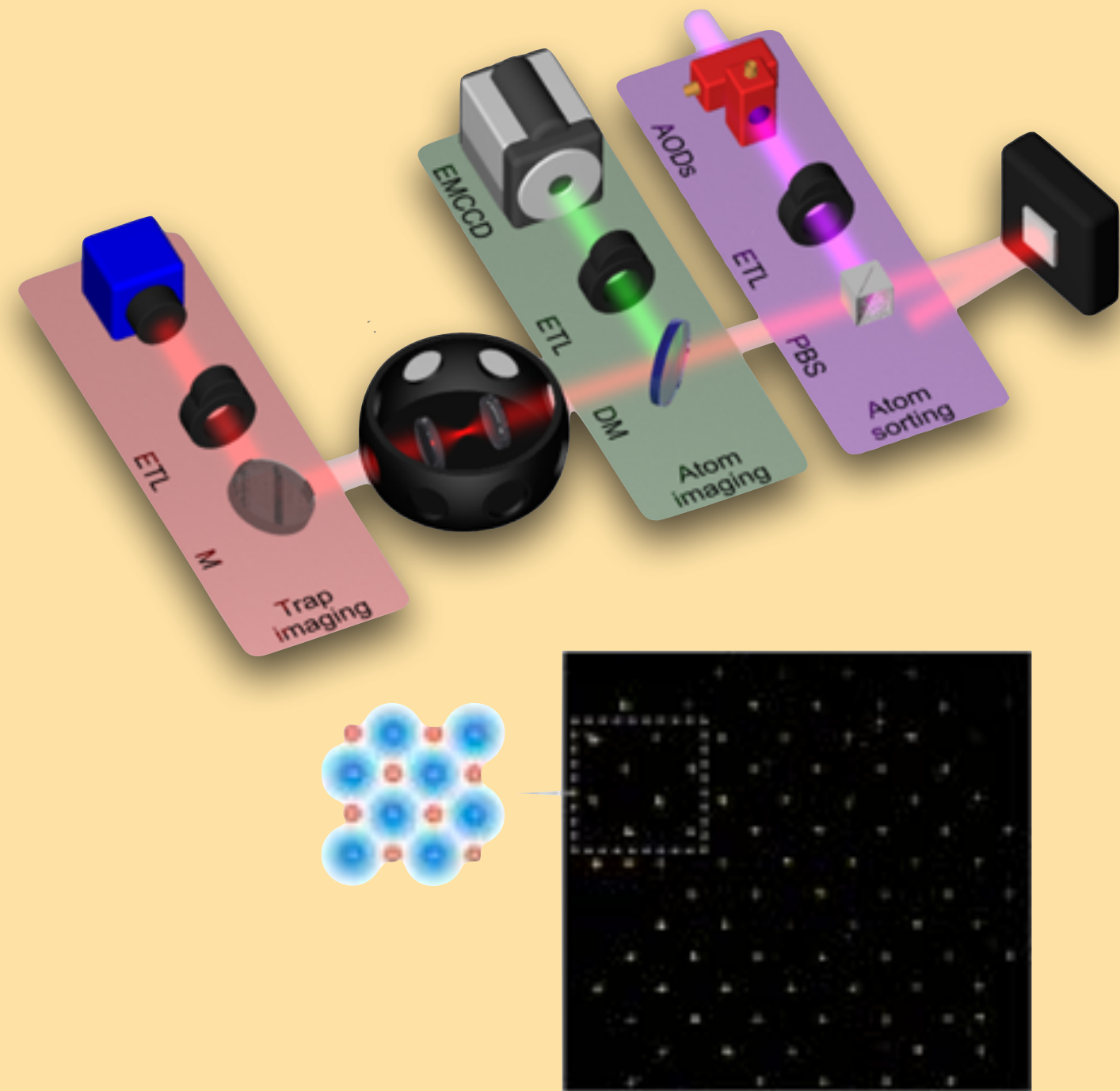
# Quantum optics - atoms @ LCF



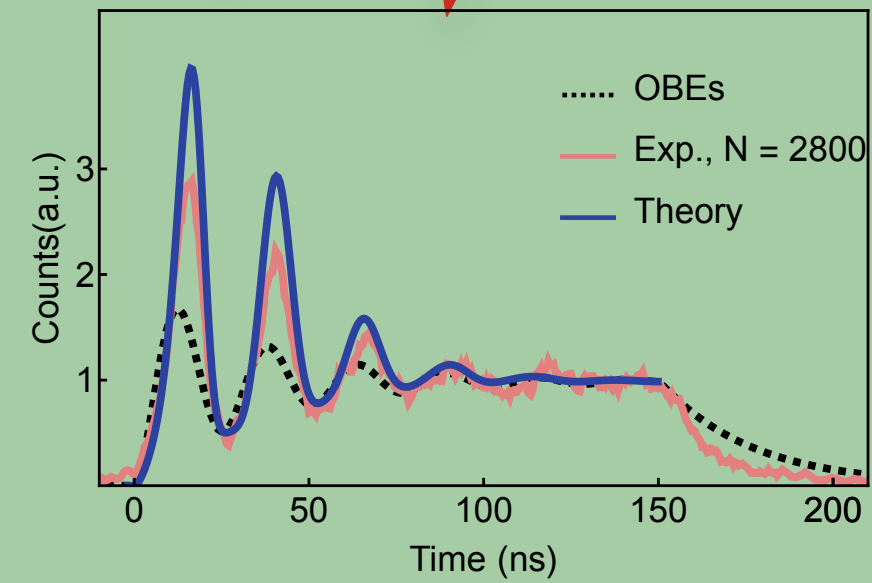
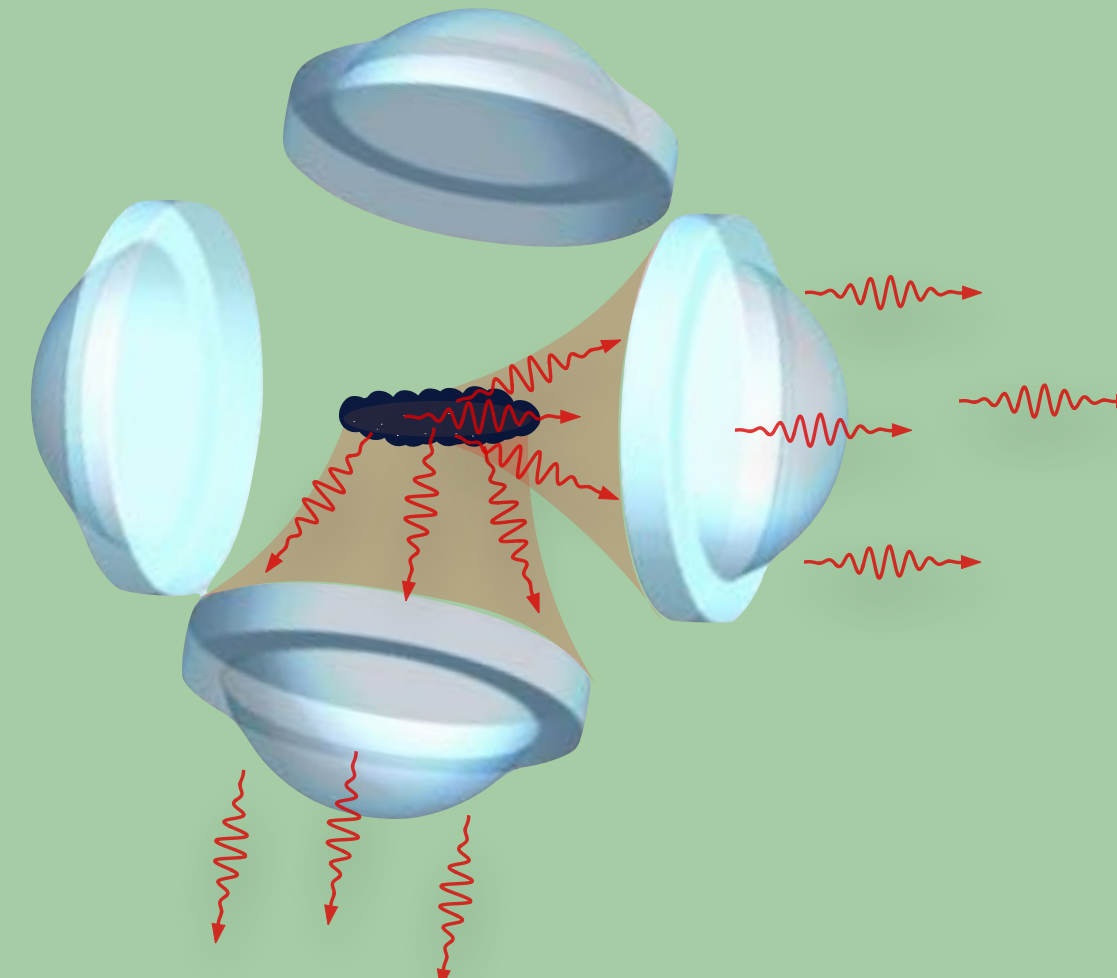


# Group thematics

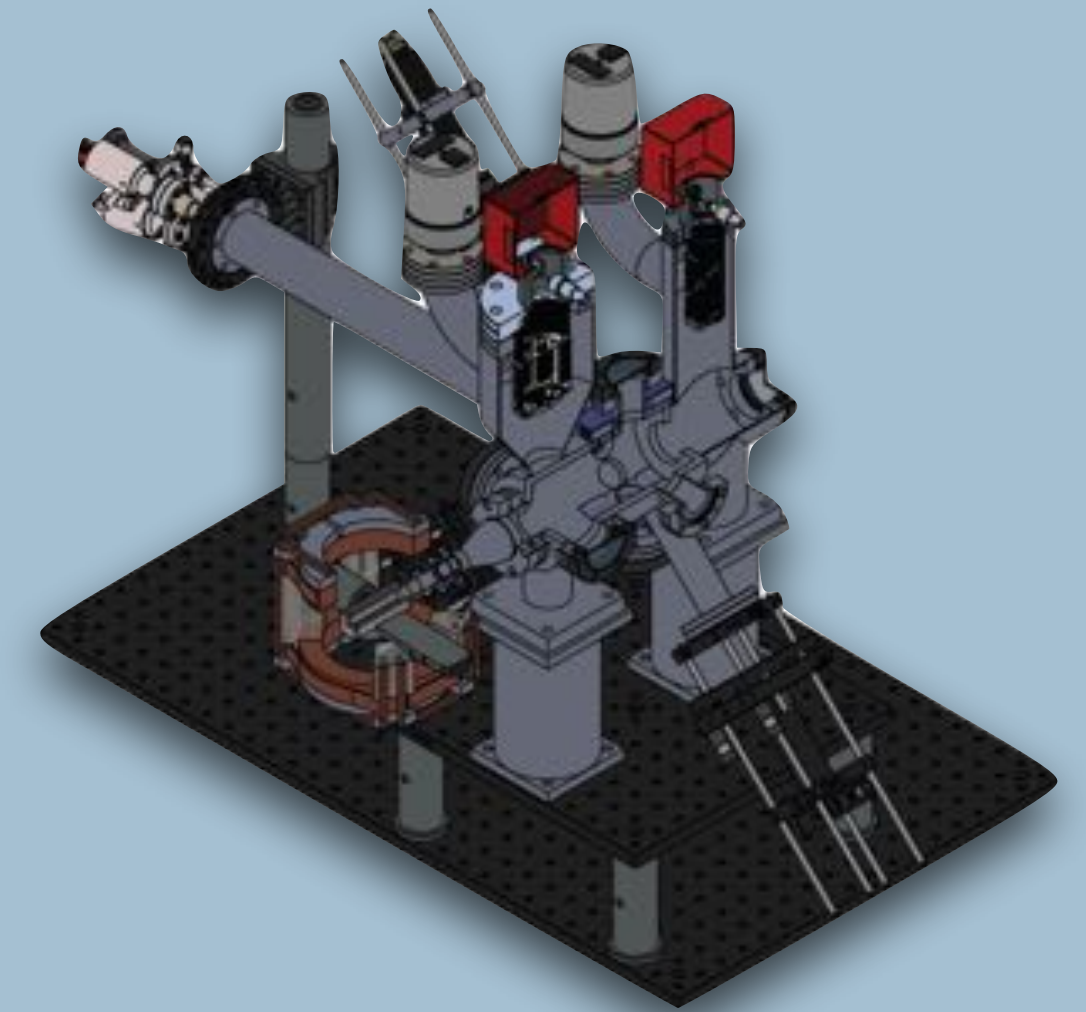
## Quantum simulation with Rydberg arrays



## Light scattering in dense clouds



## Dy tweezers arrays for light scattering (under construction)

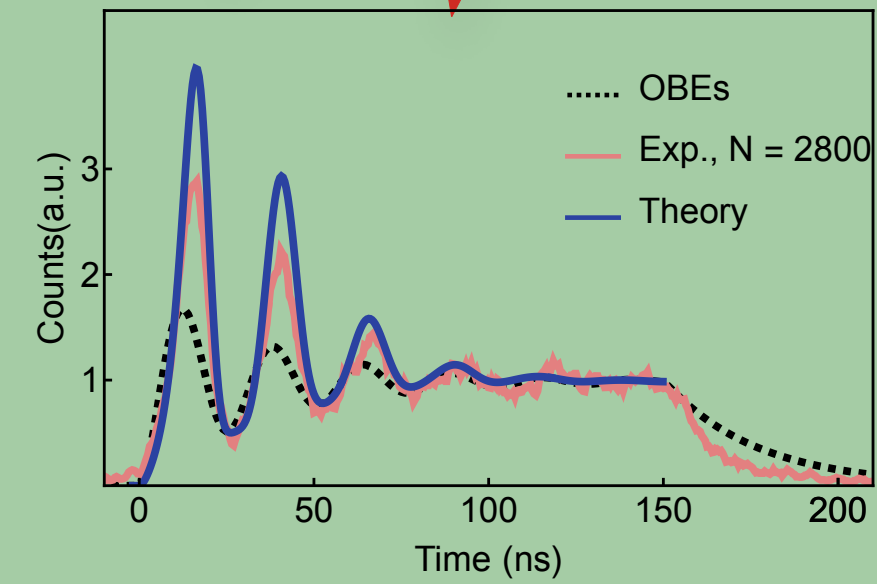
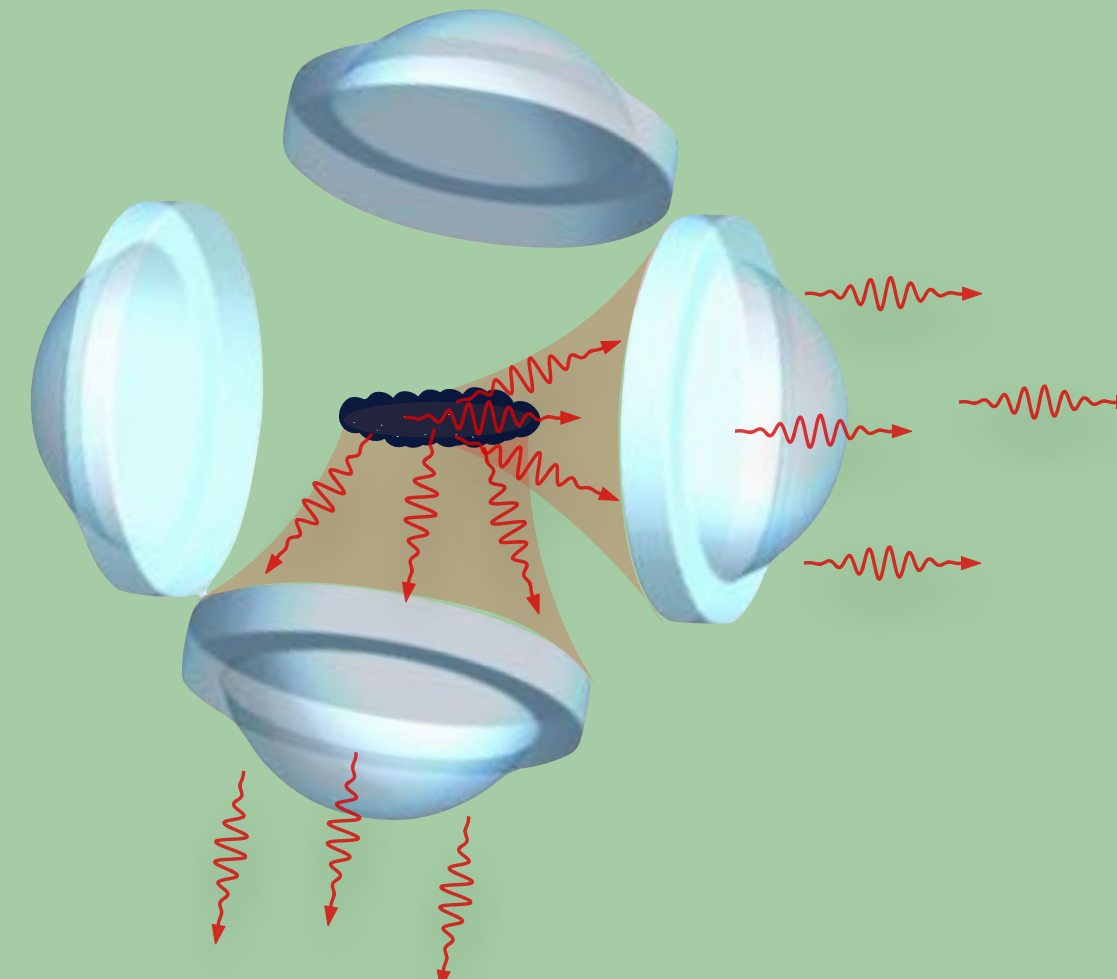


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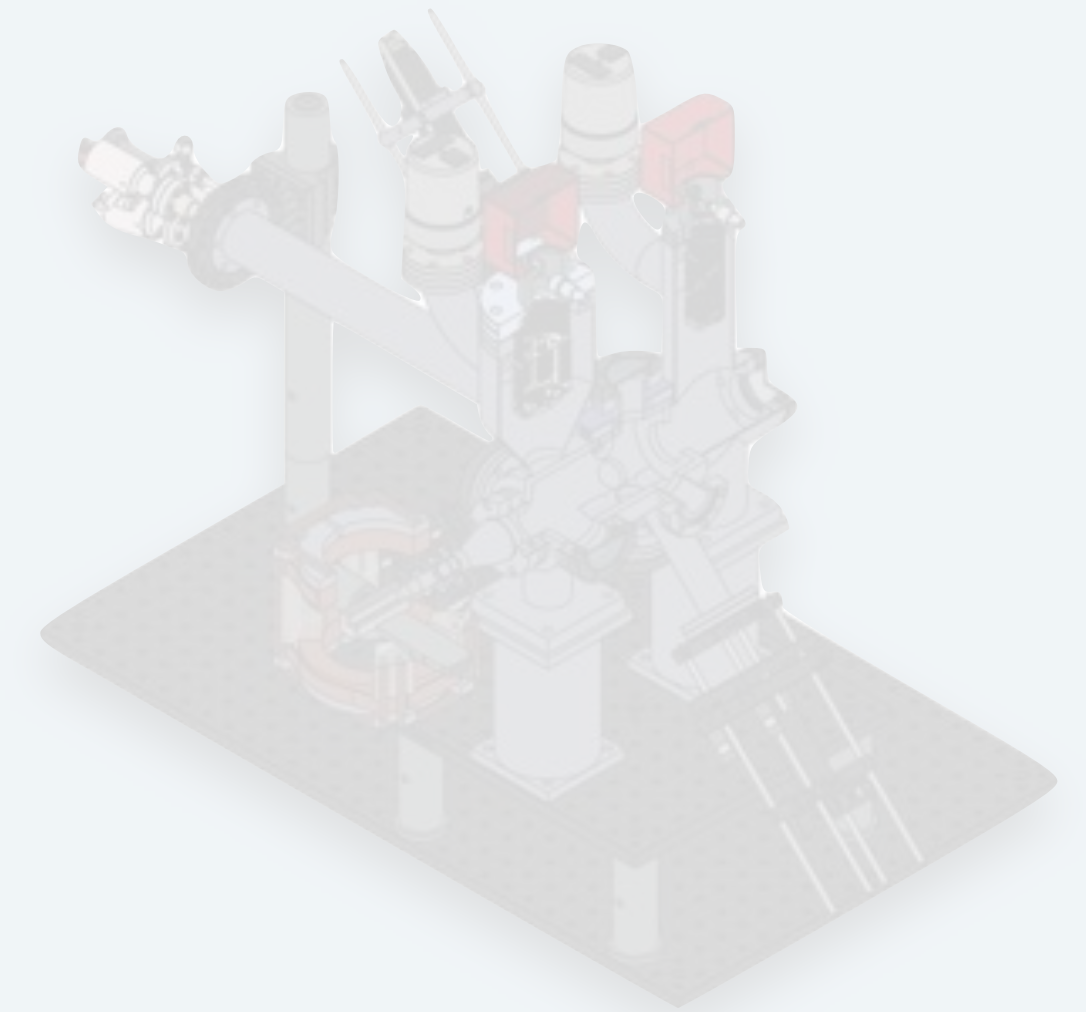
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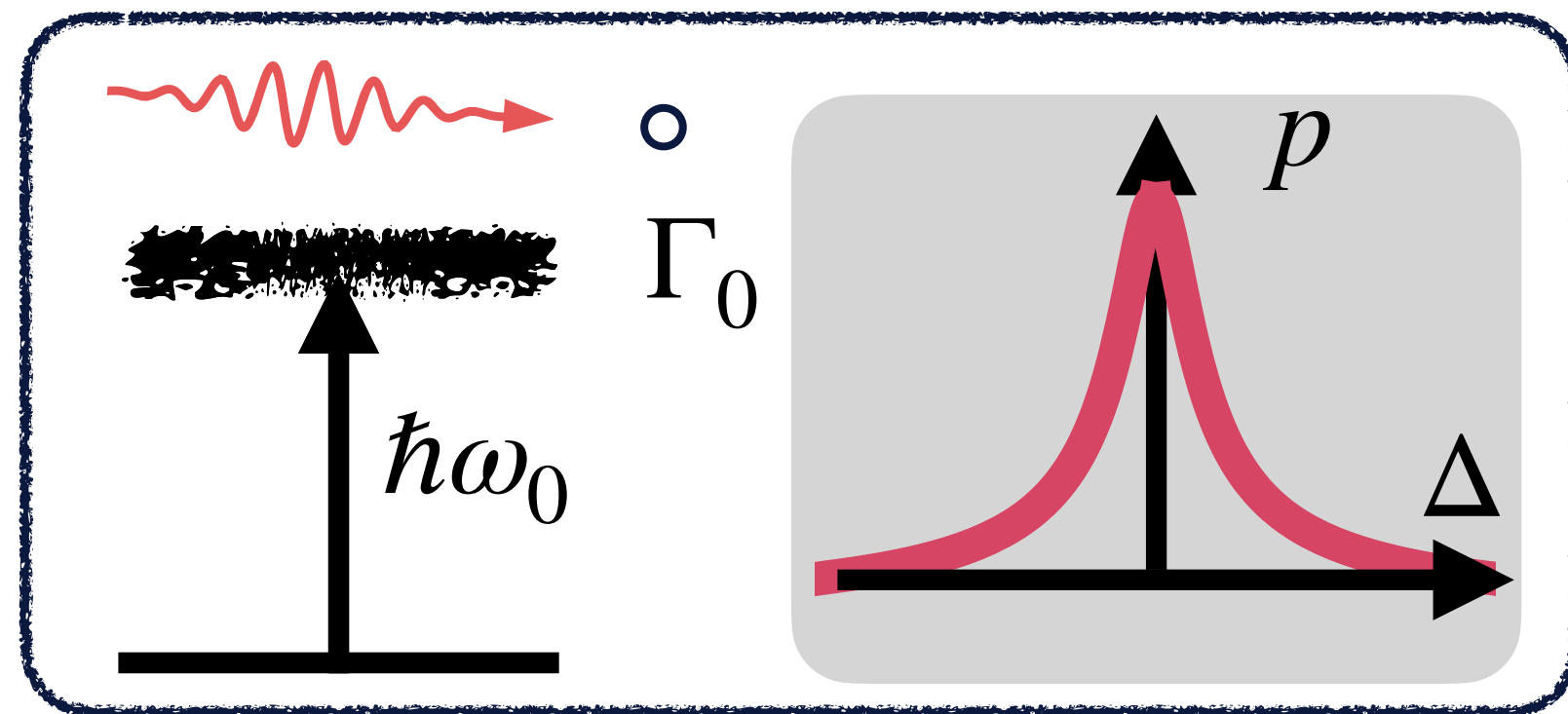
Dy tweezers arrays for light scattering  
(under construction)



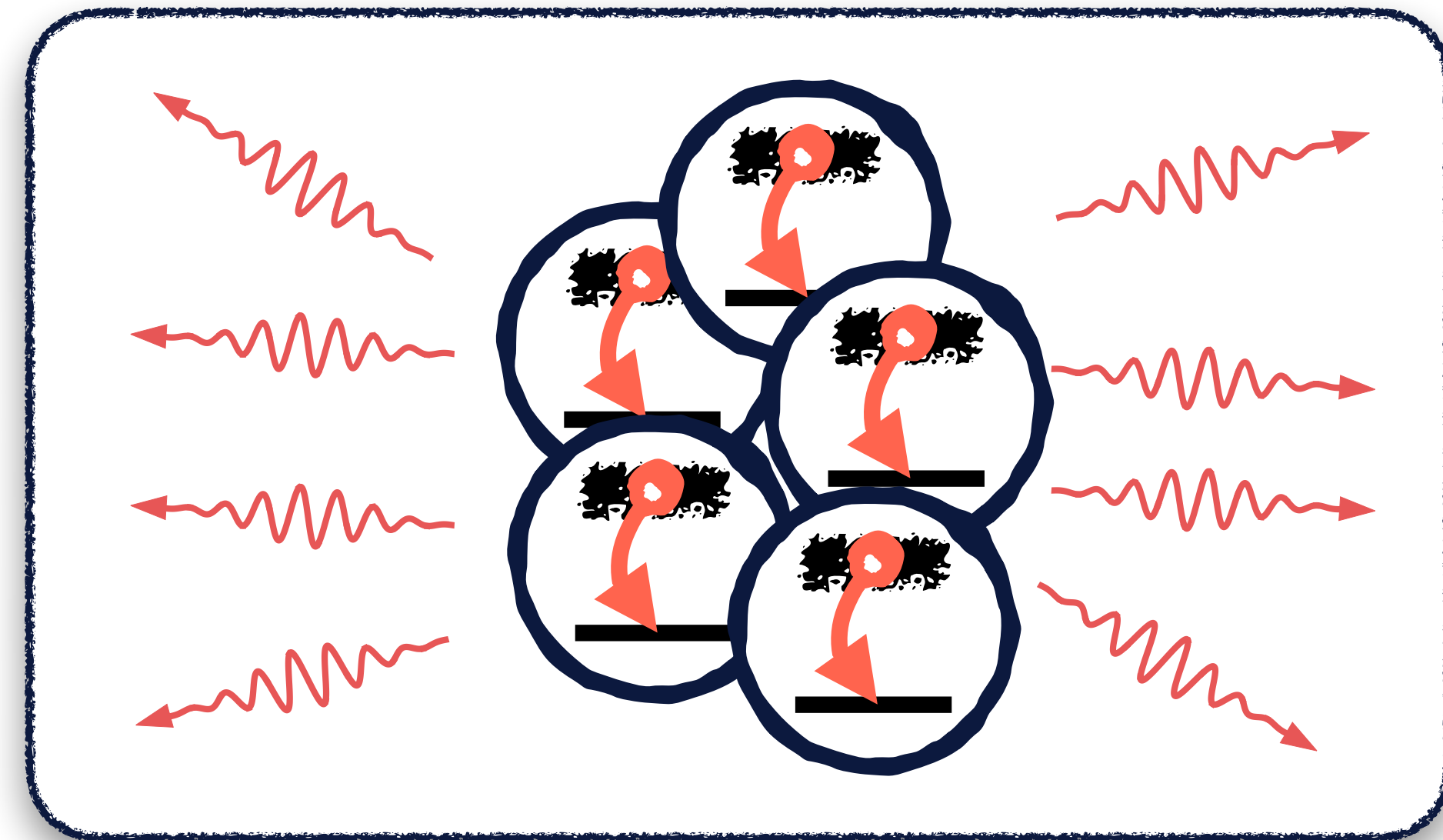


# Collective spontaneous emission

Single atom in free space



$N$ -atom spontaneous emission





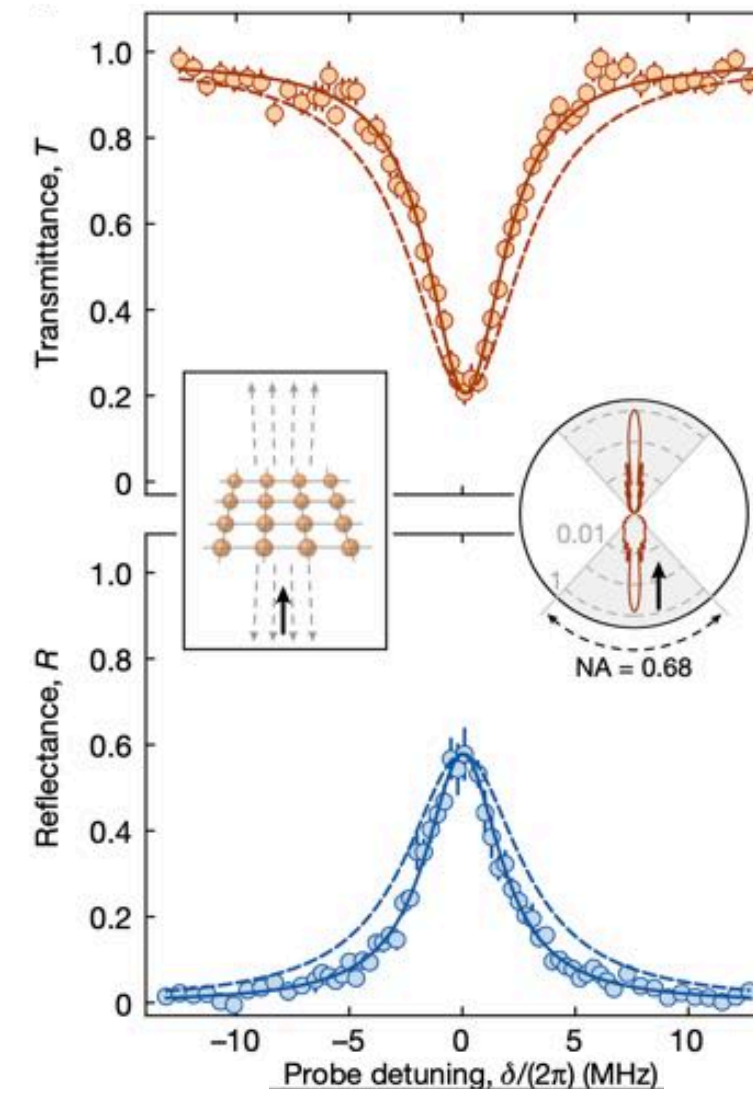
# $N$ atoms

## An atomic mirror

Bettles *et al.*, *Phys. Rev. Lett.* 116, 103602 (2016).

Shahmoon *et al.*, *Phys. Rev. Lett.* 118, 113601 (2017).

Rui *et al.*, *Nature* 583, 369 (2020).





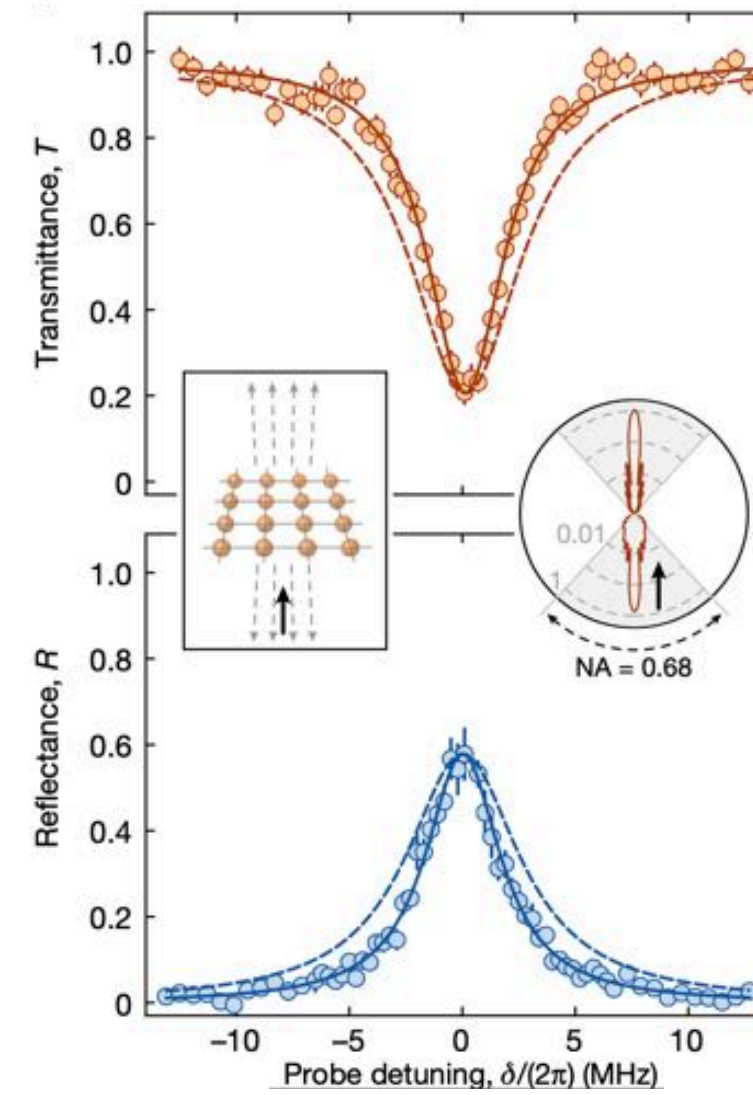
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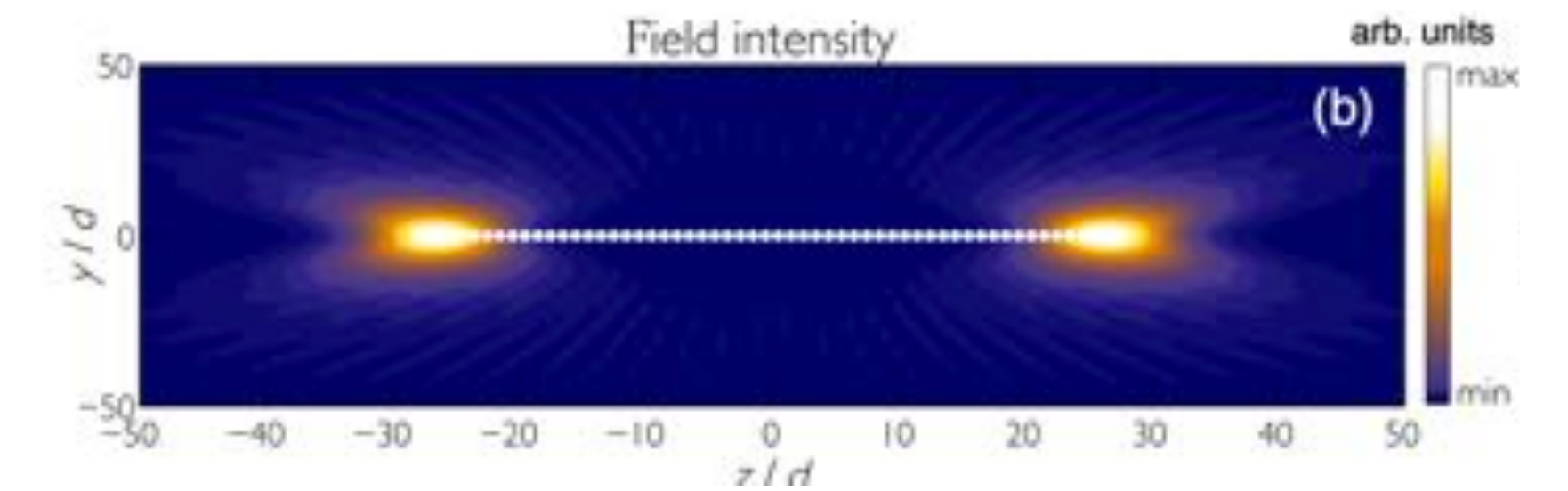
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## Subradiance as a light storage medium?

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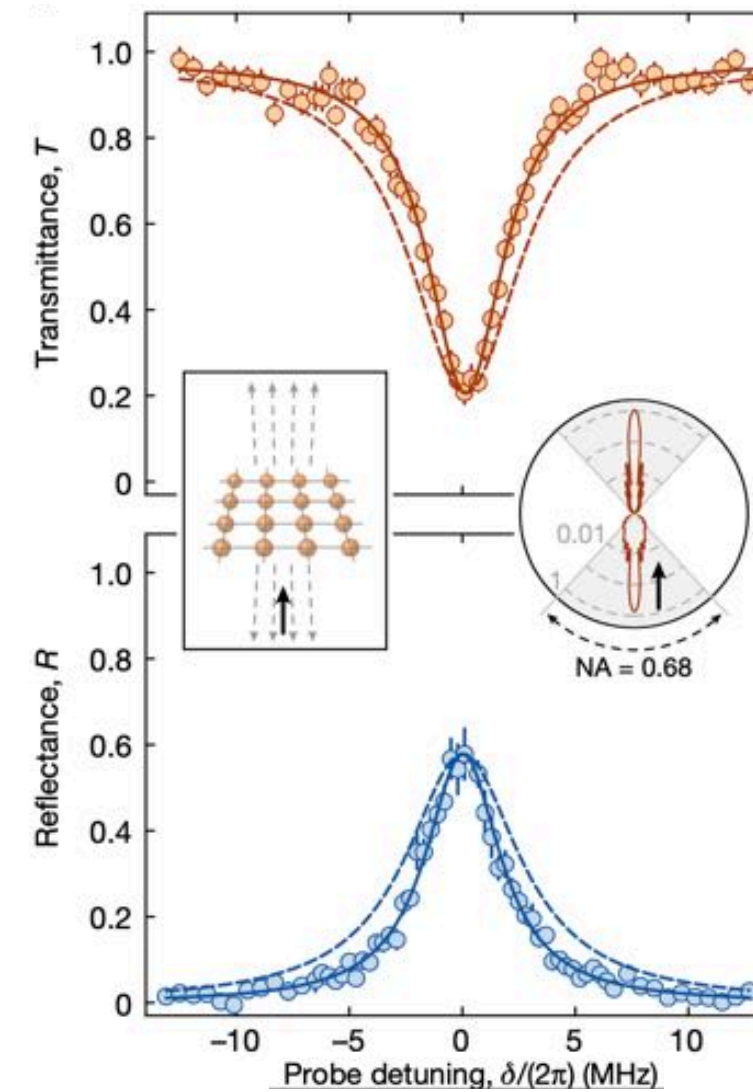
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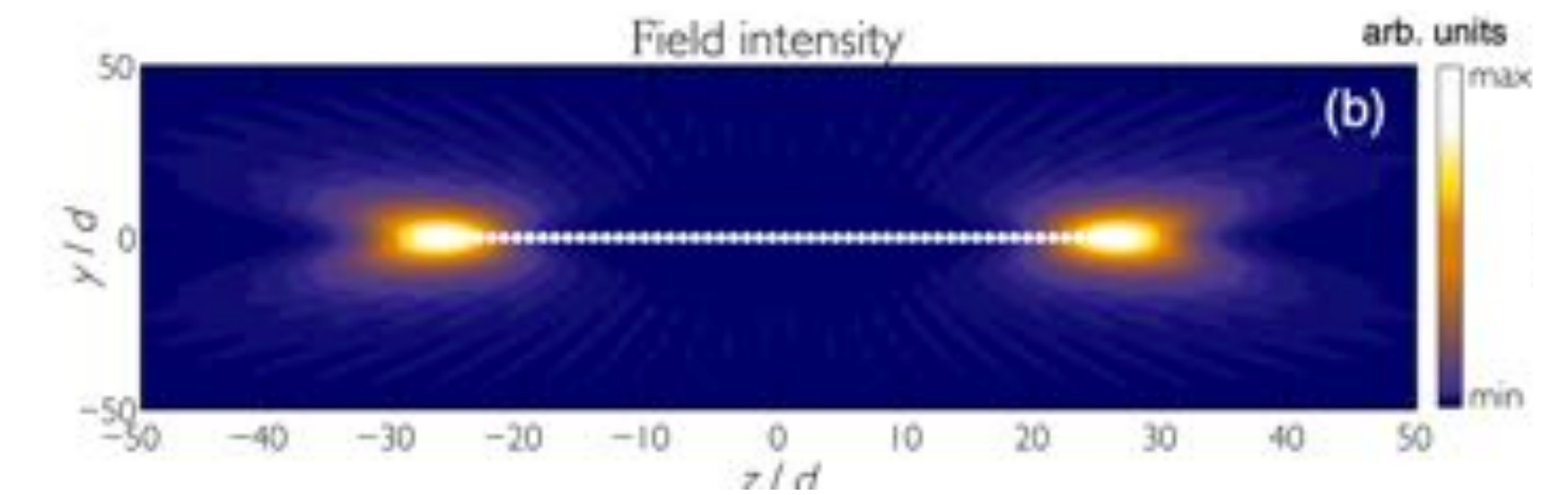
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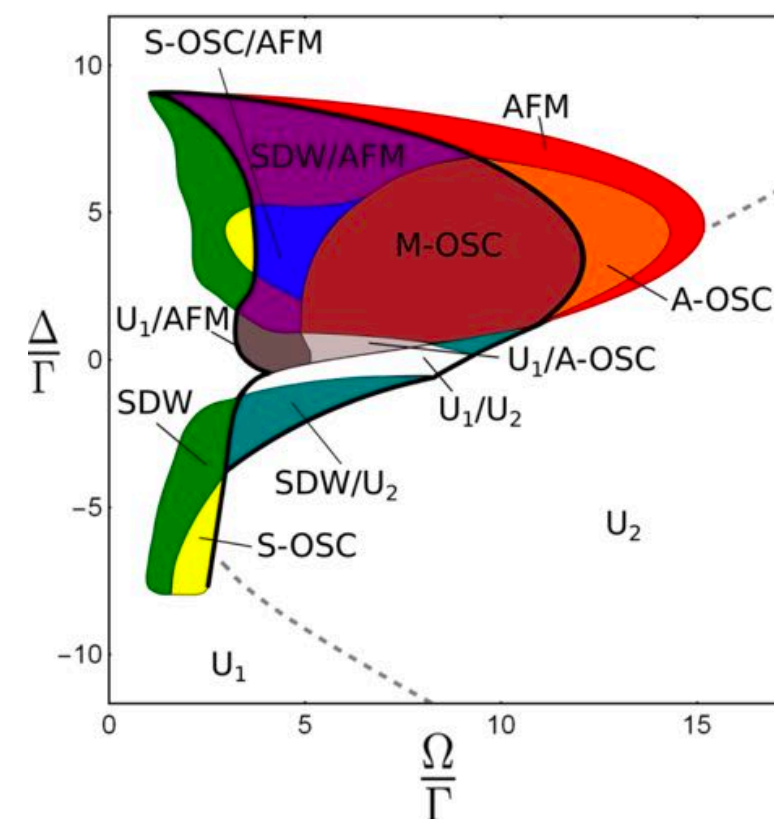
Asenjo-Garcia *et al.*, *Phys. Rev. X* 7, 031024 (2017).



## Phase transitions in driven-dissipative many-body systems.

Olmos *et al.*, *Phys Rev A* **89**, 023616 (2014).

Parmee & Cooper, *Phys Rev A* **97**, 053616 (2018).

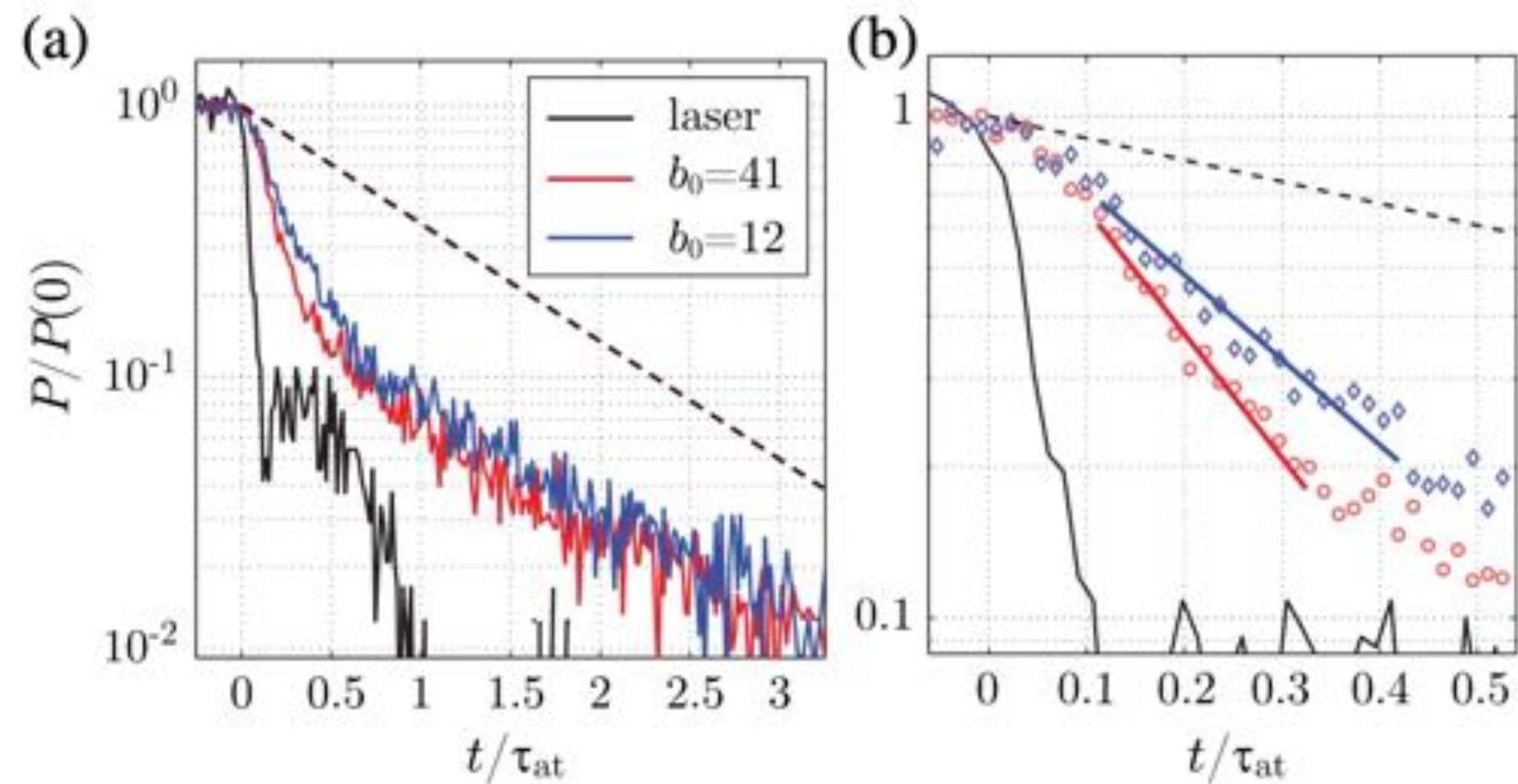




# Collective spontaneous emission

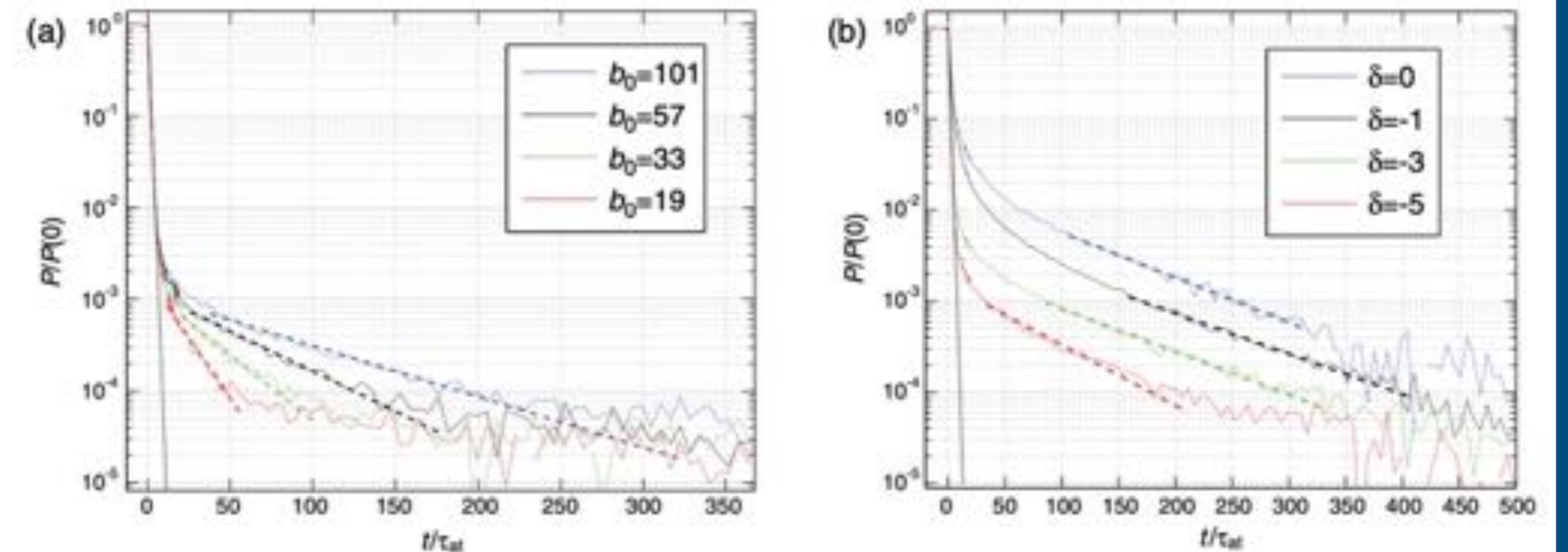
Dilute atomic clouds  $L/\lambda_0 \gg 1$  (Nice, Wisconsin, Norfolk...)

Superradiance



Araújo et al., *Phys Rev Lett* **117**, 073002 (2016).

Subradiance



Guerin et al., *Phys Rev Lett* **116**, 083601 (2016).

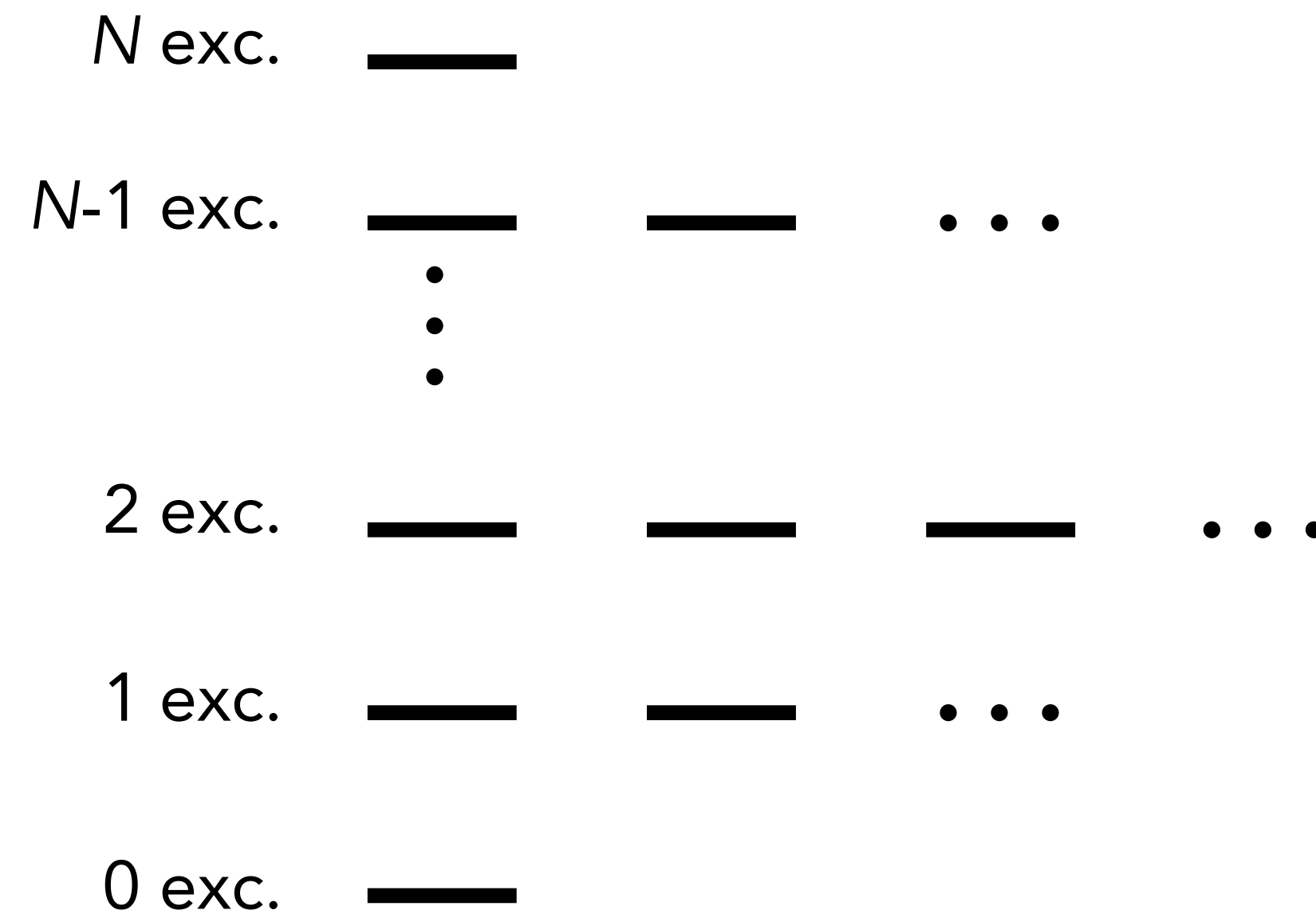
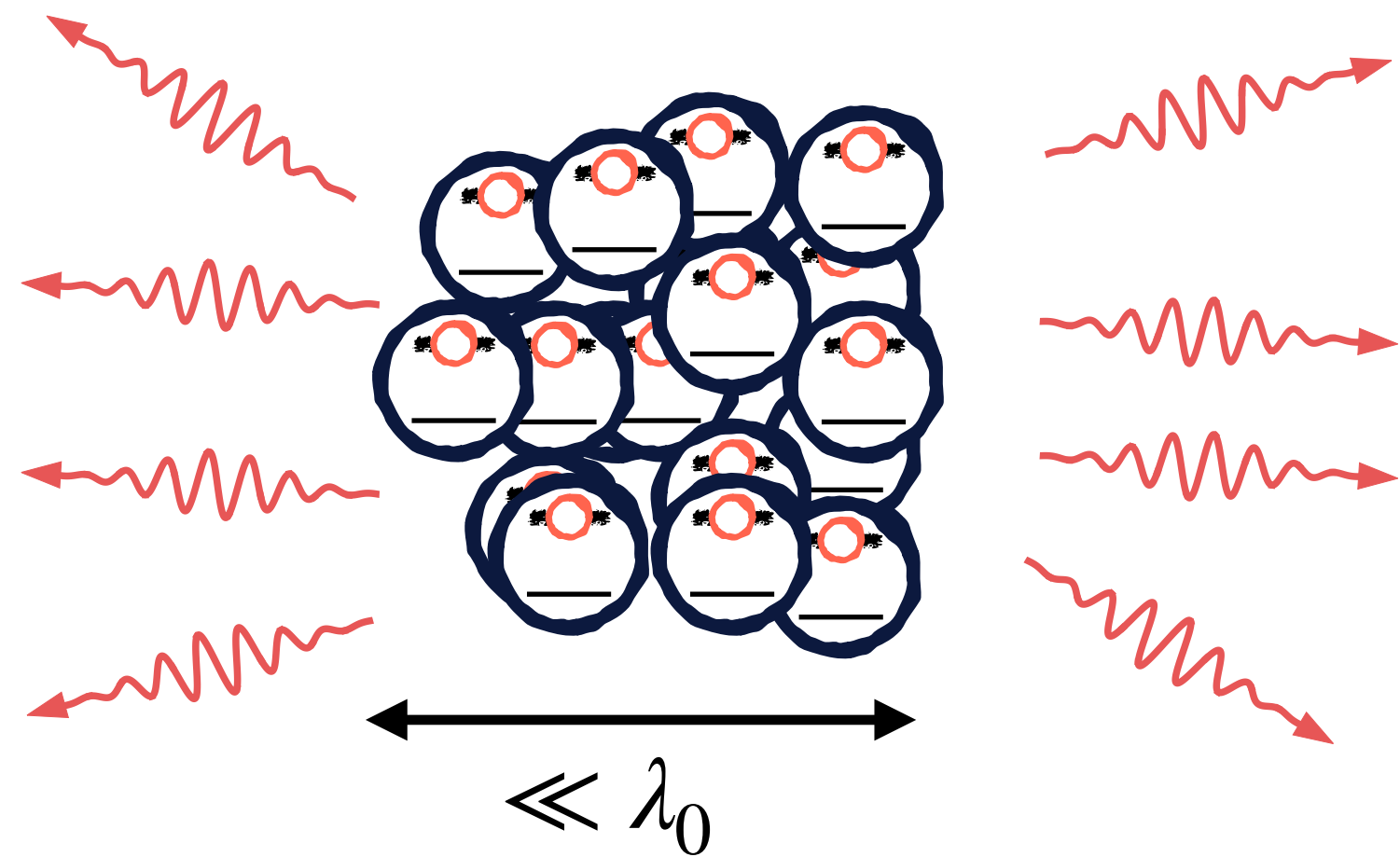
**This talk**

Small atomic clouds  $L/\lambda_0 \lesssim 1$

Quantum regime:  $n_{exc} \gg 1$

# $N$ -atom spontaneous emission

R. H. Dicke, *Phys. Rev.* **93**, 99 (1954).



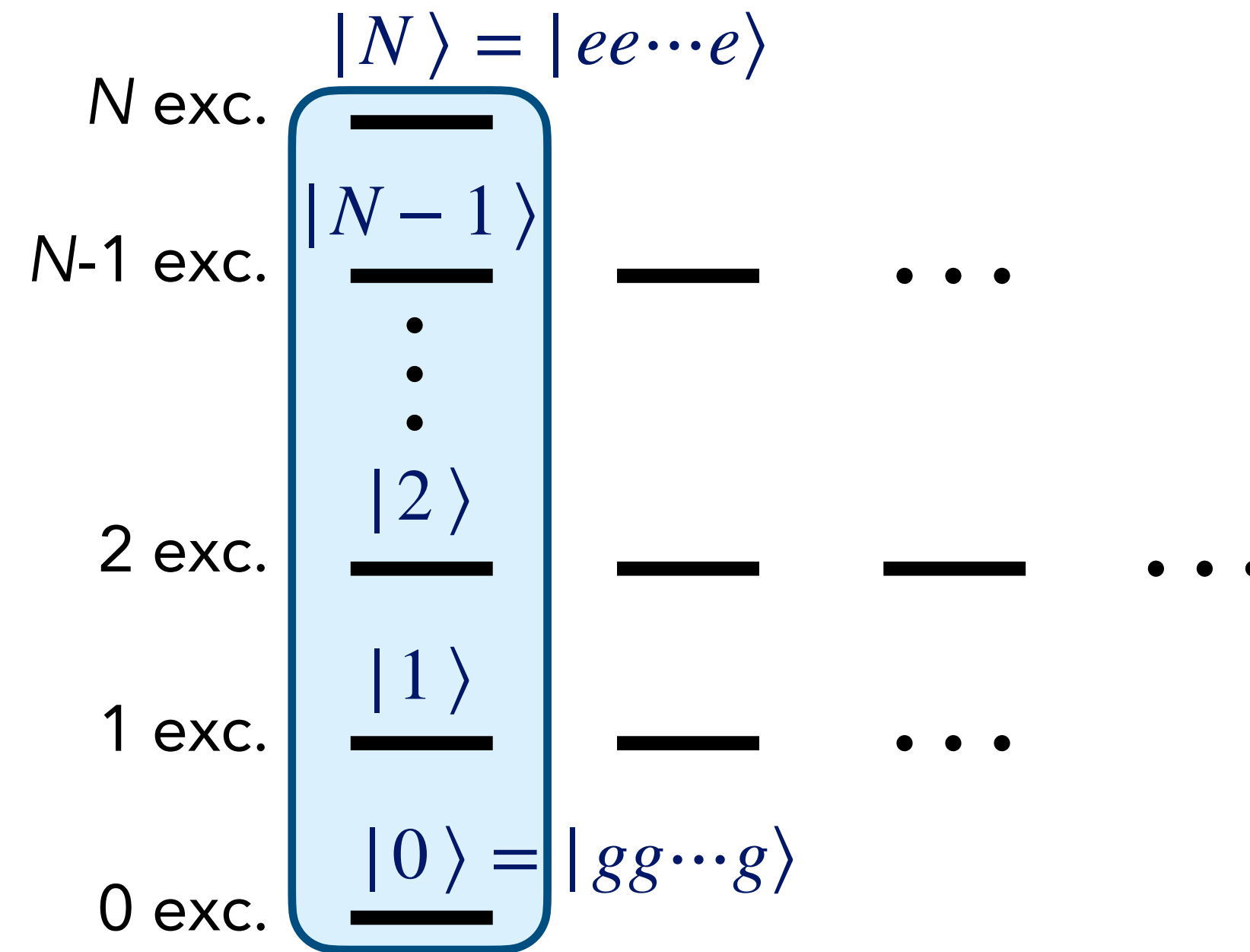
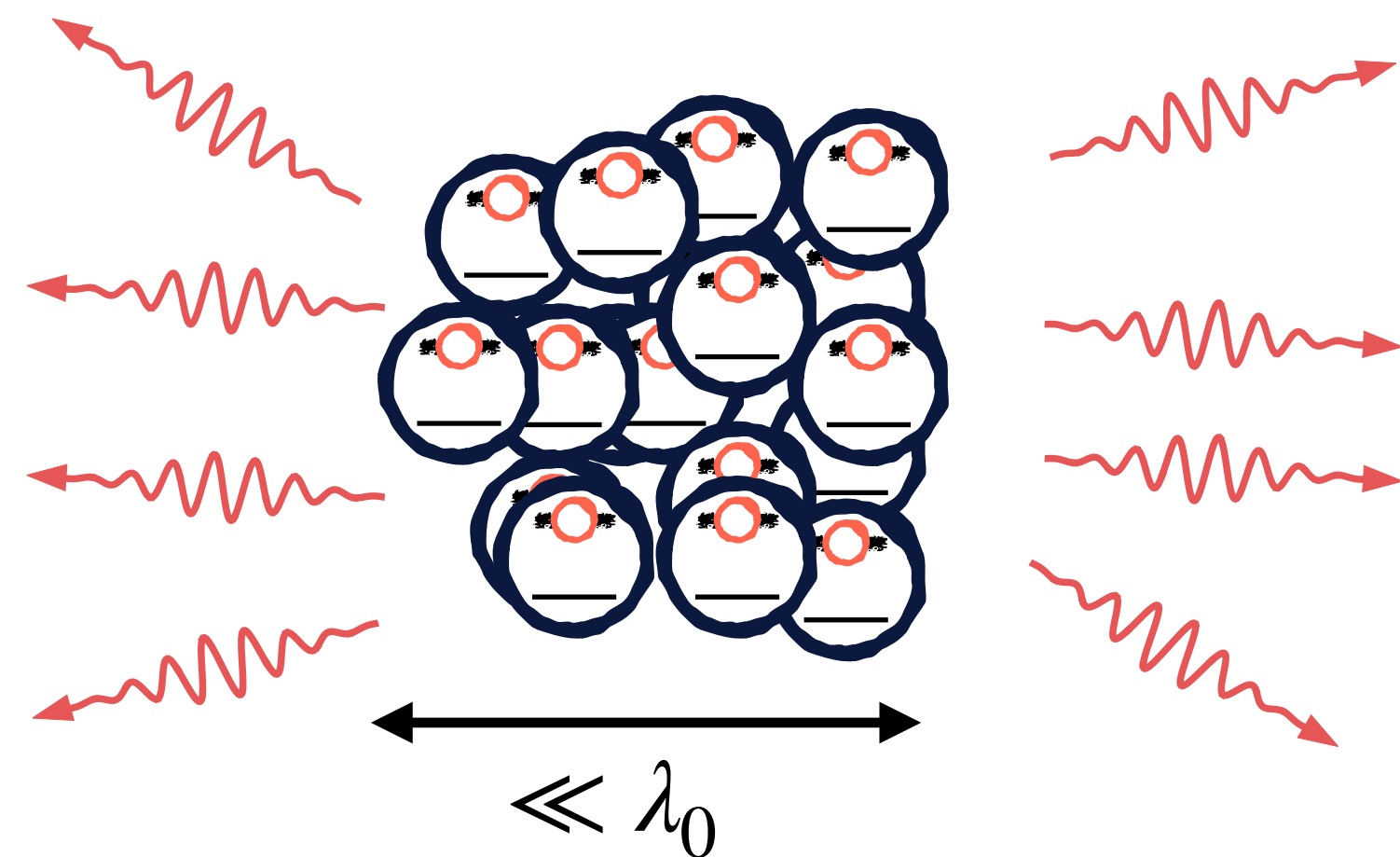
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# N-atom spontaneous emission

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Dicke states  $|n\rangle \propto (\hat{S}^+)^n |0\rangle$   $\hat{S}^+ = \sum_{i=1}^N \hat{\sigma}_i^+$

$\Gamma_N = \Gamma_0 n(N - n + 1)$  in state  $|n\rangle$

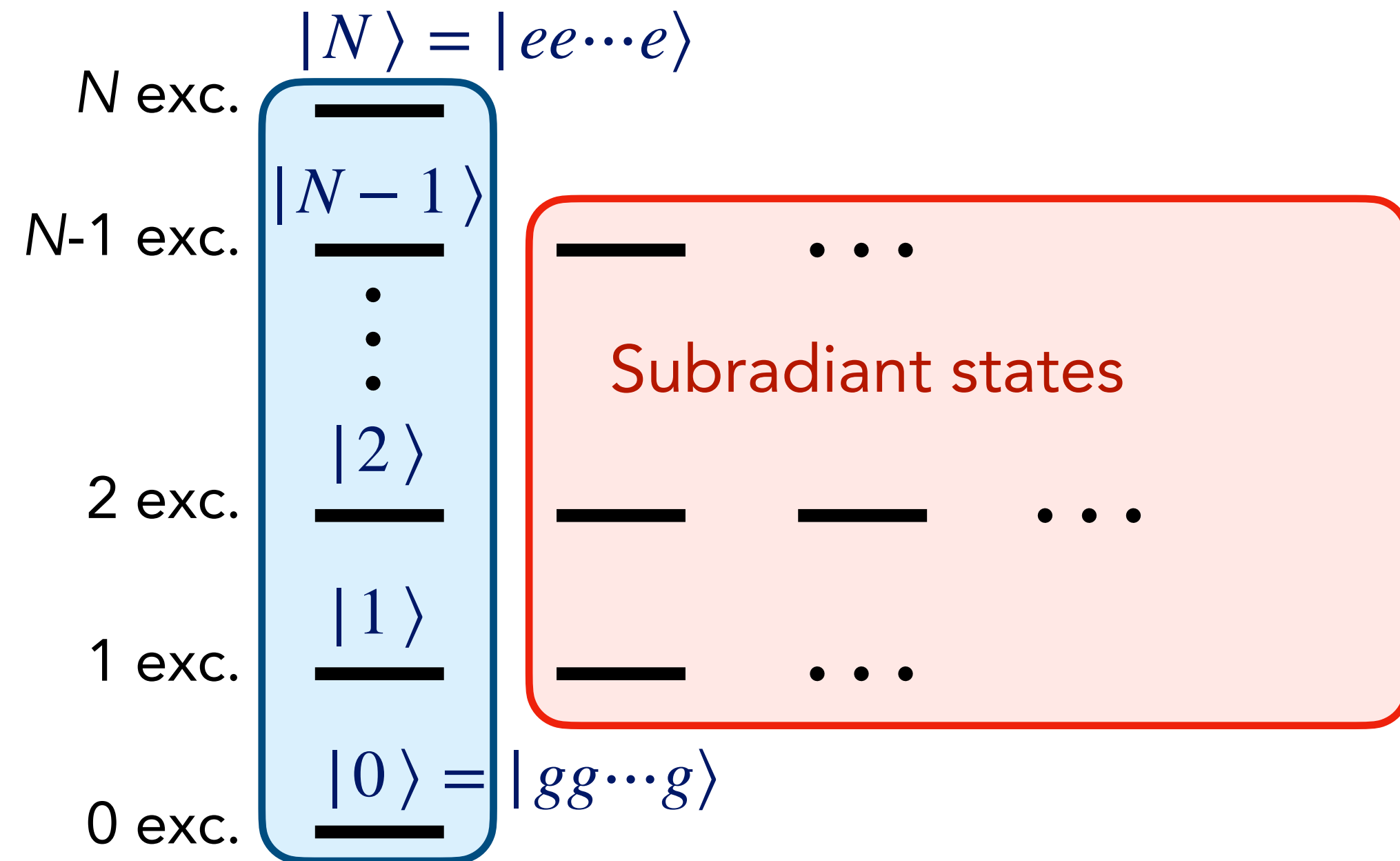
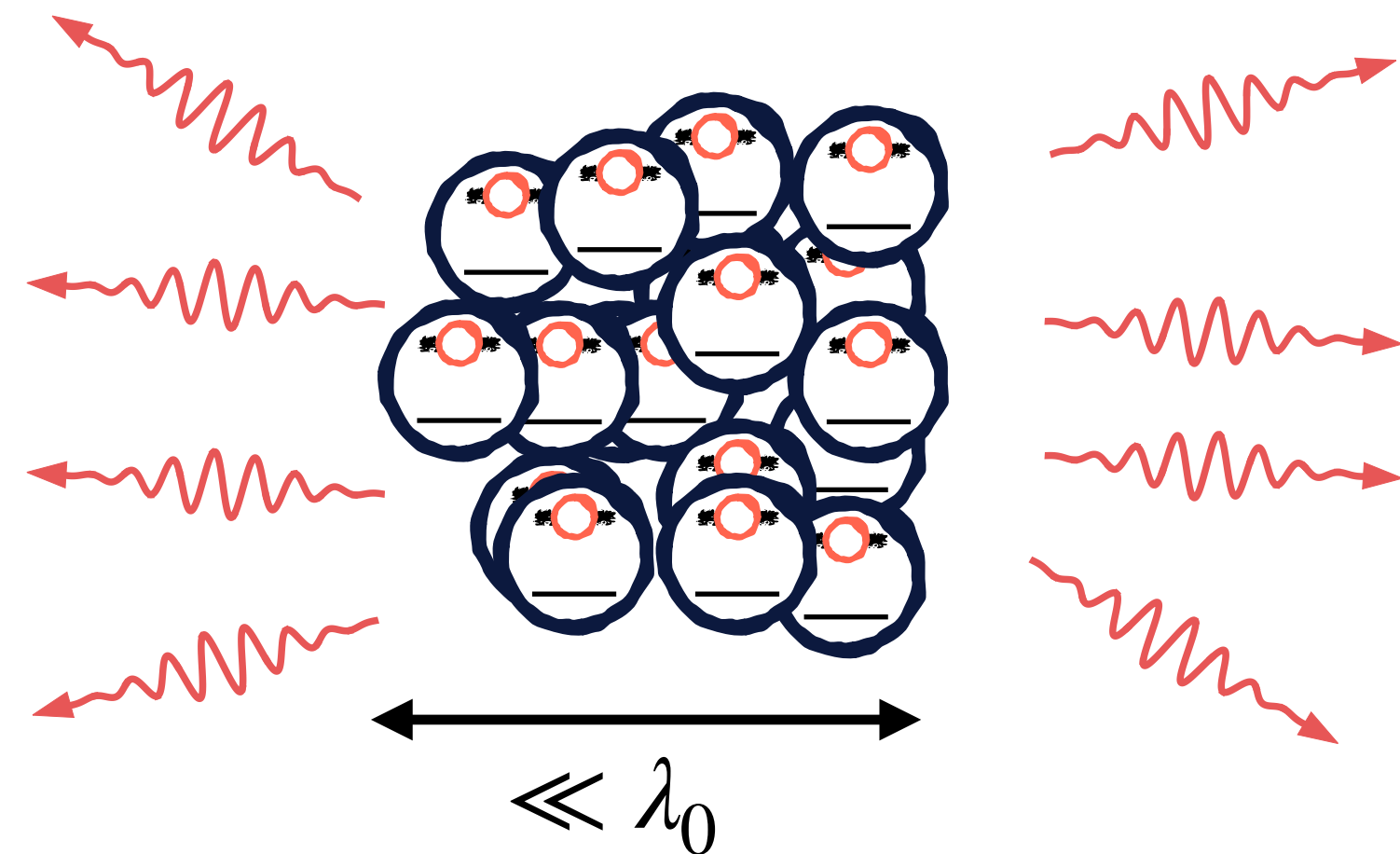
Reaches maximum for  $|n\rangle = |N/2\rangle$ , with  $\Gamma_N \simeq N^2 \Gamma_0 / 4$

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Break Dicke symmetry:

Finite-size effects, dipole-dipole interactions...

Glicenstein *Optics Letters* **47**, 1542 (2022)

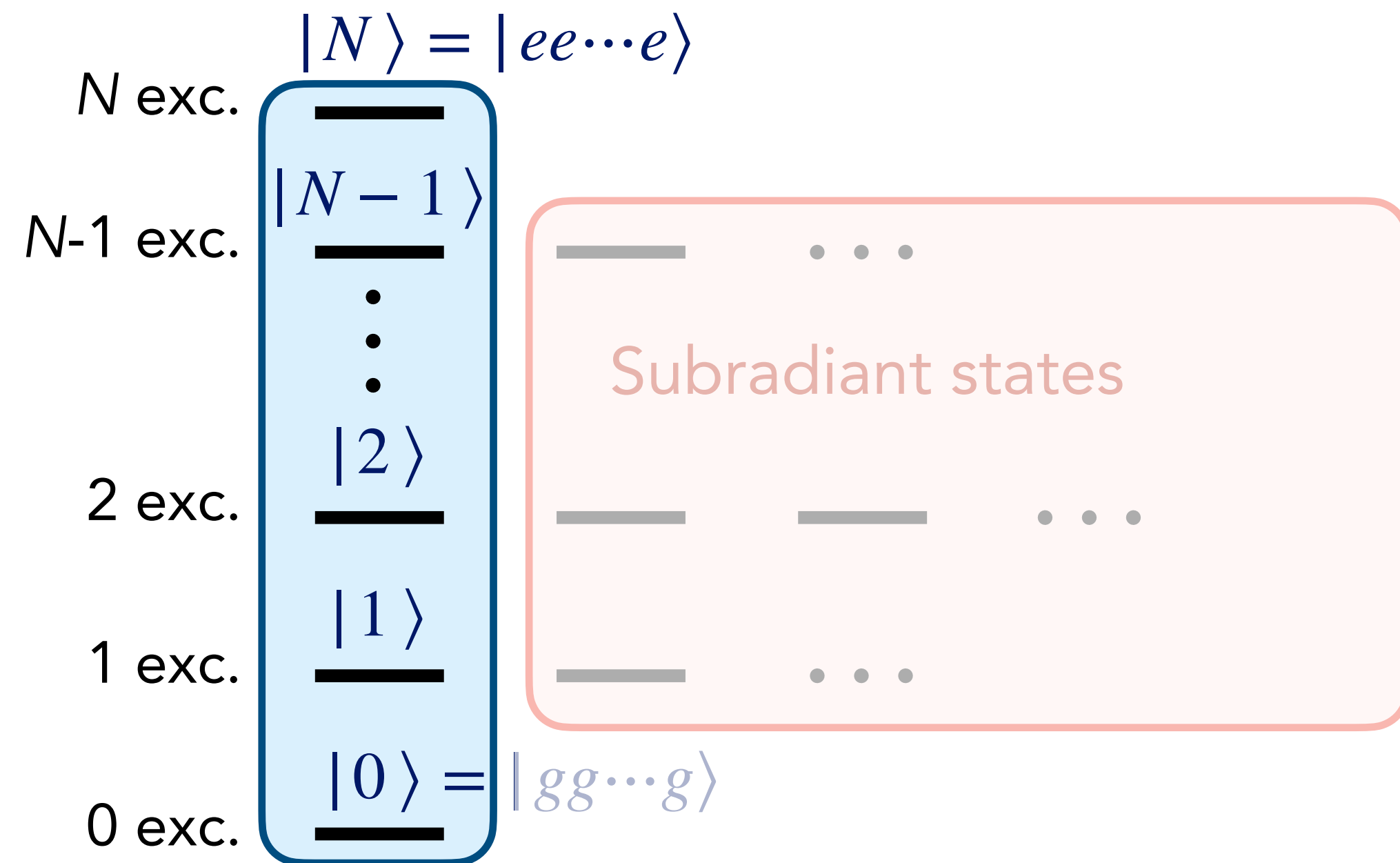
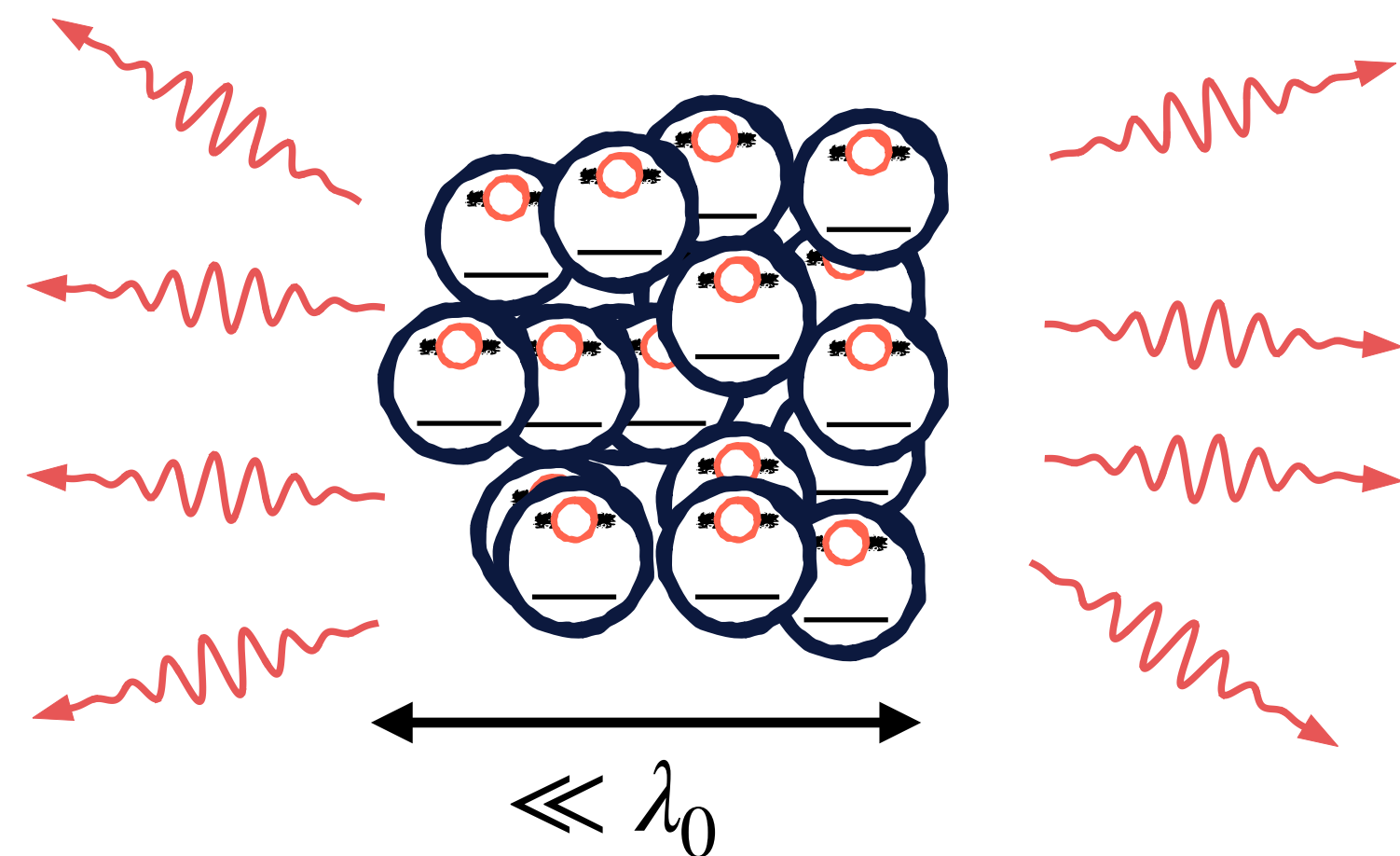
Feroli *et al.*, *PRX* **11**, 021031 (2021)

See also Nice group (Kaiser Guerin)...



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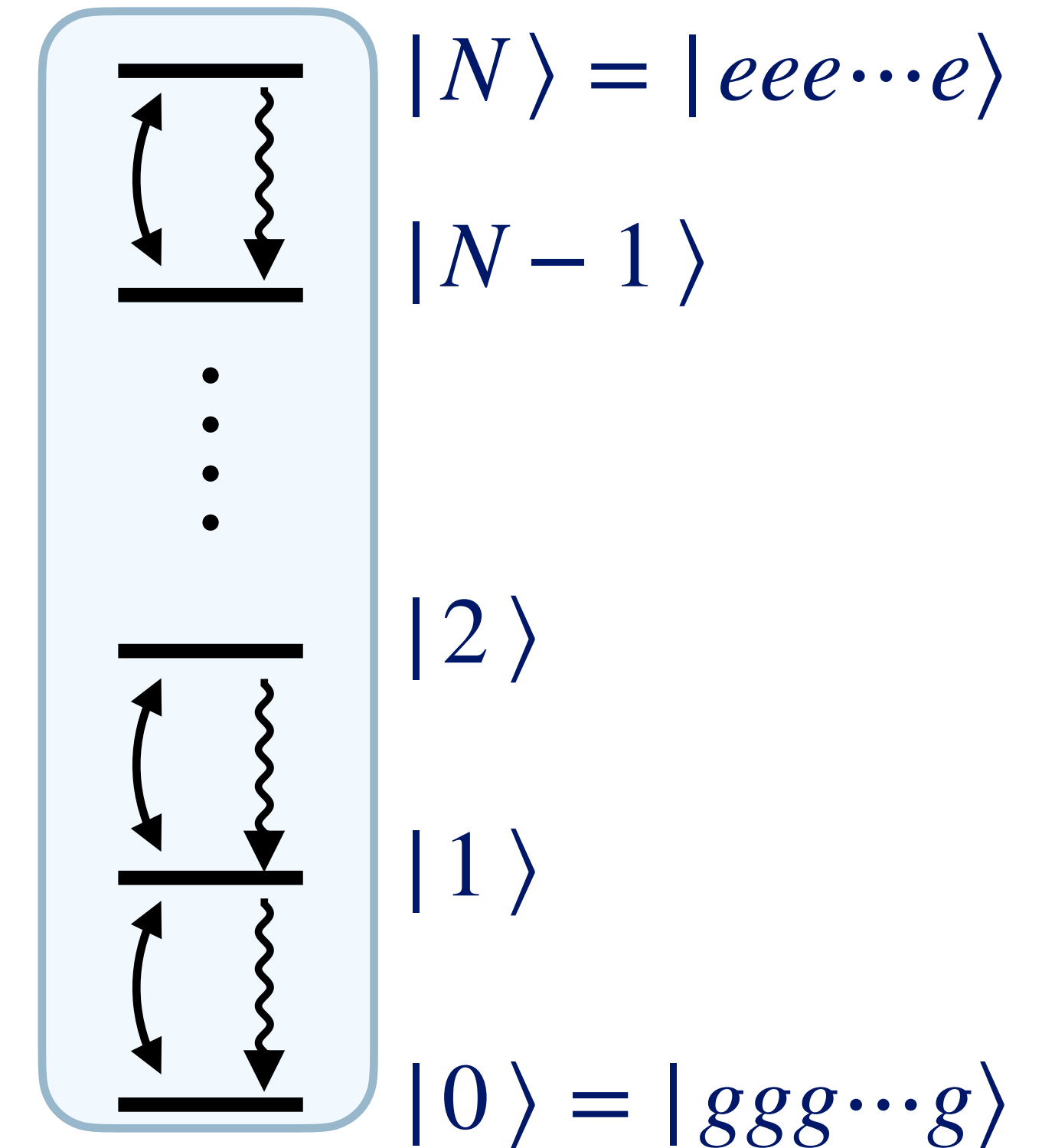
# $N$ driven atoms

Dicke symmetric conditions + classical drive

$$\dot{\hat{\rho}} = \frac{i}{\hbar} [\hat{\rho}, \hat{H}] + \frac{\Gamma_0}{2} \sum_{i,j=1}^N 2\hat{\sigma}_j^- \hat{\rho} \hat{\sigma}_i^+ - \hat{\sigma}_i^+ \hat{\sigma}_j^- \hat{\rho} - \hat{\sigma}_i^+ \hat{\sigma}_j^- \hat{\rho}$$

$$\hat{H} = \frac{\hbar\Omega_D}{2} \sum_{i=1}^N \hat{\sigma}_i^+ + \hat{\sigma}_i^-$$

Classical electric dipole of a single atom:  $\langle \hat{\sigma}_i^- \rangle$





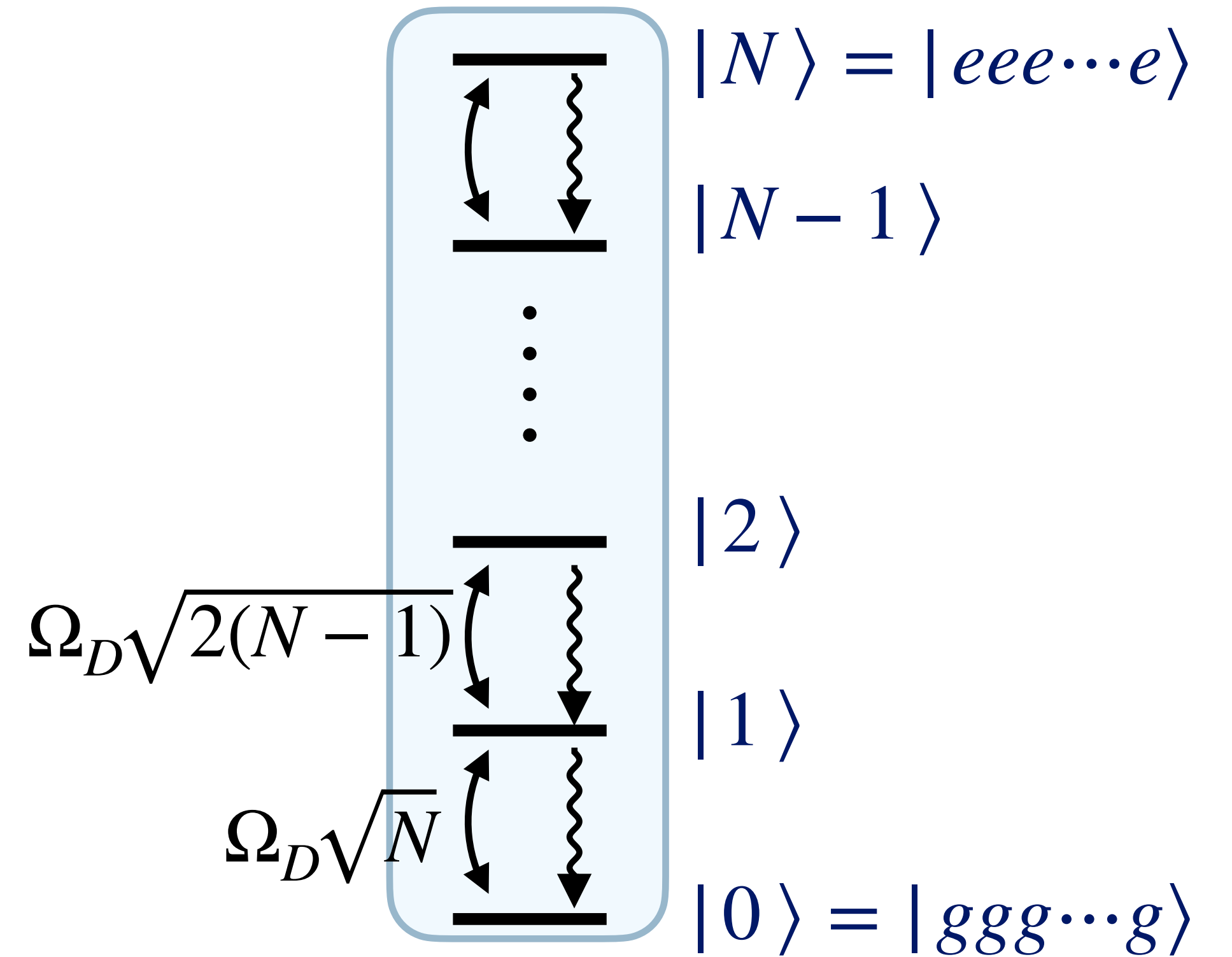
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$$\hat{H} = \frac{\hbar\Omega_D}{2} (\hat{S}^+ + \hat{S}^-) \quad \hat{S}^+ = \sum_{i=1}^N \hat{\sigma}_i^+$$

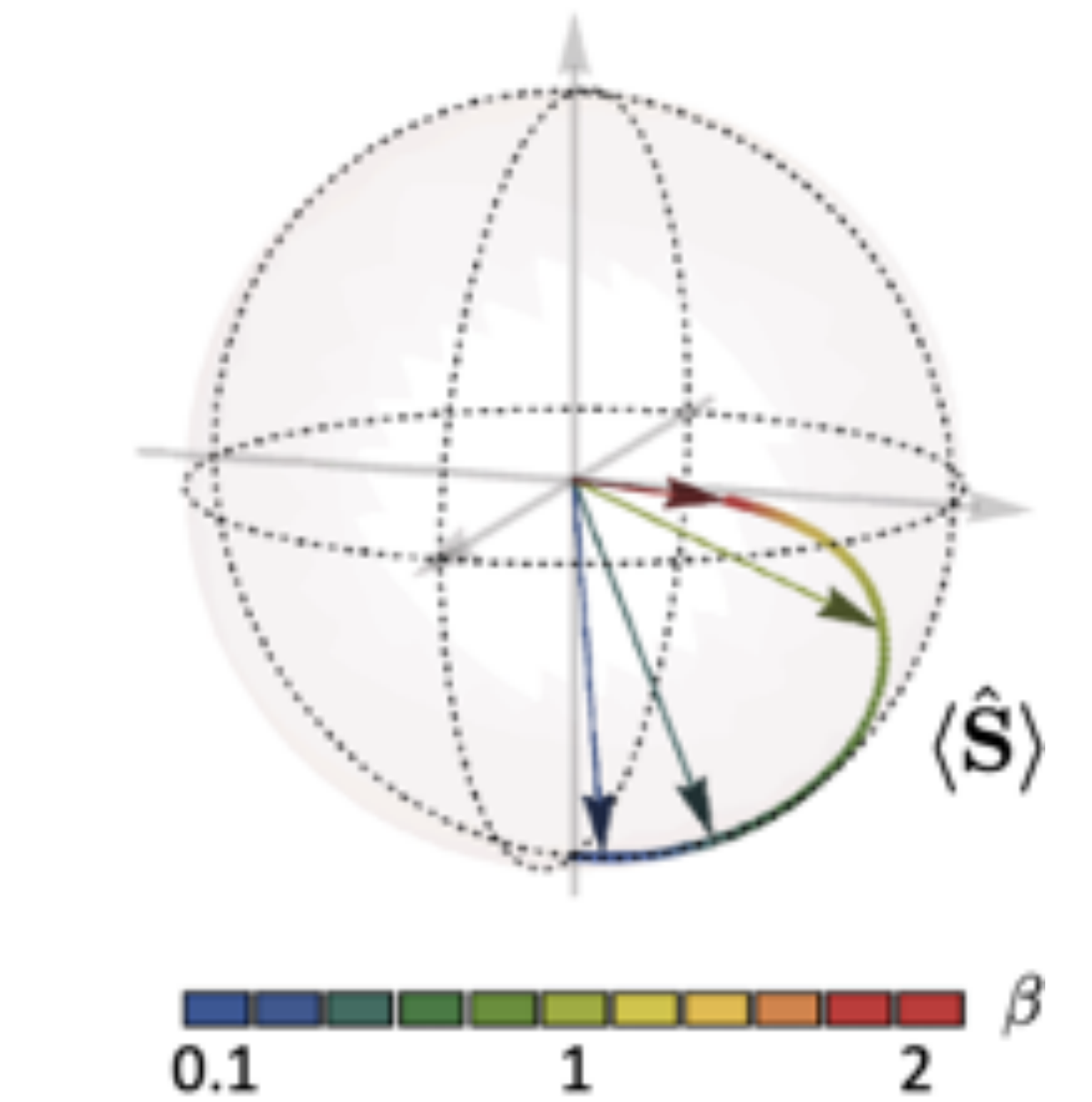
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# Driven Dicke model

$$\dot{\hat{\rho}} = i\frac{\Omega_D}{2} \left( \hat{\rho}\hat{S}^+ + \hat{\rho}\hat{S}^- - \hat{S}^+\hat{\rho} - \hat{S}^-\hat{\rho} \right) + \frac{\Gamma_0}{2} \left( 2\hat{S}^-\hat{\rho}\hat{S}^+ - \hat{S}^+\hat{S}^-\hat{\rho} - \hat{\rho}\hat{S}^+\hat{S}^- \right)$$

$$\frac{d\langle\hat{S}^-\rangle}{dt} = i\Omega_D\langle\hat{S}^z\rangle + \frac{\Gamma_0}{2} \left[ \langle\hat{S}^z\hat{S}^-\rangle + \langle\hat{S}^-\hat{S}^z\rangle \right] - \frac{\Gamma_0}{2}\langle\hat{S}^-\rangle$$





# Driven Dicke model

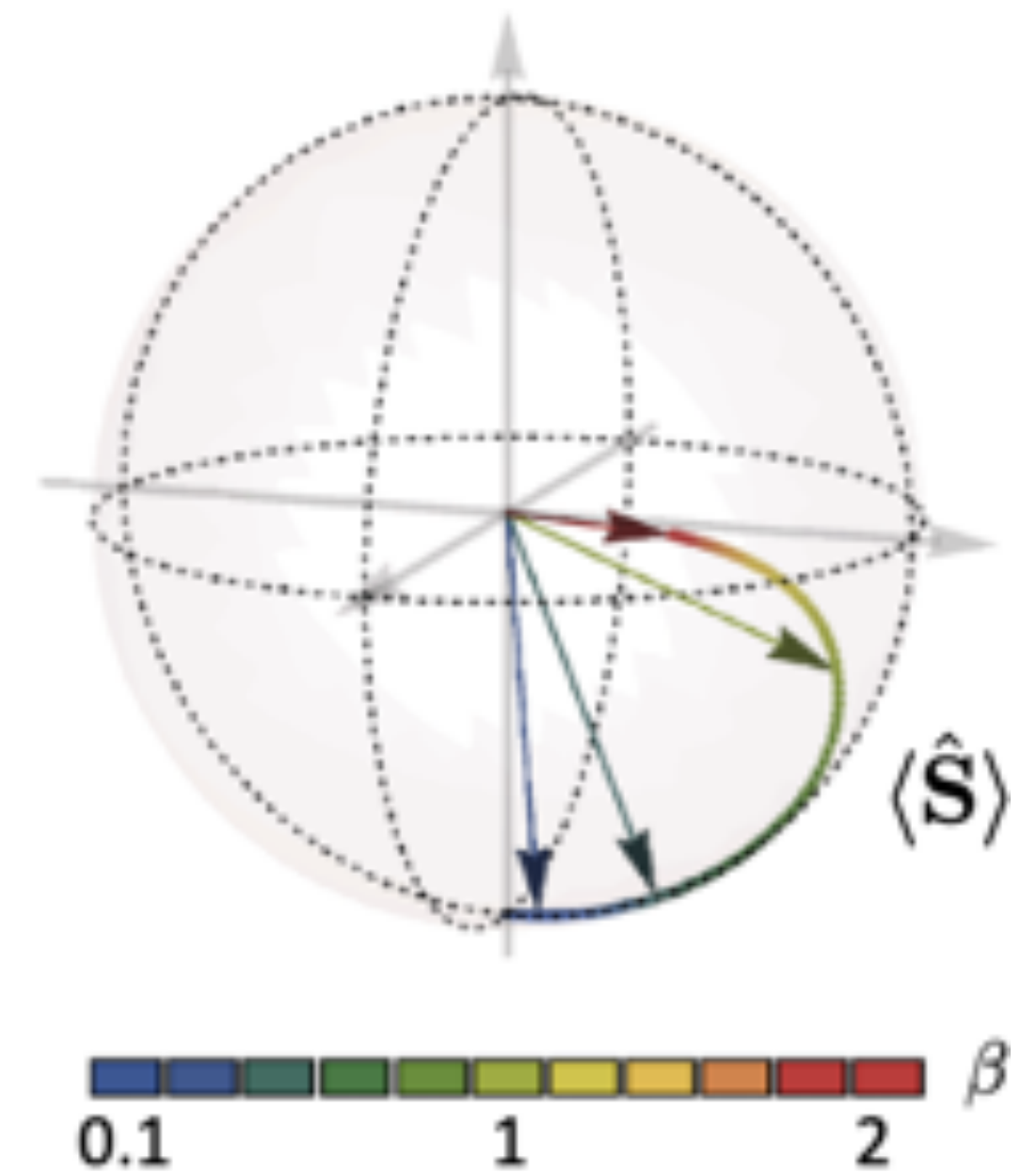
$$\dot{\hat{\rho}} = i\frac{\Omega_D}{2} \left( \hat{\rho}\hat{S}^+ + \hat{\rho}\hat{S}^- - \hat{S}^+\hat{\rho} - \hat{S}^-\hat{\rho} \right) + \frac{\Gamma_0}{2} \left( 2\hat{S}^-\hat{\rho}\hat{S}^+ - \hat{S}^+\hat{S}^-\hat{\rho} - \hat{\rho}\hat{S}^+\hat{S}^- \right)$$

$$\frac{d\langle\hat{S}^-\rangle}{dt} = i\Omega_D\langle\hat{S}^z\rangle + \frac{\Gamma_0}{2} \left[ \langle\hat{S}^z\hat{S}^-\rangle + \langle\hat{S}^-\hat{S}^z\rangle \right] - \frac{\Gamma_0}{2}\langle\hat{S}^-\rangle$$

$$\frac{d\langle\hat{S}^-\rangle}{dt} = \left( i\Omega_D + \Gamma_0\langle\hat{S}^-\rangle \right) \langle\hat{S}^z\rangle - \frac{\Gamma_0}{2}\langle\hat{S}^-\rangle \quad \text{assuming } \langle\hat{S}^-\hat{S}^z\rangle = \langle\hat{S}^-\rangle\langle\hat{S}^z\rangle$$

Effective Rabi frequency:  $\Omega_{\text{eff}} = \Omega_D - i\Gamma_0\langle\hat{S}^-\rangle$

Screening by collective dipole 



# Driven Dicke model

$$\frac{d\langle\hat{S}^{-}\rangle}{dt} = i\Omega_D\langle\hat{S}^z\rangle + \frac{\Gamma_0}{2} \left[ \langle\hat{S}^z\hat{S}^{-}\rangle + \langle\hat{S}^{-}\hat{S}^z\rangle \right] - \frac{\Gamma_0}{2}\langle\hat{S}^{-}\rangle$$

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# Driven Dicke model

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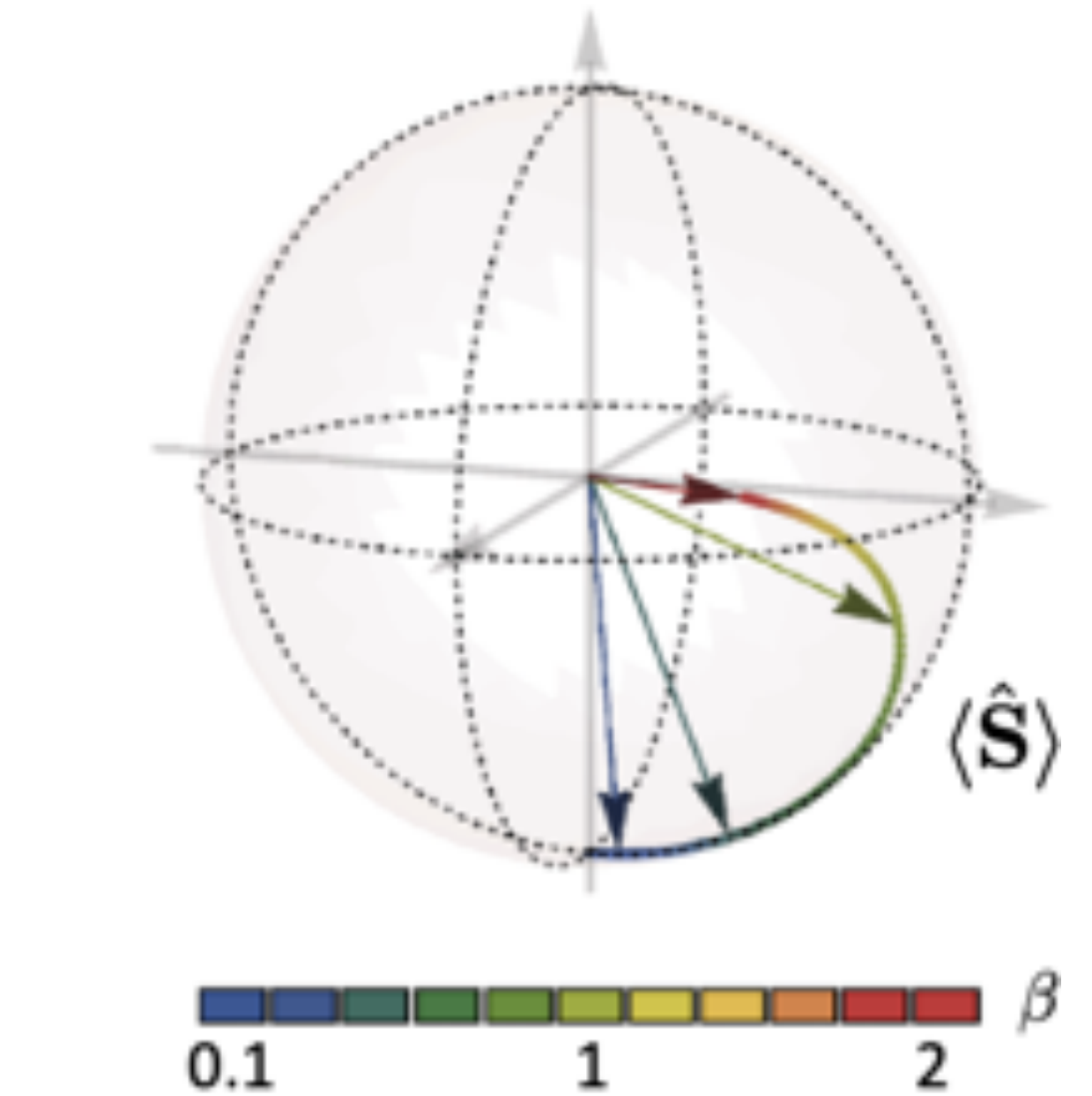
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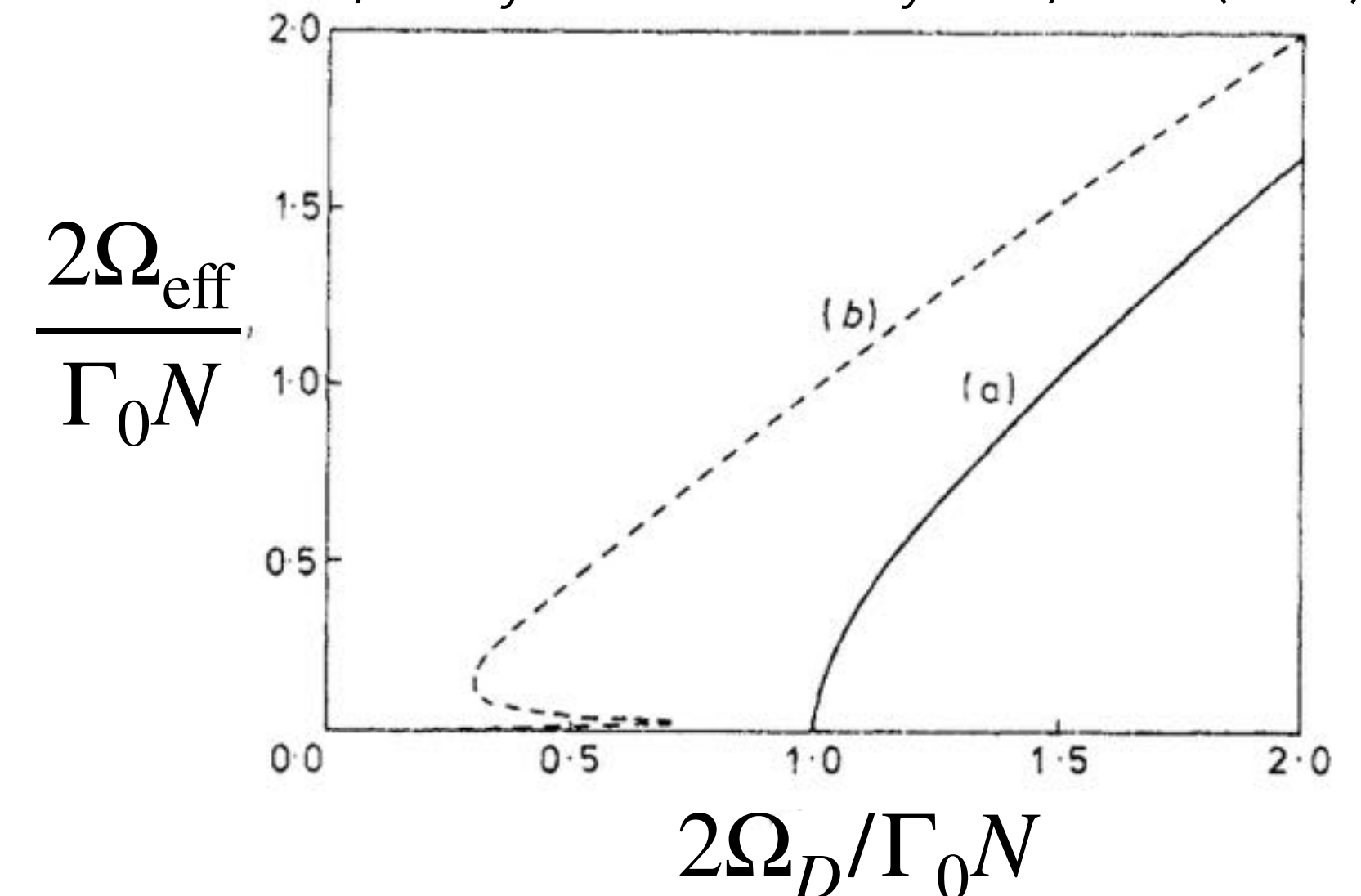
Maximum achievable dipole:  $\langle\hat{S}^{-}\rangle \simeq -iN/2$

Recover Rabi oscillations for  $\Omega_D > N\Gamma_0/2$

Scaling with  $\beta = 2\Omega_D/\Gamma_0N$



D. F. Walls, *J. Phys. B At. Mol. Phys.* **13**, 2001 (1973).





# Driven Dicke model

- $\beta < 1$  Magnetised phase

Collective dipole established

Emission rate  $\Gamma_0 \langle \hat{S}^+ \hat{S}^- \rangle \propto \Omega_D^2 / \Gamma_0$

- $\beta > 1$  Superradiant phase

Superradiant collective spont. em. dominates

Emission rate  $\Gamma_0 \langle \hat{S}^+ \hat{S}^- \rangle \propto N^2 \Gamma_0$

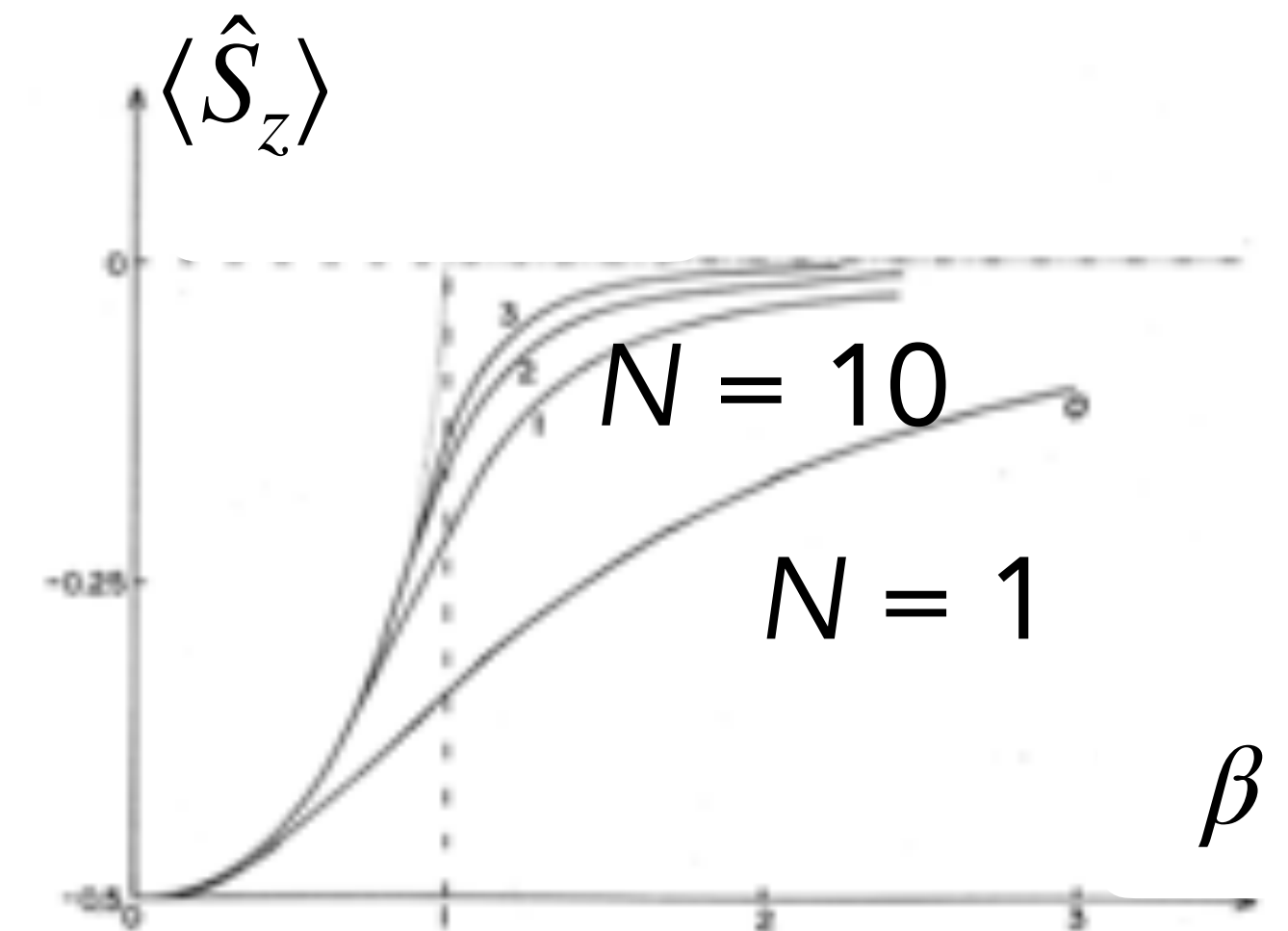
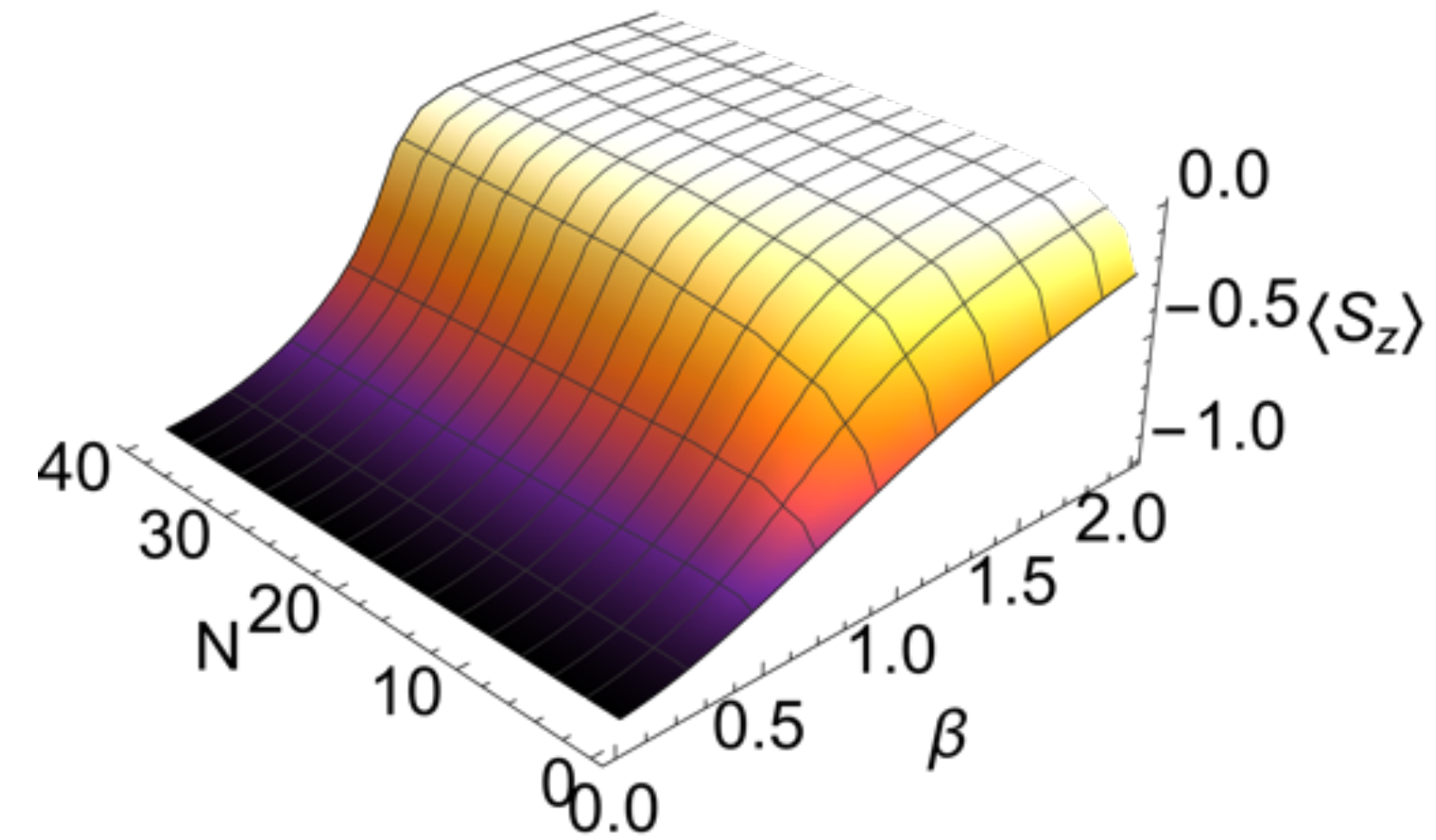
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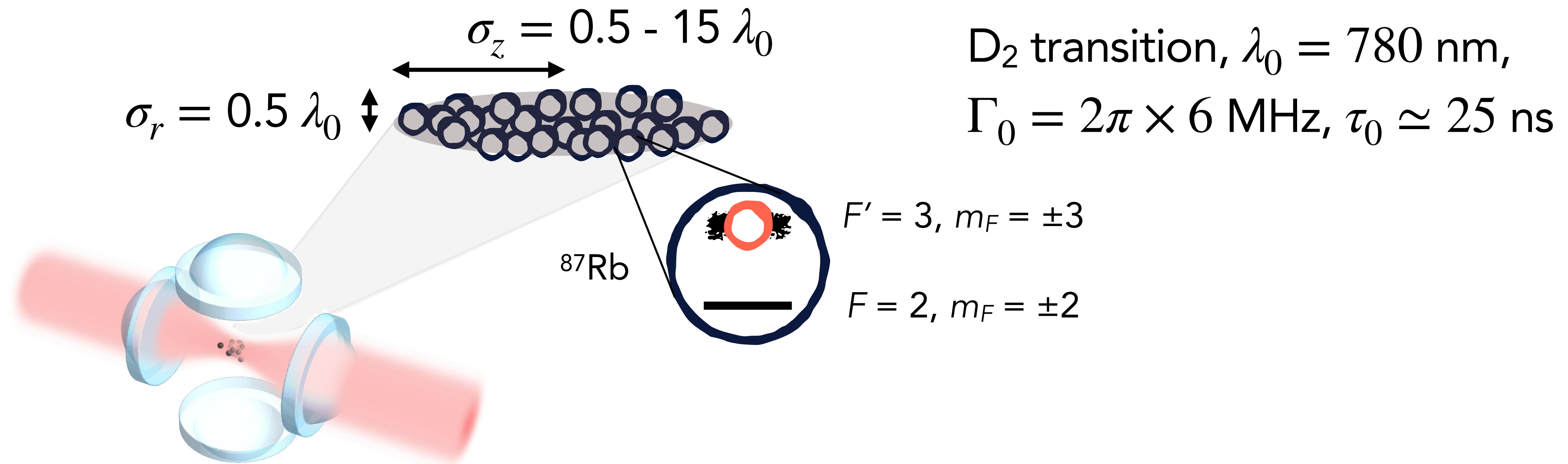
Phase transition for  $N \rightarrow \infty$

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Superradiant collective spont. em. dominates  
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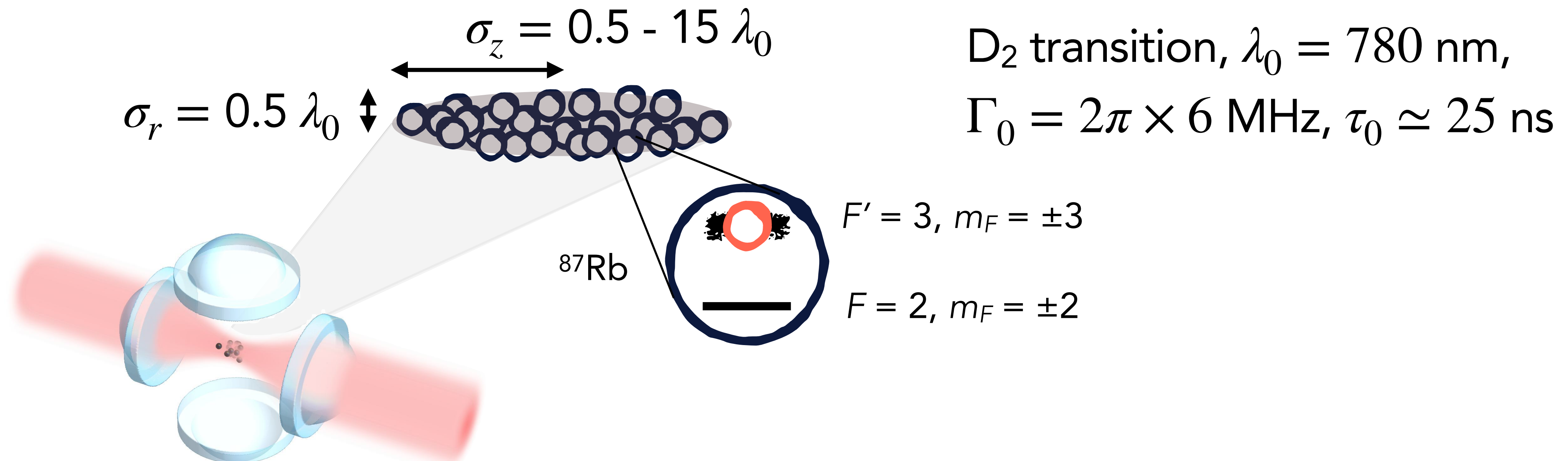
Narducci et al., *Phys Rev A* **18**, 1571–1576 (1978).

# Dense clouds of 2-level atoms





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Tight optical dipole trap (waist  $1.5 - 3 \mu\text{m}$ )

Clouds of  $N = 100 - 5000$  atoms

$T \sim 500 \mu\text{K}$ : no Doppler broadening, frozen distribution

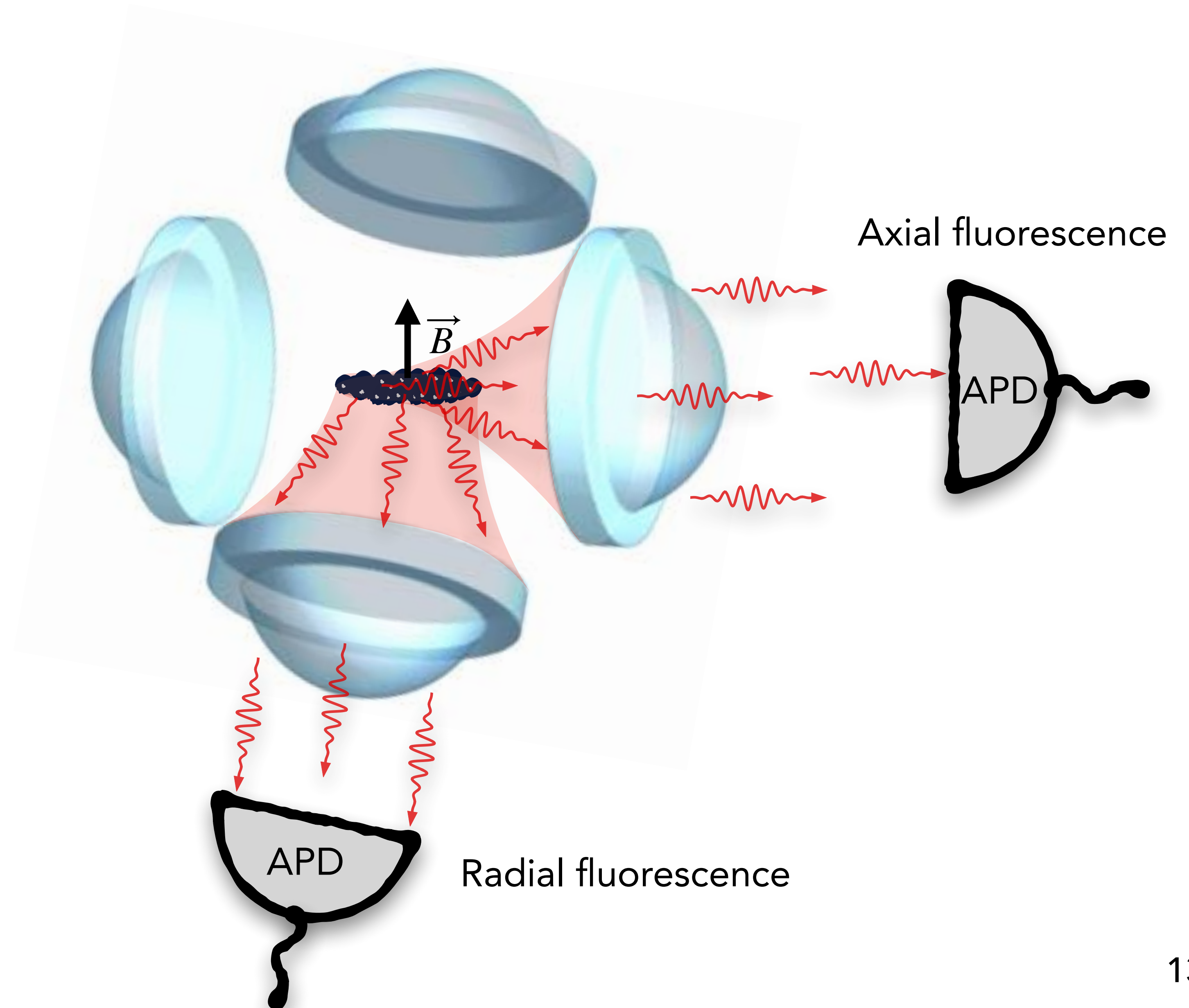
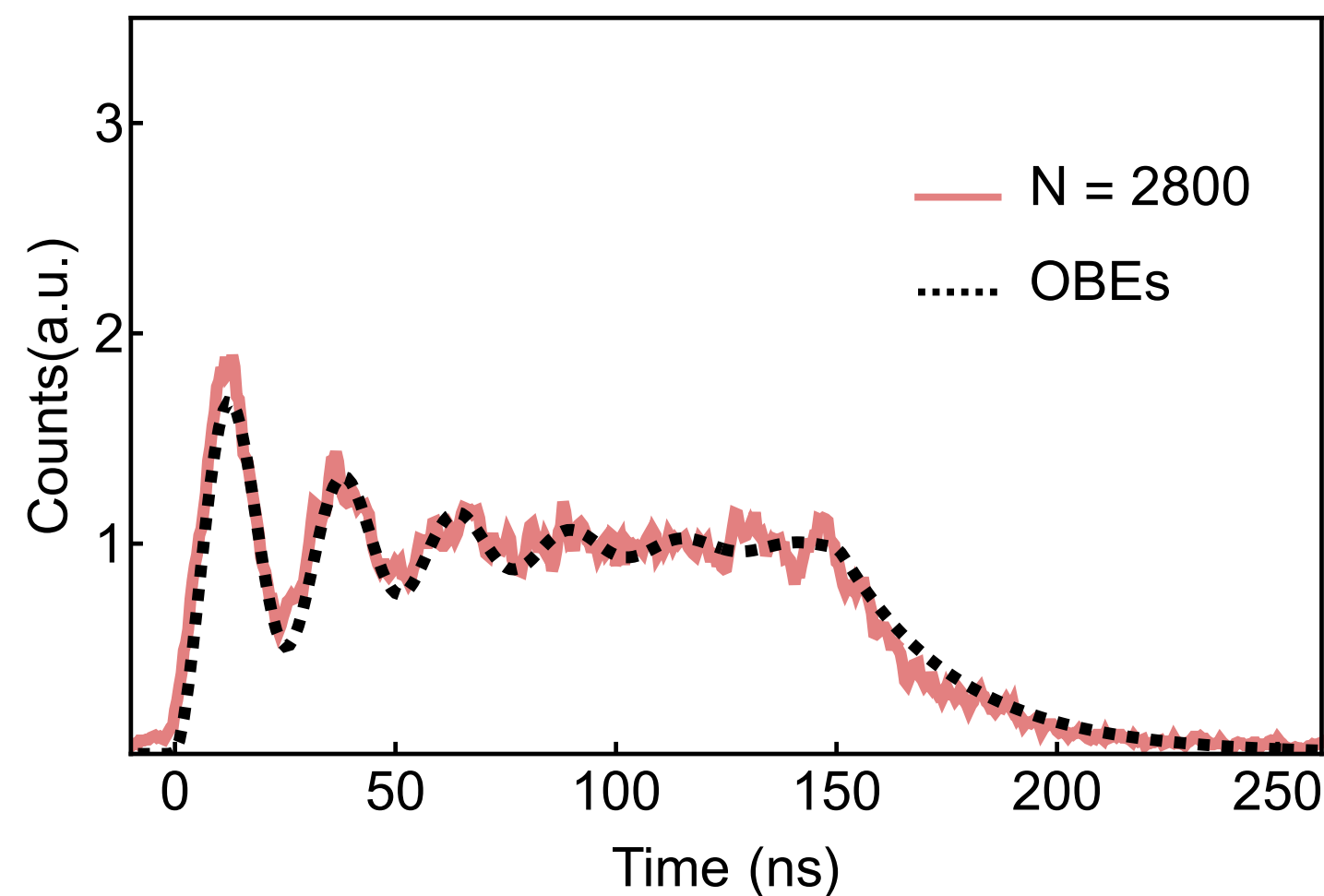
Optical pumping + magnetic field isolate 2 levels

# Dense clouds of 2-level atoms

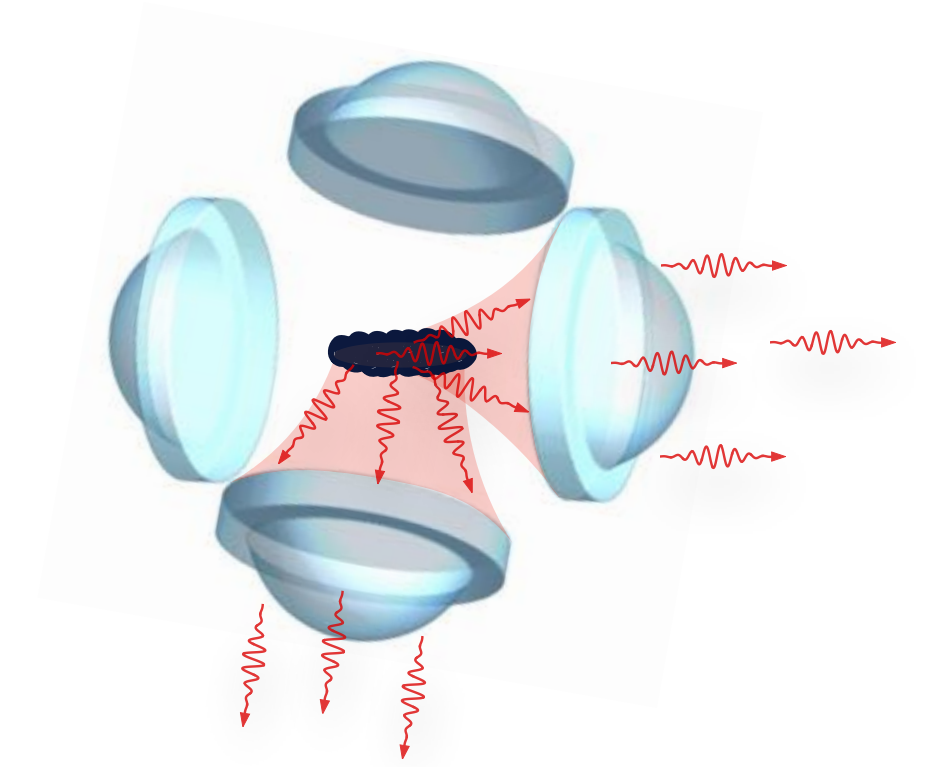
## Observations

Single photon detection (NA=0.45)

Along two directions



# $N$ -atom spontaneous emission





# $N$ -atom spontaneous emission

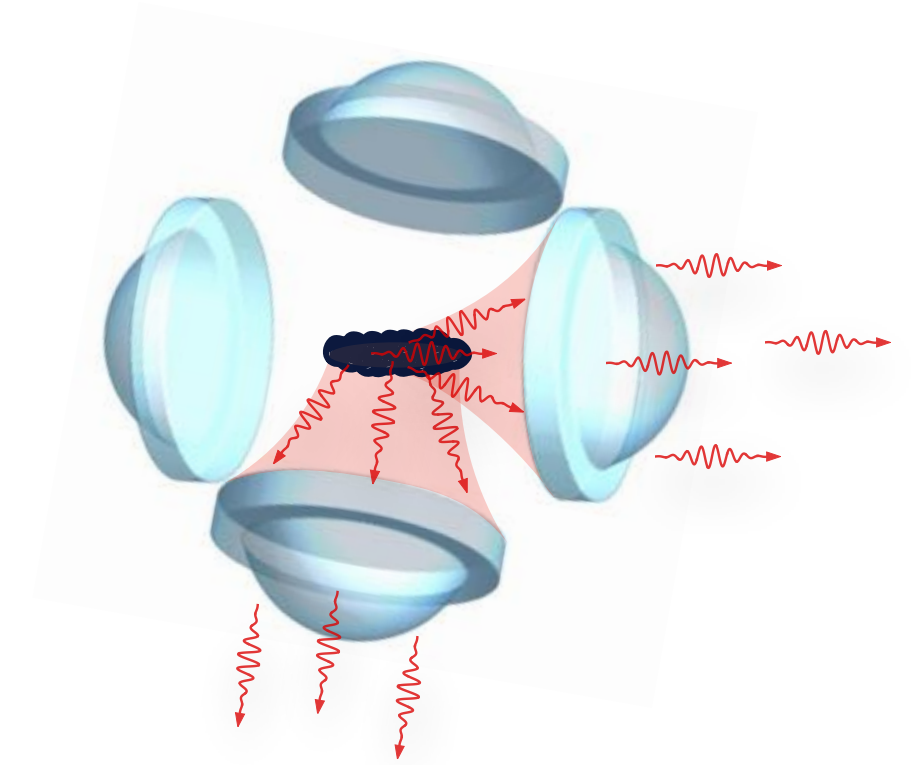
Intensity

$$I(\mathbf{r}) = \langle \hat{E}^-(\mathbf{r}) \hat{E}^+(\mathbf{r}) \rangle$$

$$\hat{E}^+(\mathbf{r}) \propto \sum_n G(\mathbf{r}, \mathbf{r}_i, \omega_0) \hat{\sigma}_n^-$$

$$I_N(\mathbf{k}) = I_1(\mathbf{k}) \sum_n \left[ \langle \hat{e}_n \rangle + \sum_{m \neq n} e^{i\mathbf{k} \cdot (\mathbf{r}_m - \mathbf{r}_n)} \langle \hat{\sigma}_m^+ \hat{\sigma}_n^- \rangle \right]$$

Dissipation



# N-atom spontaneous emission

Intensity

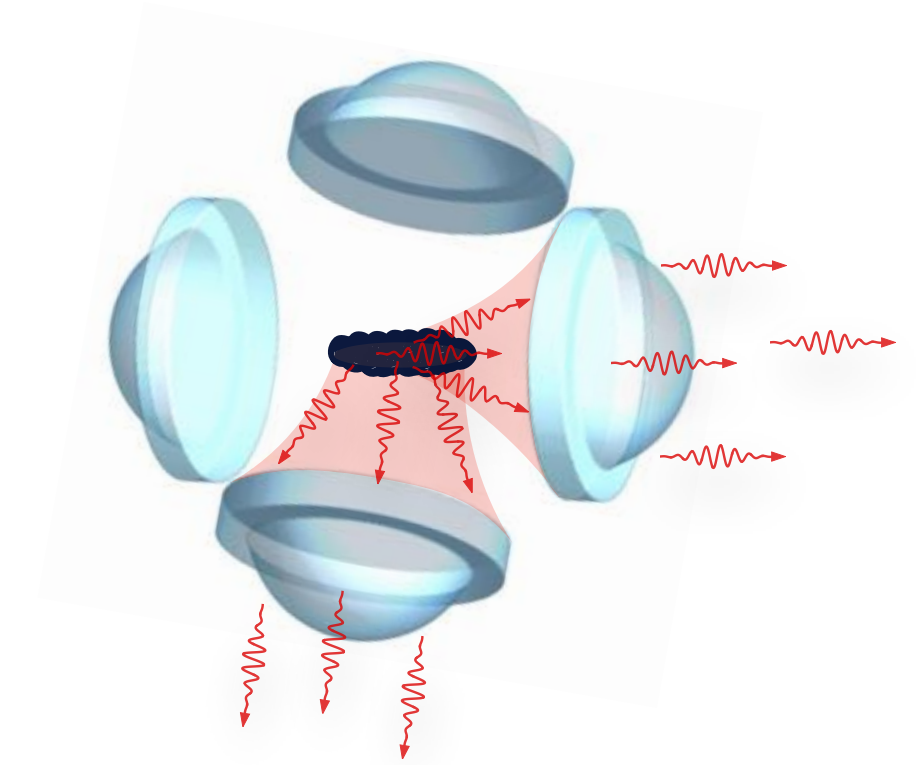
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Dissipation

Correlations



# N-atom spontaneous emission

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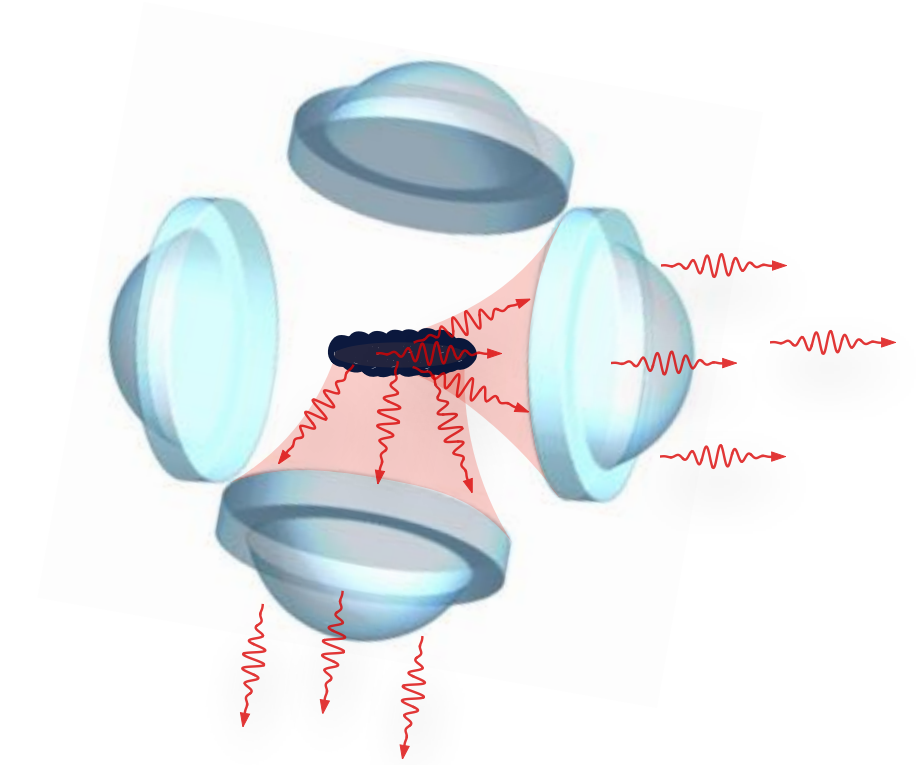
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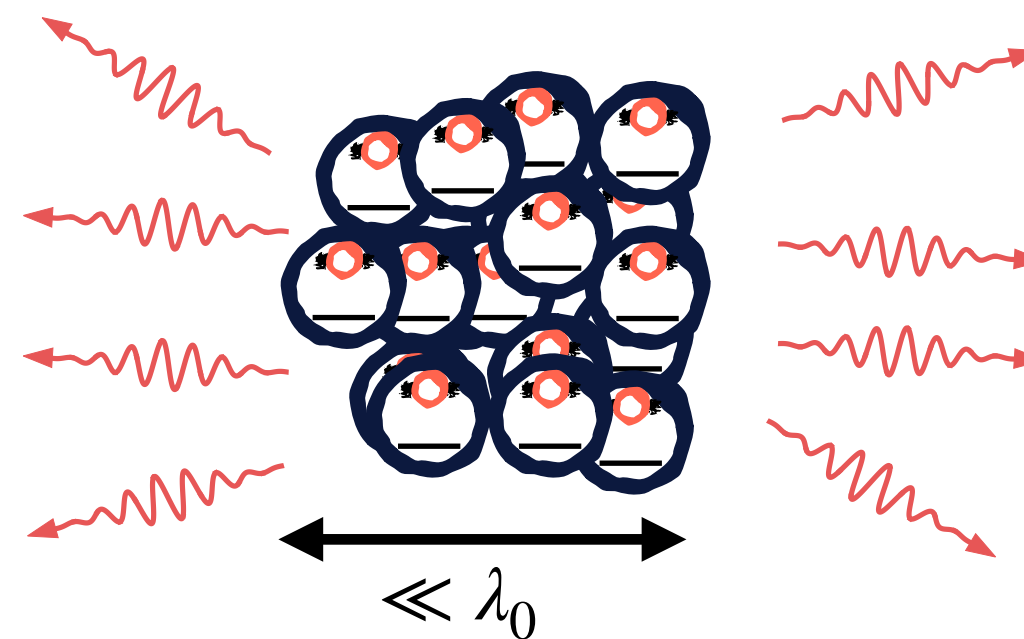
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Dissipation

Correlations



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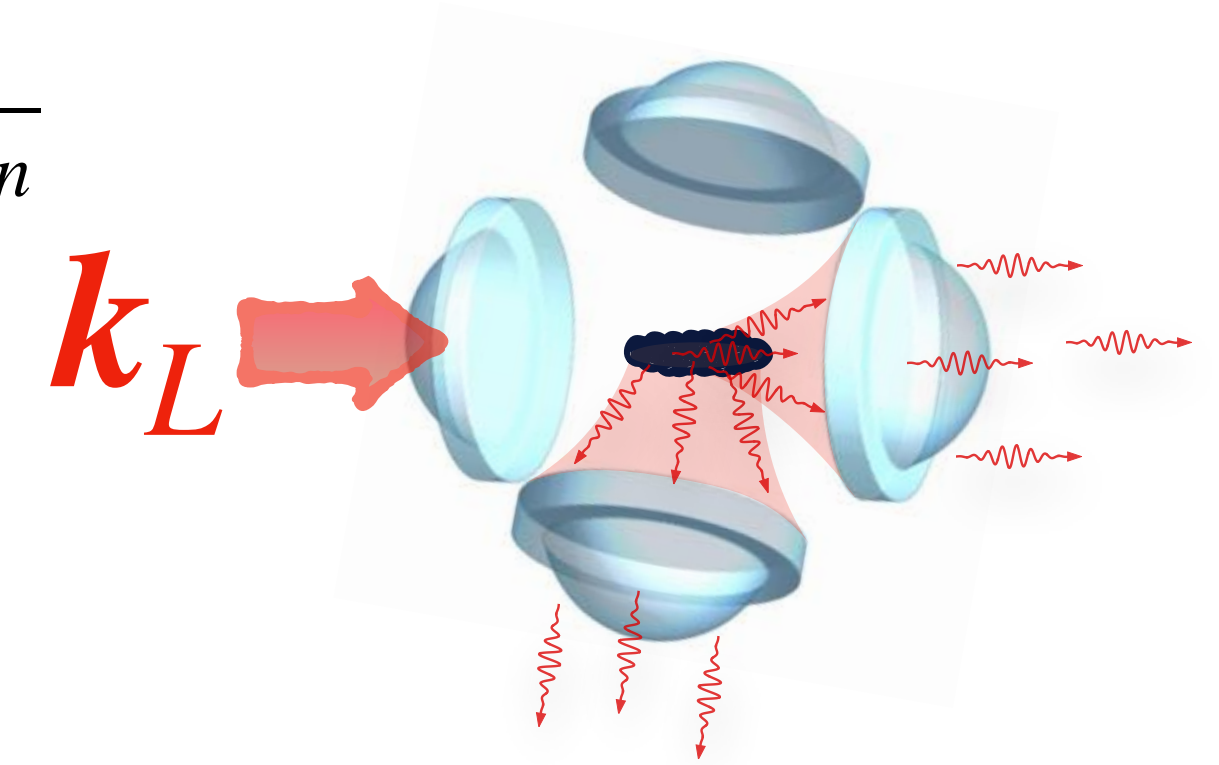
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↑
↑  
 Dissipation                      Correlations



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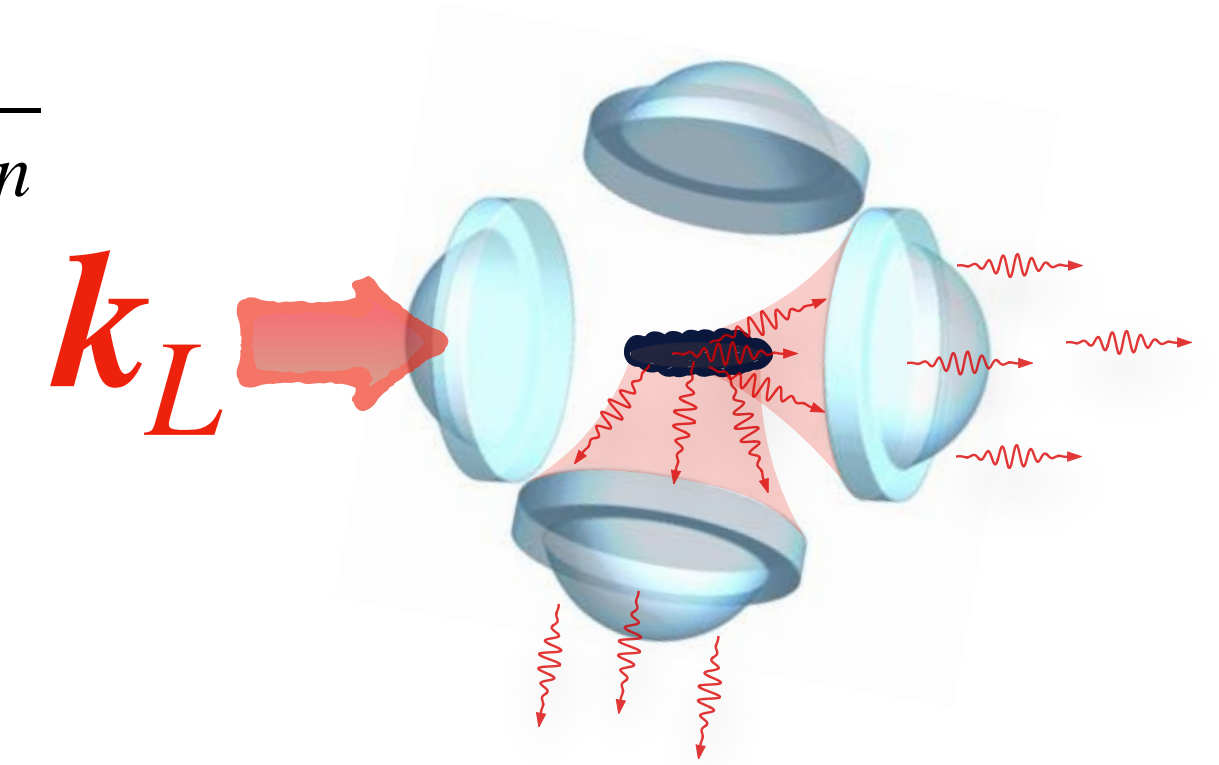
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↑ Dissipation
 ↑ Correlations



Spin wave:  $\langle \hat{\sigma}_m^+ \hat{\sigma}_n^- \rangle = e^{-i\mathbf{k}_{SW} \cdot (\mathbf{r}_m - \mathbf{r}_n)} \langle \hat{\sigma}_1^+ \hat{\sigma}_2^- \rangle$

E.g. laser driving:  $|\psi\rangle = \prod_n (\cos \theta |g_n\rangle + e^{-i\mathbf{k}_L \cdot \mathbf{r}_n} \sin \theta |e_n\rangle)$   $\langle \hat{\sigma}_1^+ \hat{\sigma}_2^- \rangle = \cos \theta \sin \theta$

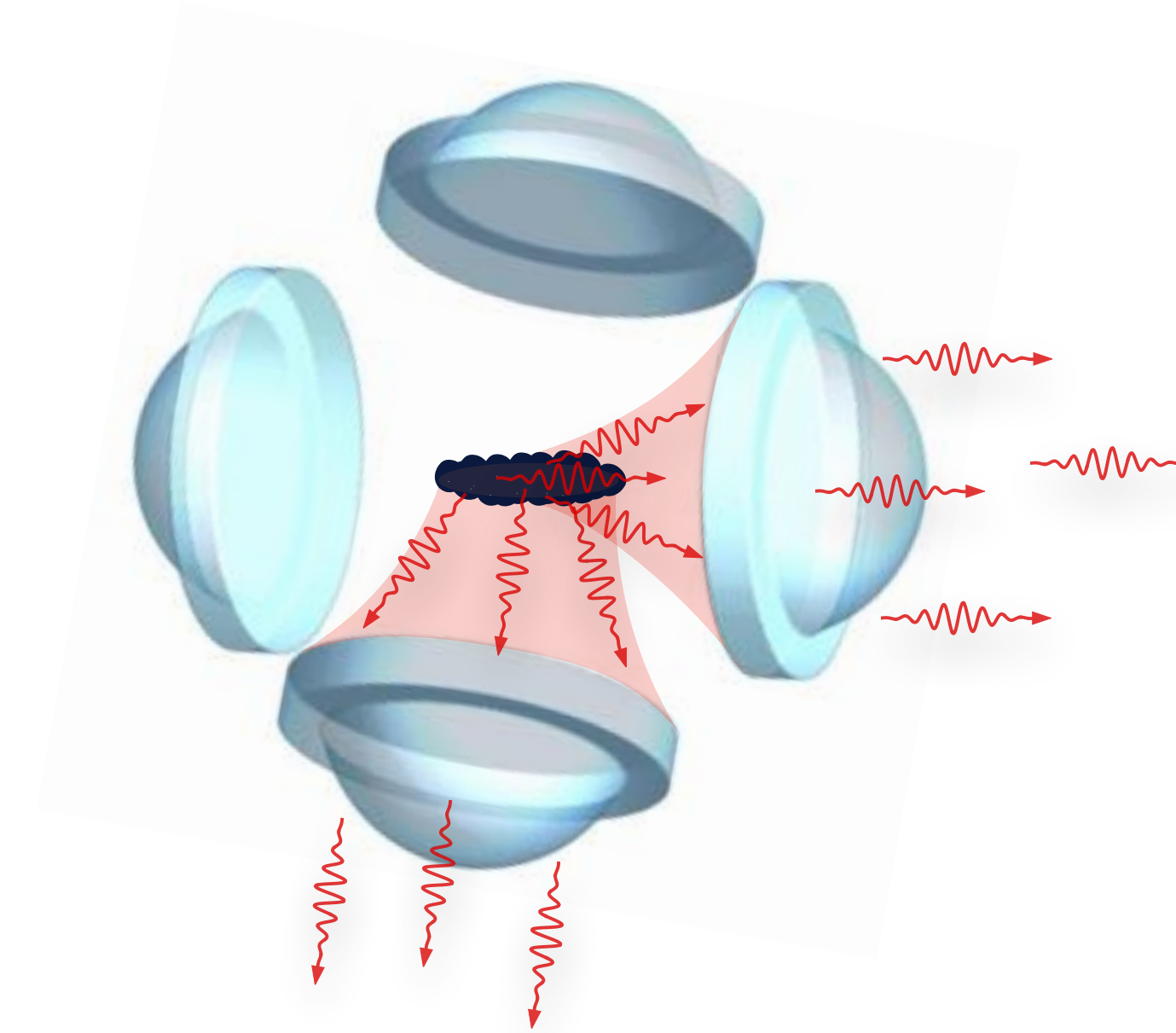
$$I_N(\mathbf{k}) = I_1(\mathbf{k}) \left[ \sum_n \langle \hat{e}_n \rangle + \langle \hat{\sigma}_1^+ \hat{\sigma}_2^- \rangle \sum_{m,n} e^{i(\mathbf{k} - \mathbf{k}_{SW}) \cdot (\mathbf{r}_m - \mathbf{r}_n)} \right]$$

Gross & Haroche, *Physics Reports* **93**, 301 (1982).

Allen & Eberly, *Optical resonance and two-level atoms*, Courier Corp. (1987)

# $N$ -atom spontaneous emission

$$I_N(\mathbf{k}) = I_1(\mathbf{k}) \left[ N\langle \hat{e} \rangle + N^2 \langle \hat{\sigma}_1^+ \hat{\sigma}_2^- \rangle \left| \frac{1}{N} \sum_n e^{i(\mathbf{k} - \mathbf{k}_{SW}) \cdot \mathbf{r}_n} \right|^2 \right]$$



Gross & Haroche, *Physics Reports* **93**, 301 (1982).

Allen & Eberly, *Optical resonance and two-level atoms*, Courier Corp. (1987)



# N-atom spontaneous emission

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$$\mu(\mathbf{k}_{SW}) = \frac{P_{\text{coop}}}{NP_1} \sim \frac{\Delta\Omega}{4\pi} \quad C = \mu N$$

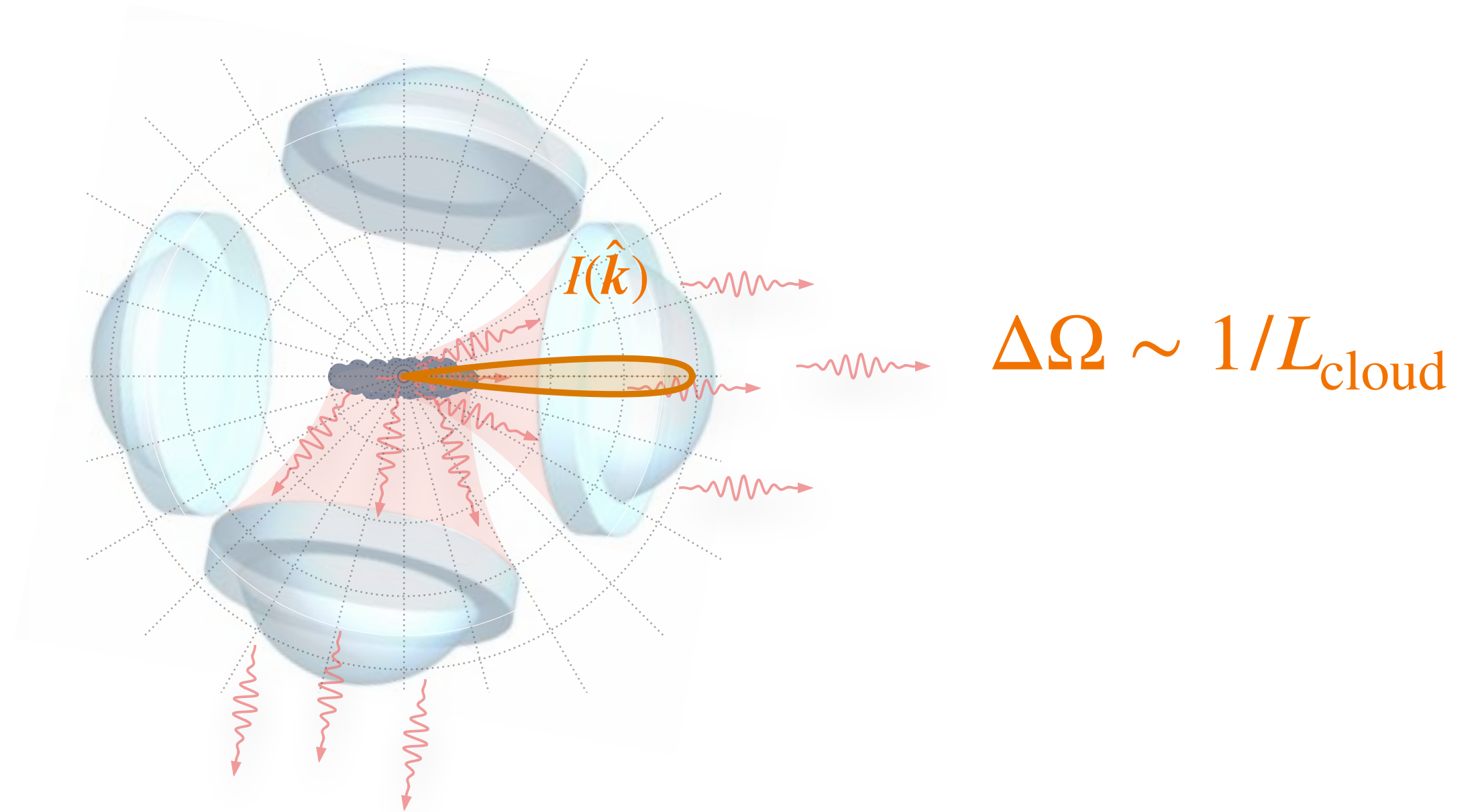
Dicke case:  $\mu = 1$

General case: Effective atom number  
 $N \rightarrow \tilde{N} = \mu N$

For large, dilute clouds, one has  $\mu N = OD$

Gross & Haroche, *Physics Reports* **93**, 301 (1982).

Allen & Eberly, *Optical resonance and two-level atoms*, Courier Corp. (1987)



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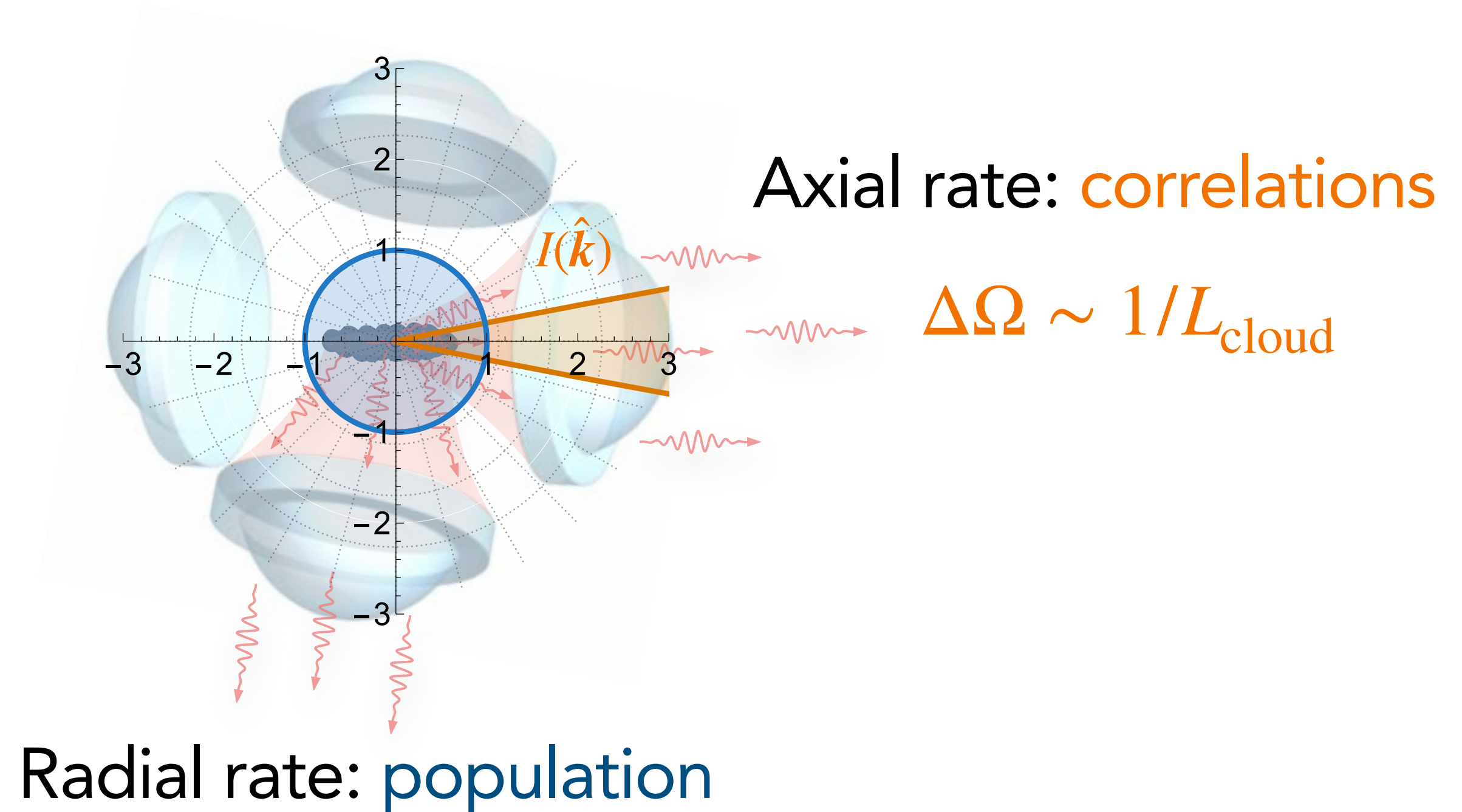
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For large, dilute clouds, one has  $\mu N = OD$

Gross & Haroche, *Physics Reports* **93**, 301 (1982).

Allen & Eberly, *Optical resonance and two-level atoms*, Courier Corp. (1987)



# Analogy with clouds in cavity

Small clouds

Clouds in cavity

Collective coupling to:

Diffraction pattern

Cavity mode

Set by:

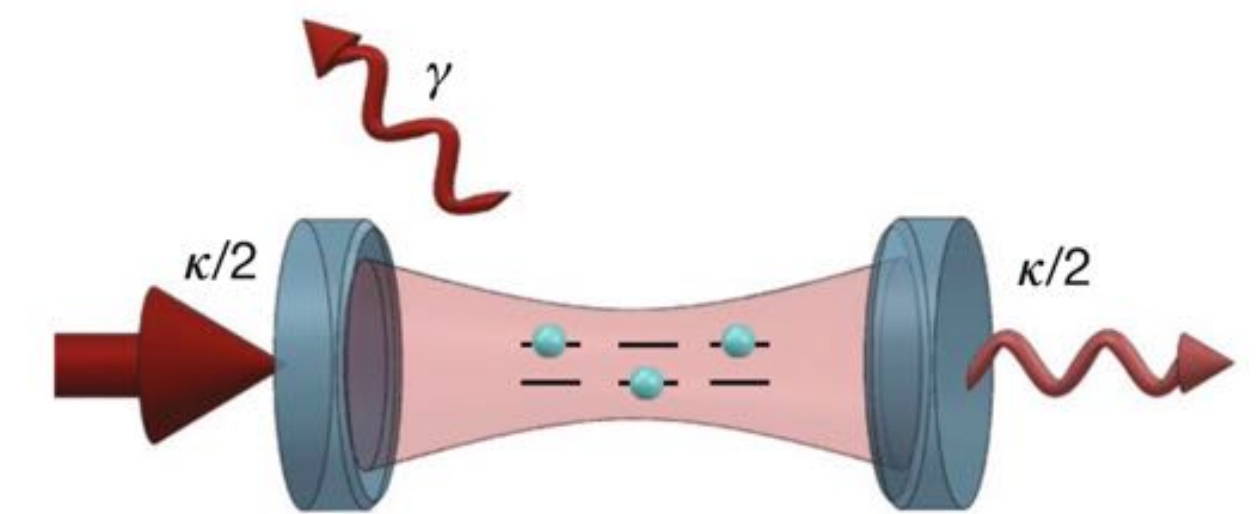
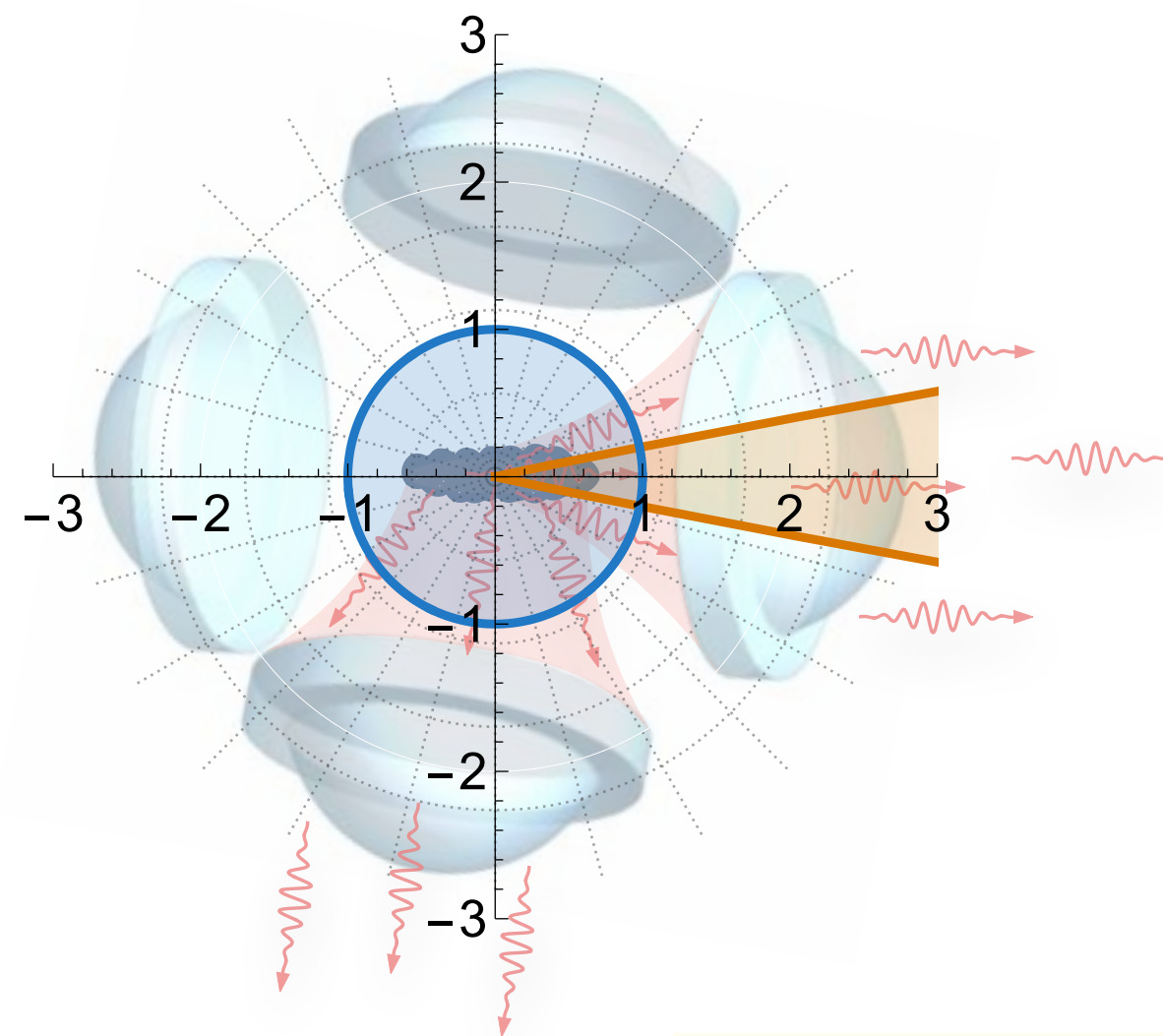
Cloud shape / structure factor

Cavity geometry

Cooperativity  $C_N = P_{\text{coop}}/P_1$

$$N \times \mu$$

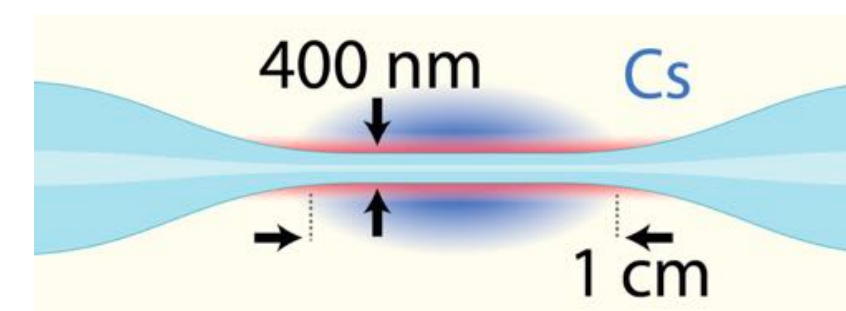
$$N \times (g^2/\kappa\gamma)$$



from: *Nature* **580**, 602 (2020)

Similarities with nanofiber-coupled atoms

See e.g. arXiv: 2211.08940 (2022)





# Driving axially: Dicke symmetry restored

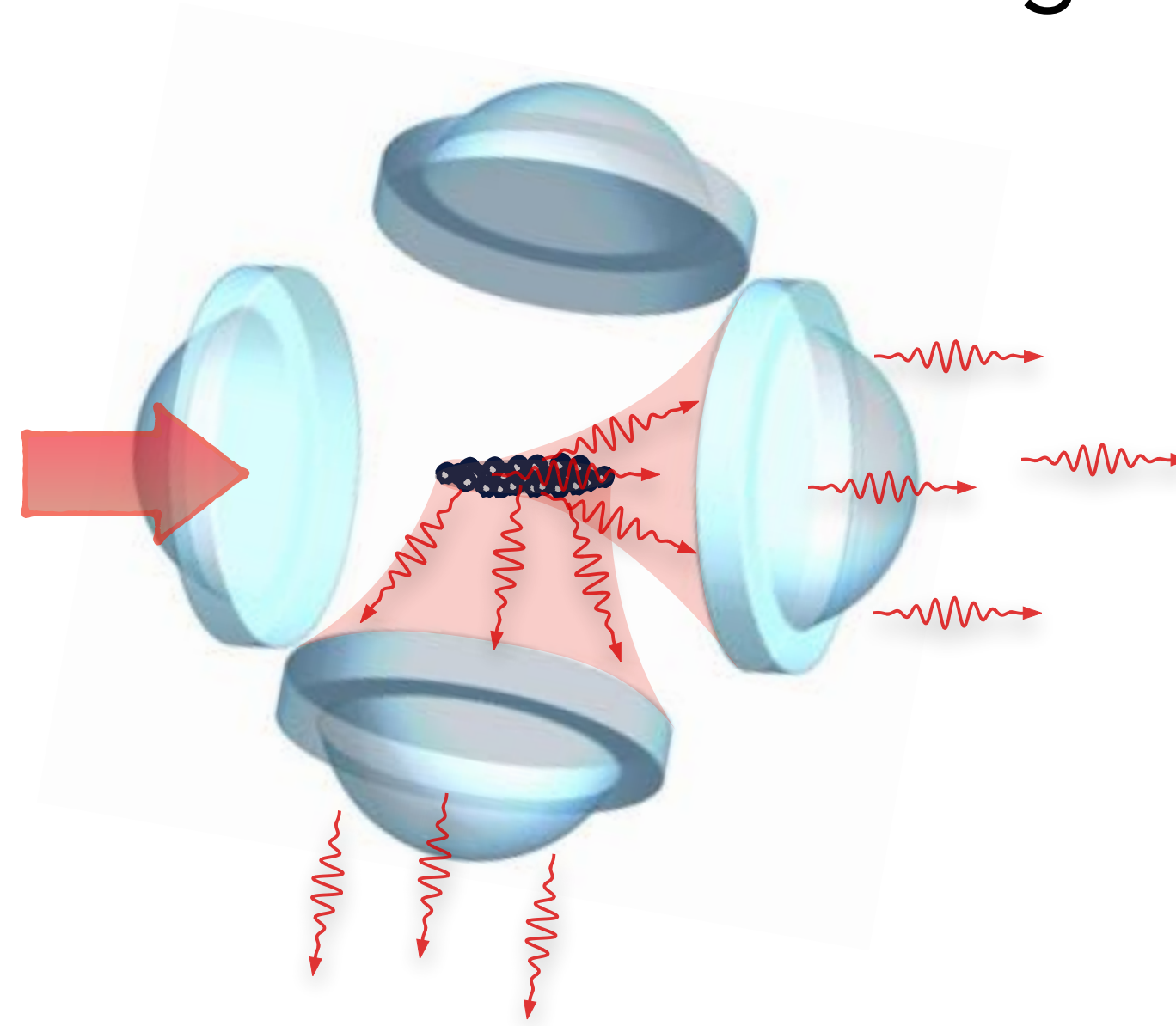
Dicke case: Indistinguishable coupling to

- Emission mode ✓
- Driving mode ✓

Driving "distinguishably"

$k_L$

Driving "indistinguishably"



Superradiant mode  
Atoms indistinguishably coupled

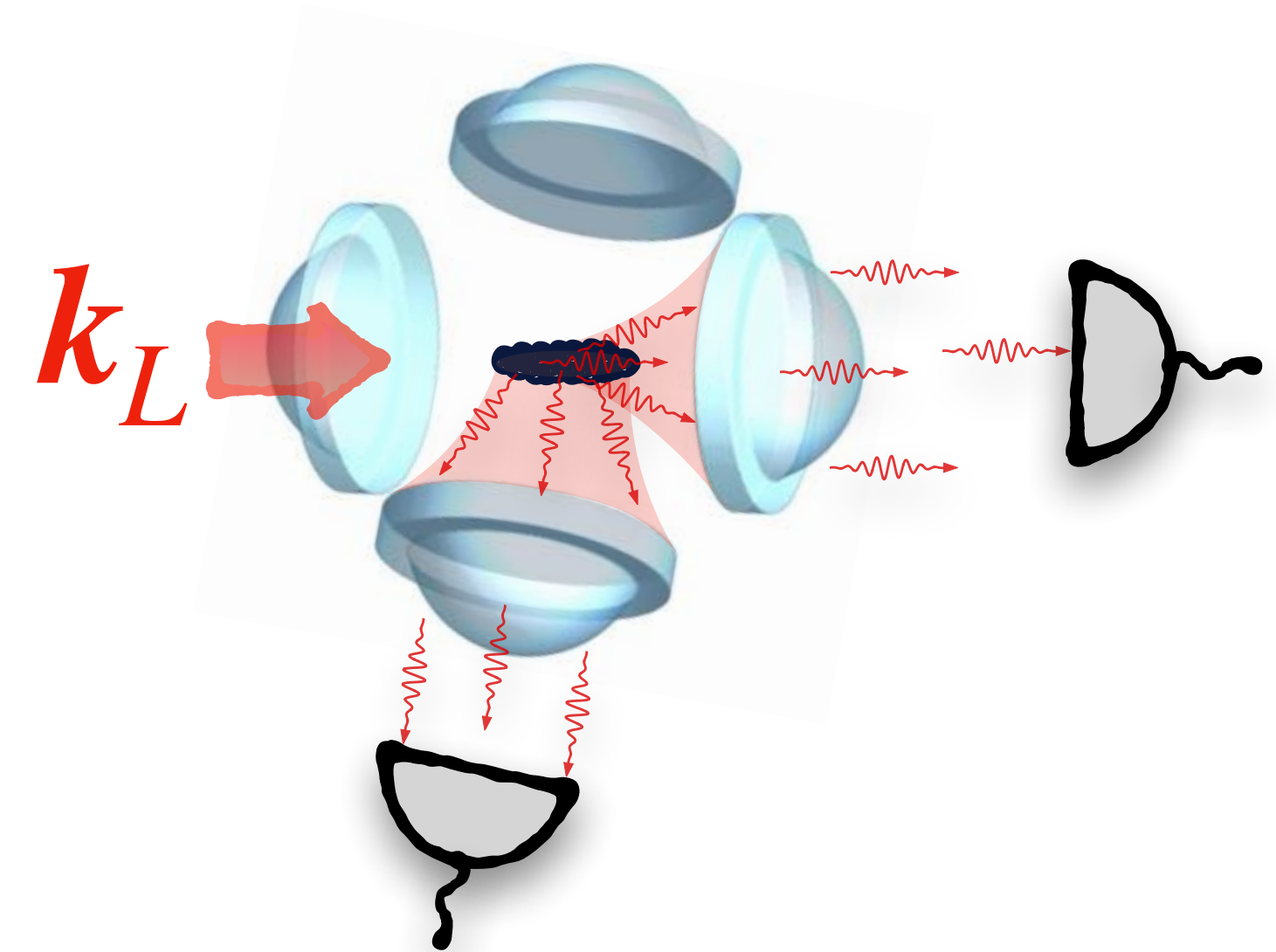
# Driving axially: Dicke symmetry restored

Classically: dipole  $d_n \propto \langle \sigma_n^- \rangle$

$$\sim \Gamma_0 \frac{e^{ik_0|\mathbf{r}_m - \mathbf{r}_n|}}{4\pi k_0 |\mathbf{r}_m - \mathbf{r}_n|}$$

$$\frac{d\langle \hat{\sigma}_n^- \rangle}{dt} = i\Omega_D e^{ik_L \cdot \mathbf{r}_n} \langle \hat{\sigma}_n^z \rangle - \frac{\Gamma_0}{2} \langle \hat{\sigma}_n^- \rangle + \sum_{m \neq n} \frac{\Gamma_{mn}}{2} [\langle \hat{\sigma}_n^z \hat{\sigma}_m^- \rangle + \langle \hat{\sigma}_m^- \hat{\sigma}_n^z \rangle]$$

\*Ignoring real part of DDI



# Driving axially: Dicke symmetry restored

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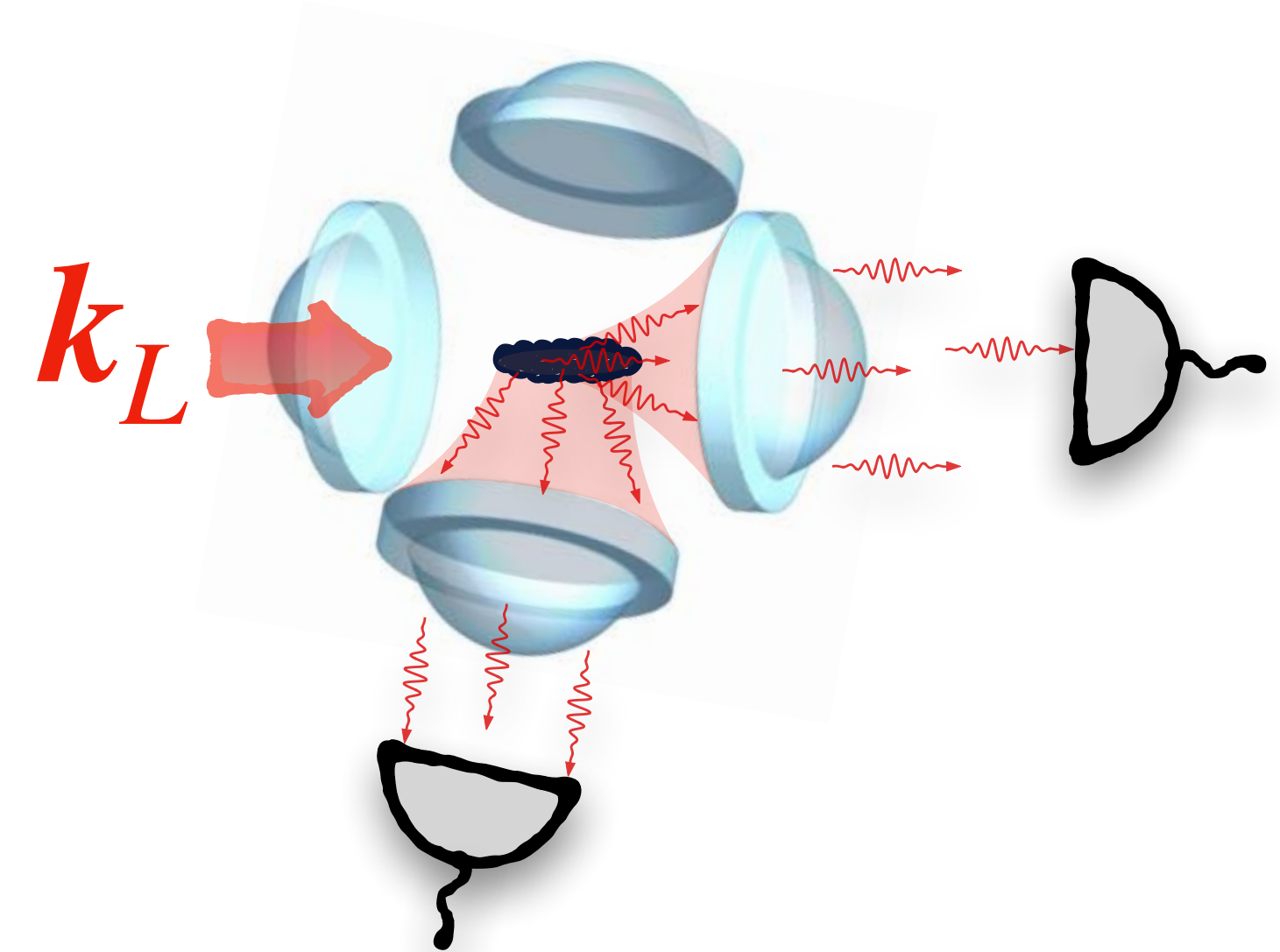
$$\sim \Gamma_0 \frac{e^{ik_0|\mathbf{r}_m - \mathbf{r}_n|}}{4\pi k_0 |\mathbf{r}_m - \mathbf{r}_n|}$$

$$\frac{d\langle \hat{\sigma}_n^- \rangle}{dt} = i\Omega_D e^{ik_L \cdot \mathbf{r}_n} \langle \hat{\sigma}_n^z \rangle - \frac{\Gamma_0}{2} \langle \hat{\sigma}_n^- \rangle + \sum_{m \neq n} \frac{\Gamma_{mn}}{2} [\langle \hat{\sigma}_n^z \hat{\sigma}_m^- \rangle + \langle \hat{\sigma}_m^- \hat{\sigma}_n^z \rangle]$$

\*Ignoring real part of DDI

Mean-field ansatz:  $\langle \hat{\sigma}_n^- \rangle = \langle \hat{\sigma}^- \rangle e^{ik_L \cdot \mathbf{r}_n}$        $\langle \hat{\sigma}_n^z \rangle = \langle \hat{\sigma}^z \rangle$       Allen & Eberly (1987)

$$\frac{d\langle \hat{\sigma}^- \rangle}{dt} = (i\Omega_D + \Gamma_0 \mu N \langle \hat{\sigma}^- \rangle) \langle \hat{\sigma}^z \rangle - \frac{\Gamma_0}{2} \langle \hat{\sigma}^- \rangle$$



# Driving axially: Dicke symmetry restored

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\*Ignoring real part of DDI

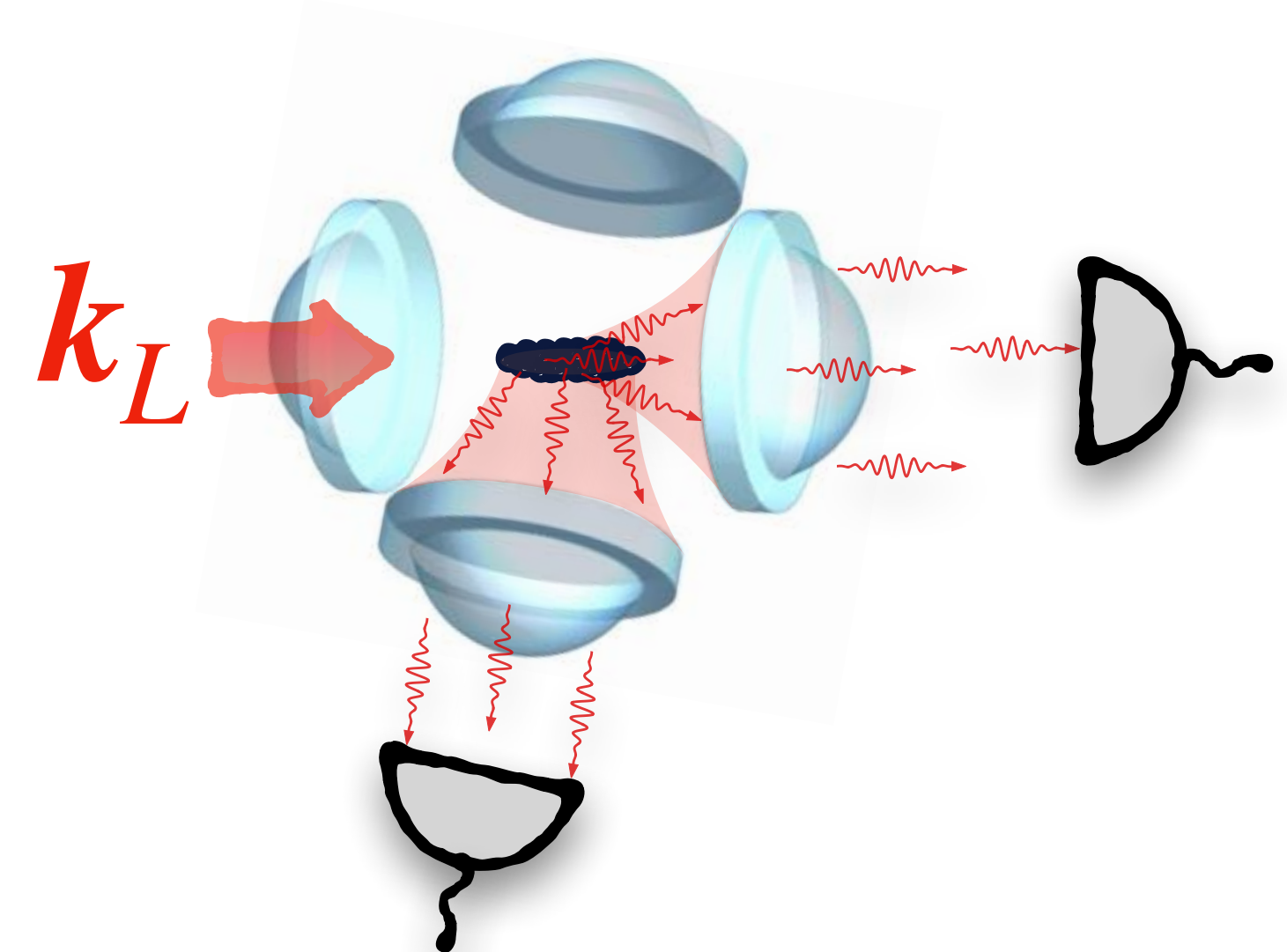
Mean-field ansatz:  $\langle \hat{\sigma}_n^- \rangle = \langle \hat{\sigma}^- \rangle e^{ik_L \cdot \mathbf{r}_n}$      $\langle \hat{\sigma}_n^z \rangle = \langle \hat{\sigma}^z \rangle$     Allen & Eberly (1987)

$$\frac{d\langle \hat{\sigma}^- \rangle}{dt} = (i\Omega_D + \Gamma_0 \mu N \langle \hat{\sigma}^- \rangle) \langle \hat{\sigma}^z \rangle - \frac{\Gamma_0}{2} \langle \hat{\sigma}^- \rangle$$

$$N \rightarrow \tilde{N} = \mu N$$

$$\frac{d\langle \hat{\sigma}^- \rangle}{dt} = (i\Omega_D + \Gamma_0 N \langle \hat{\sigma}^- \rangle) \langle \hat{\sigma}^z \rangle - \frac{\Gamma_0}{2} \langle \hat{\sigma}^- \rangle$$

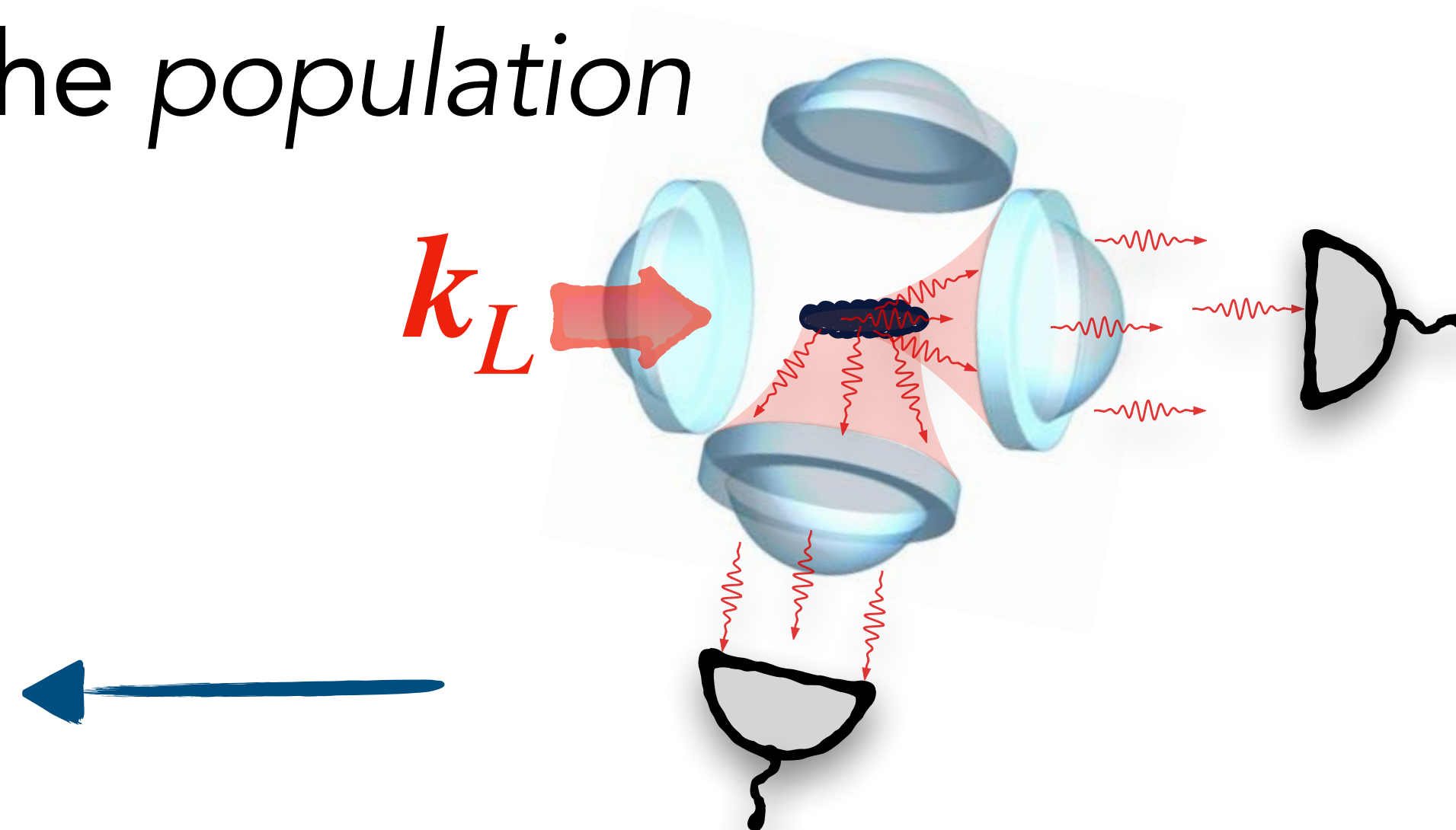
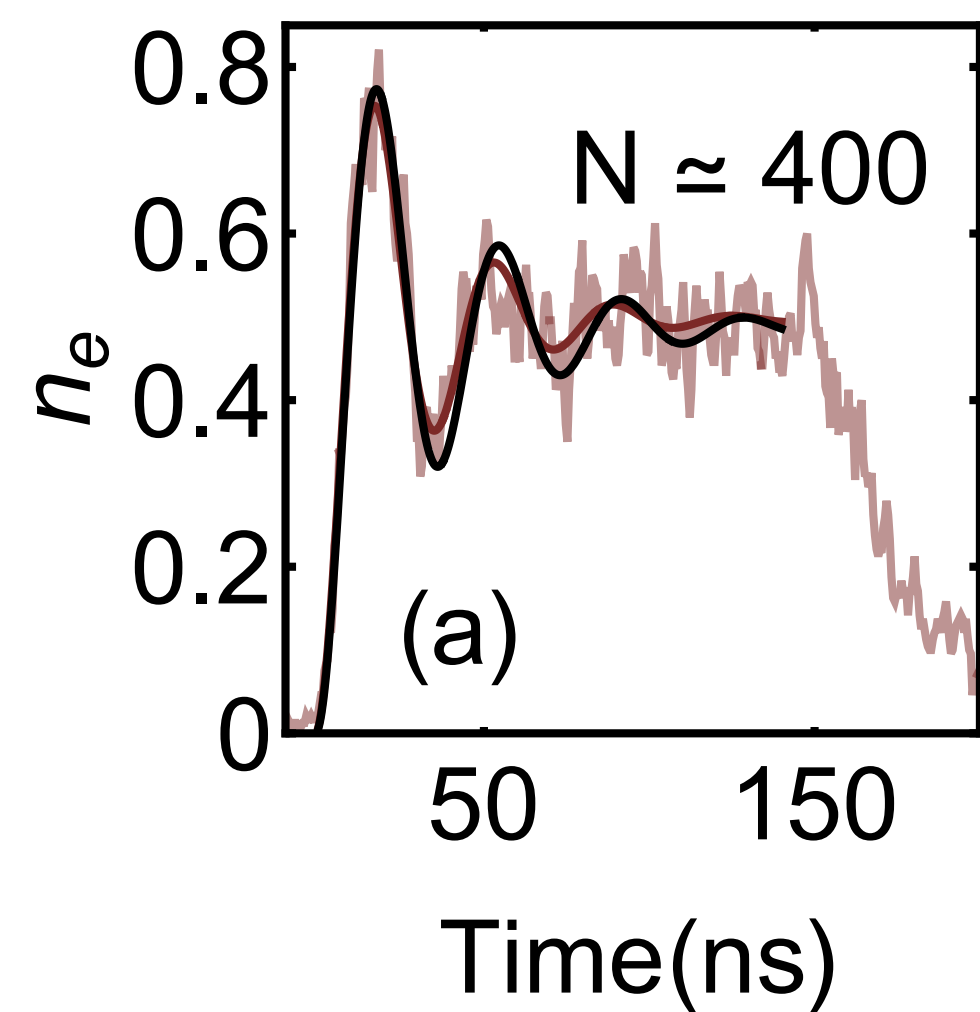
Dicke case





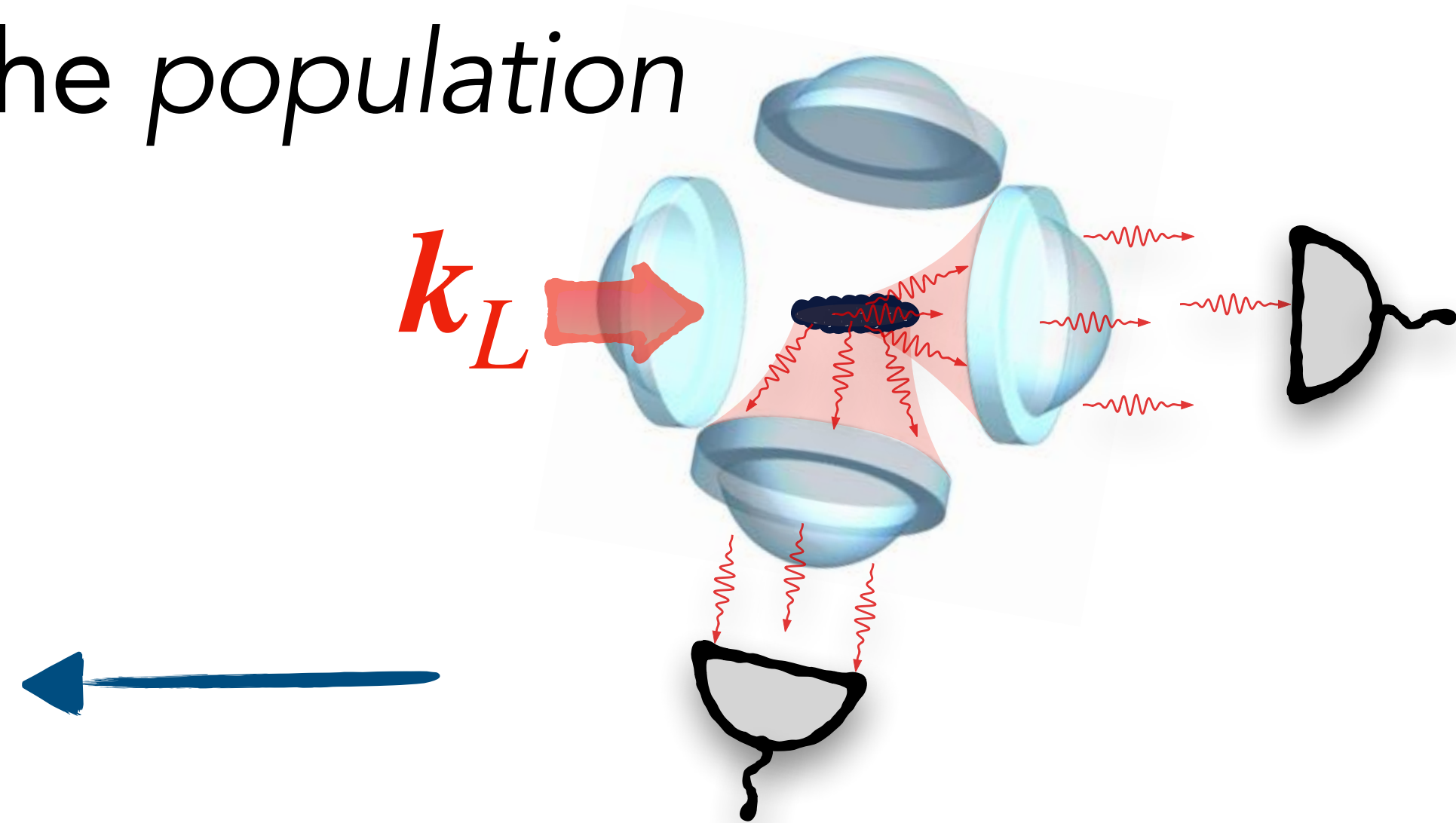
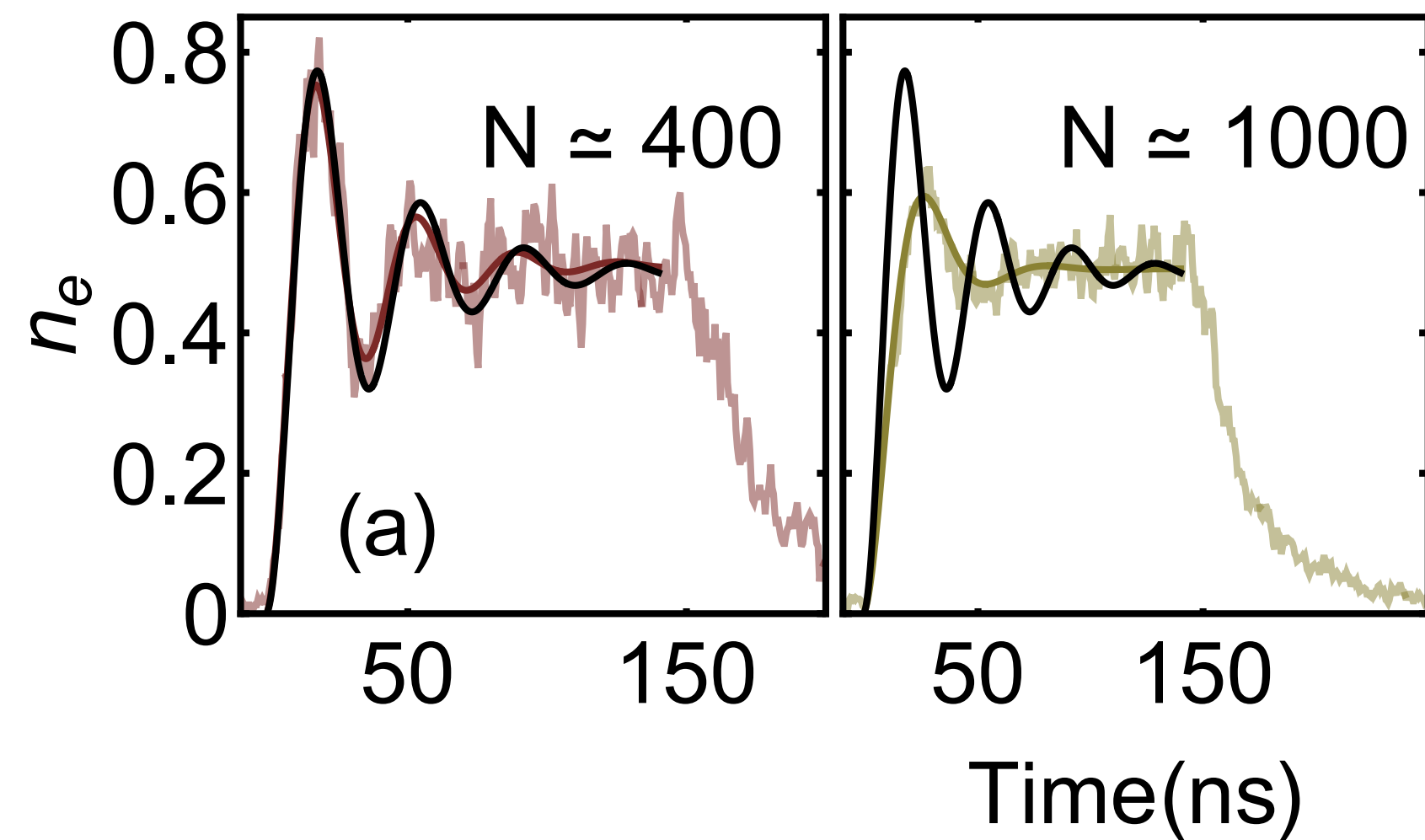
# Rabi oscillations driving Axially

Modification of Rabi oscillations in the *population*



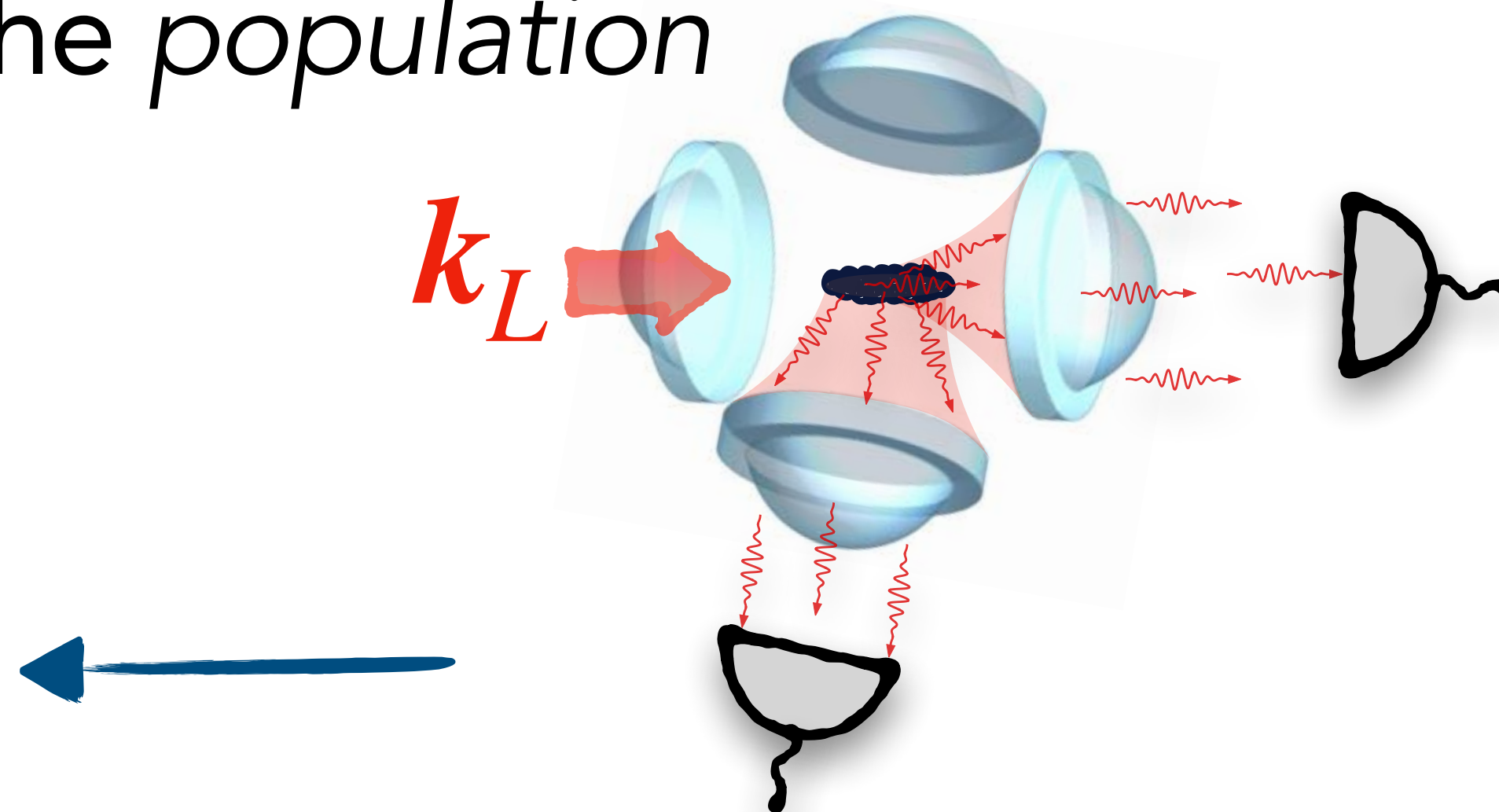
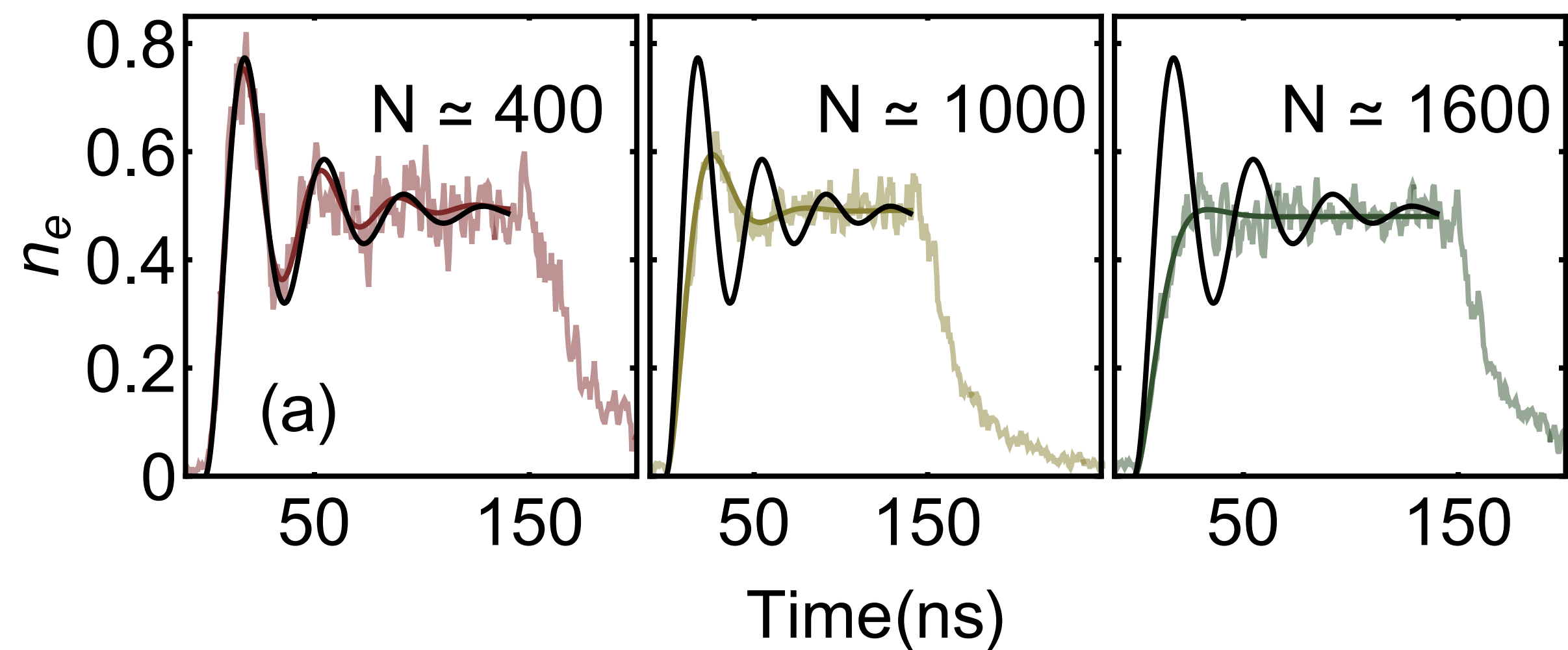
# Rabi oscillations driving Axially

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# Rabi oscillations driving Axially

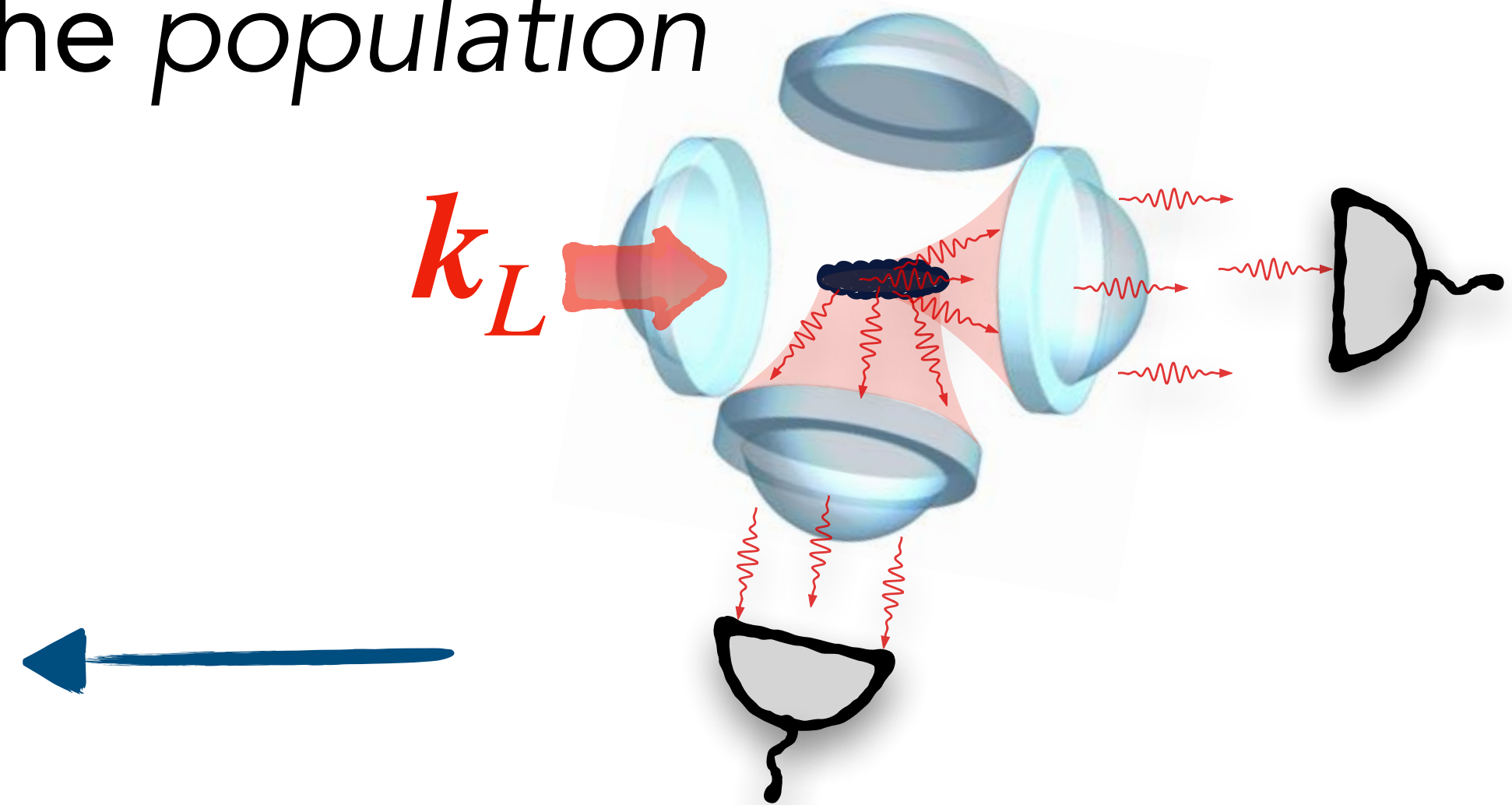
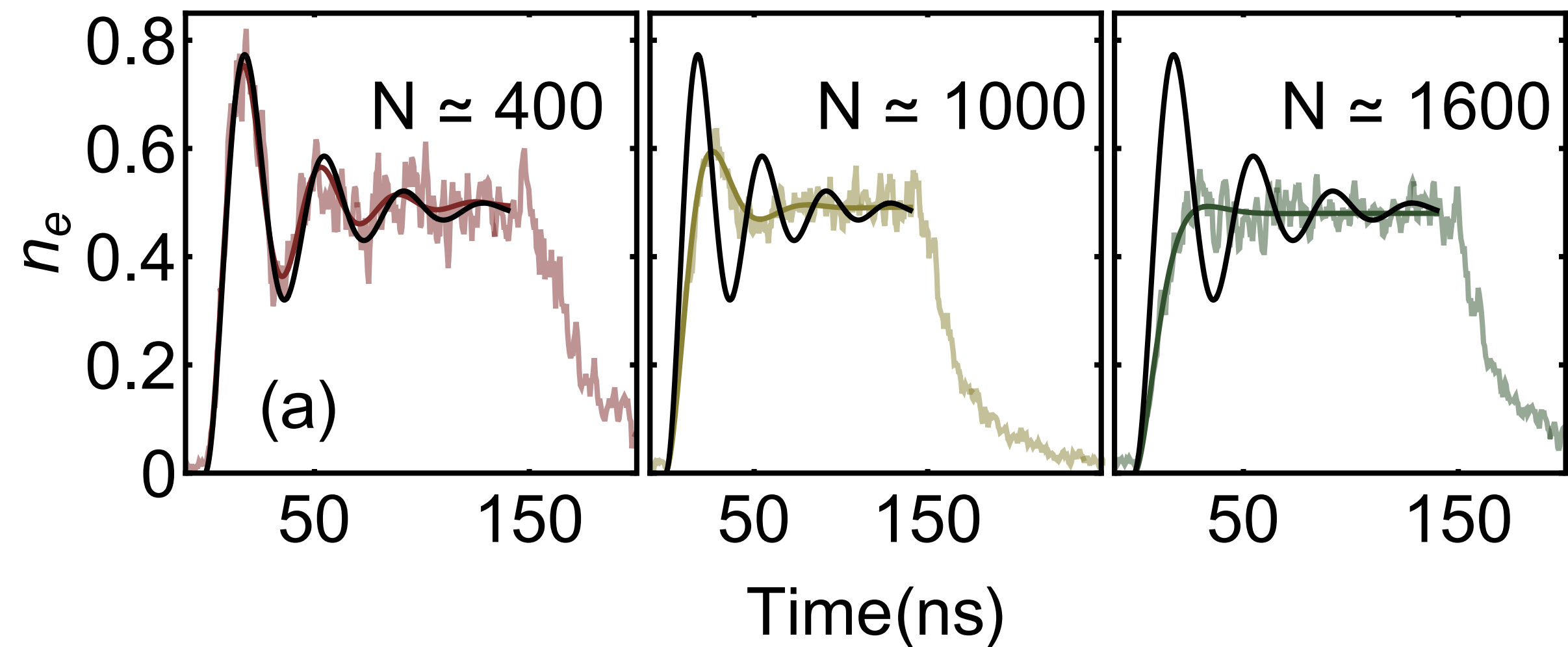
Modification of Rabi oscillations in the *population*



$\Omega_{\text{eff}}$  suppressed at large  $N$

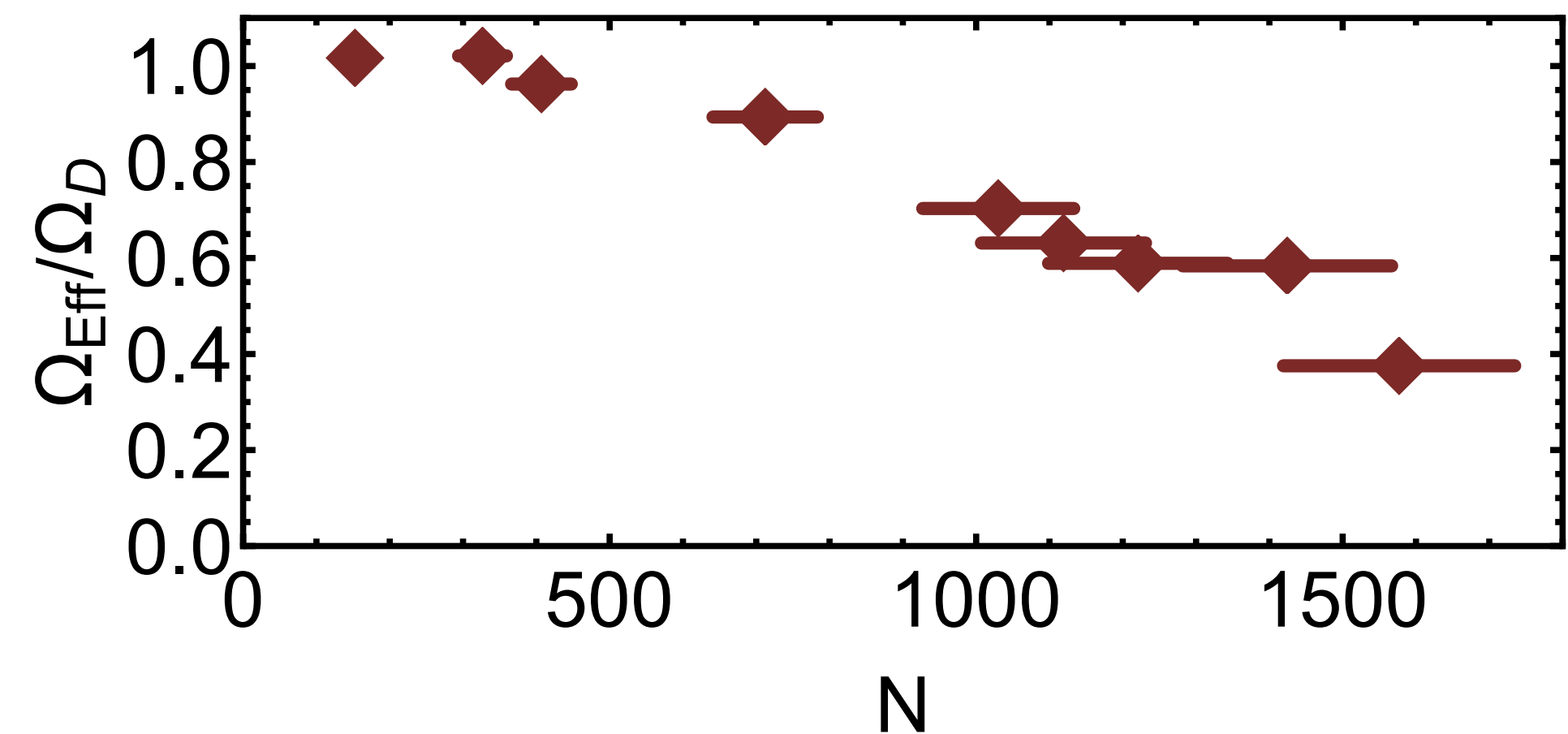
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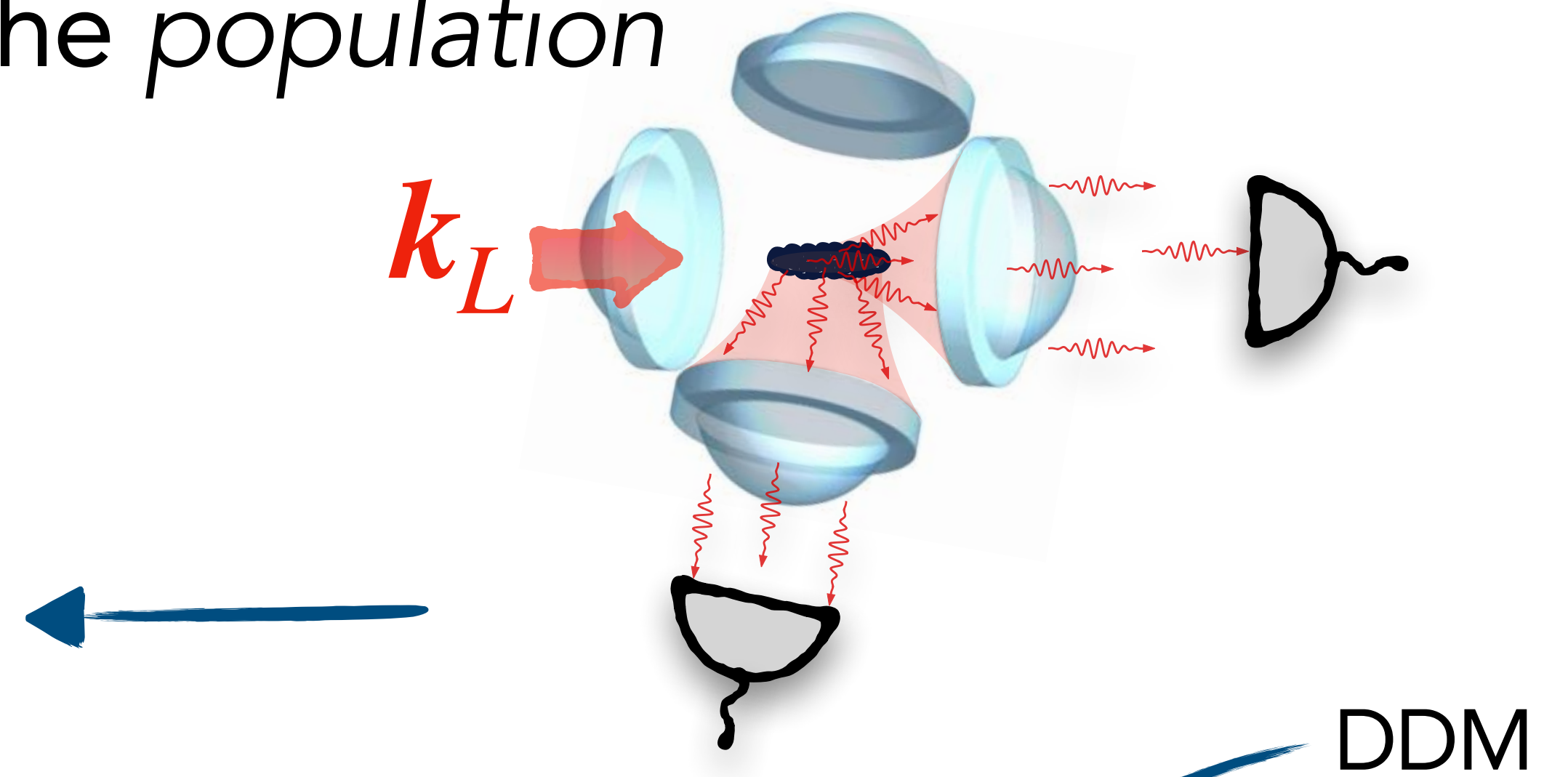
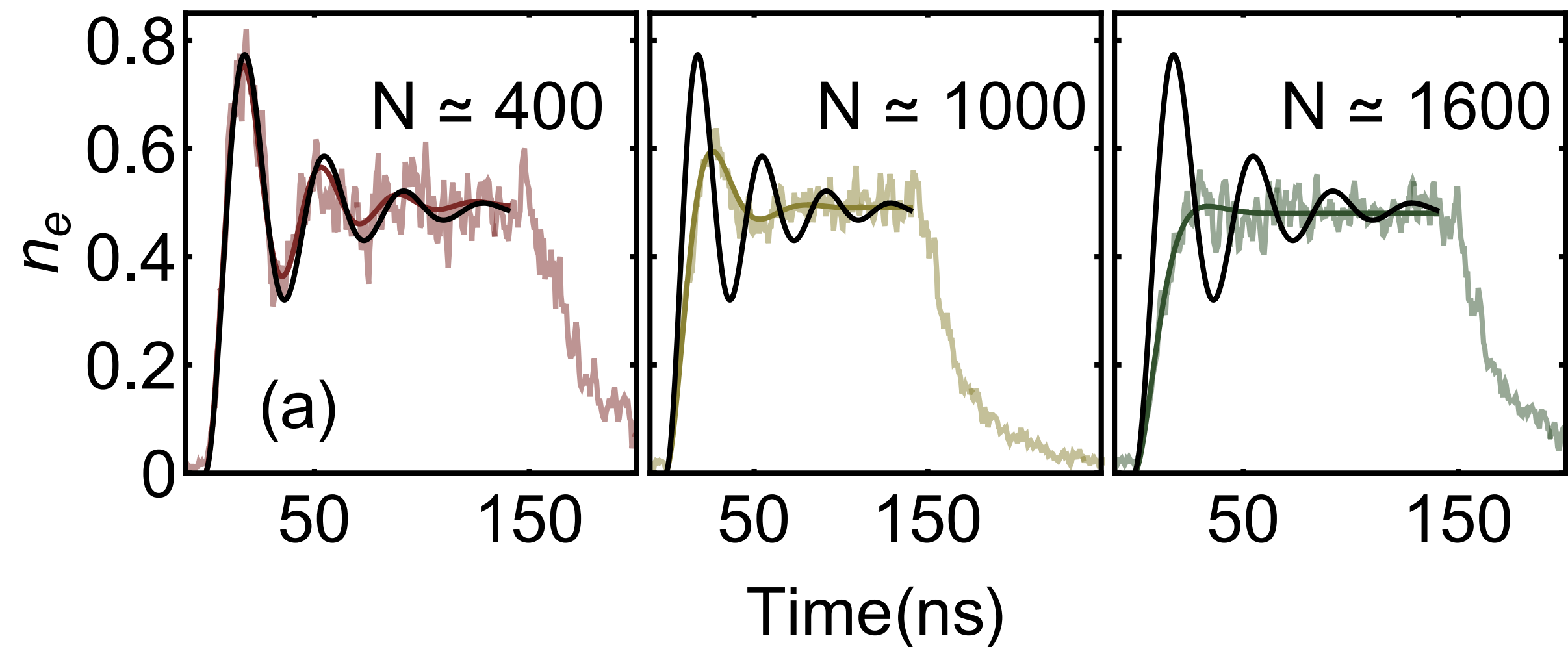
Effective Rabi frequency extracted by fit





# Rabi oscillations driving Axially

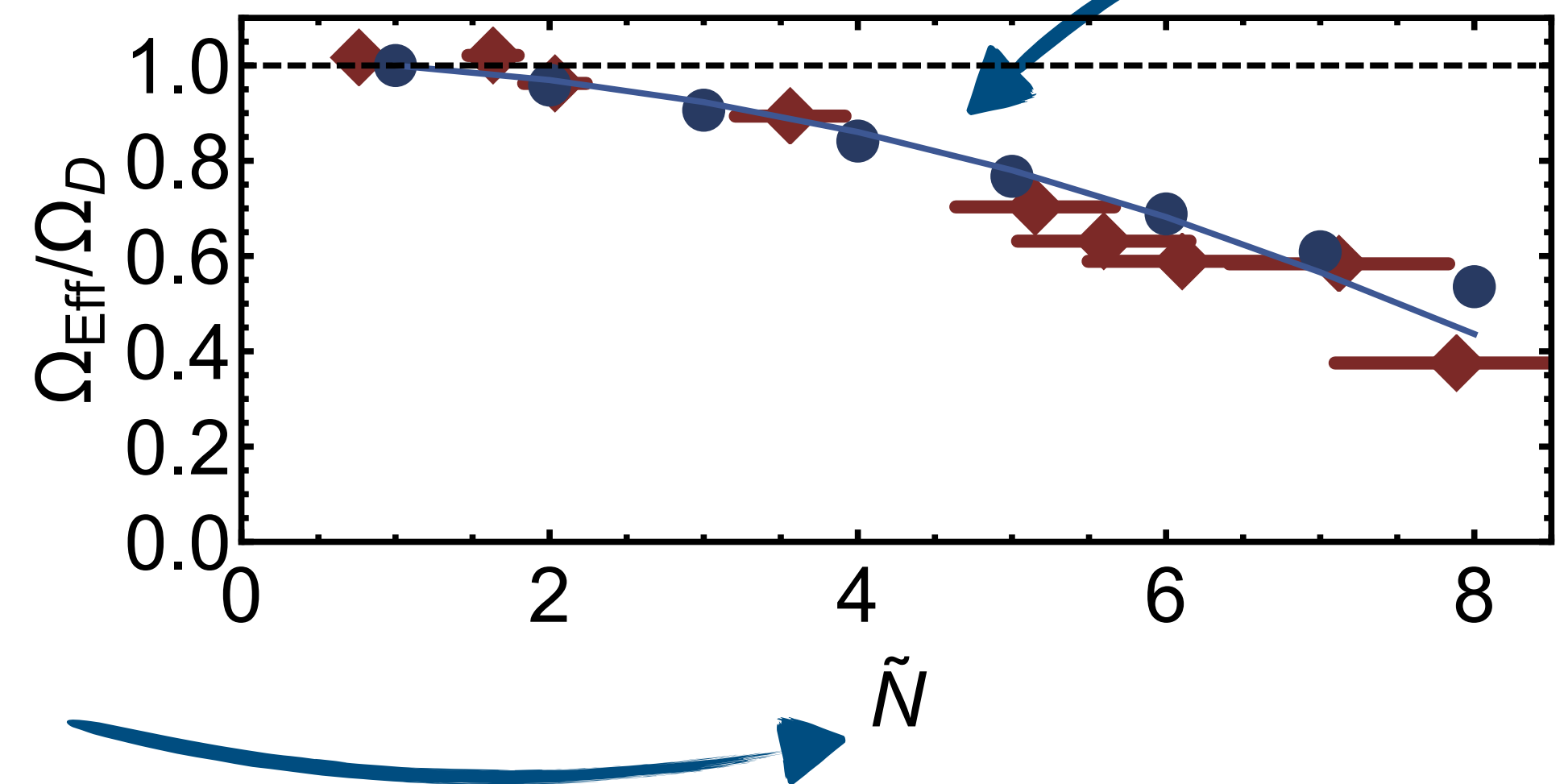
Modification of Rabi oscillations in the *population*



$\Omega_{\text{eff}}$  suppressed at large  $N$

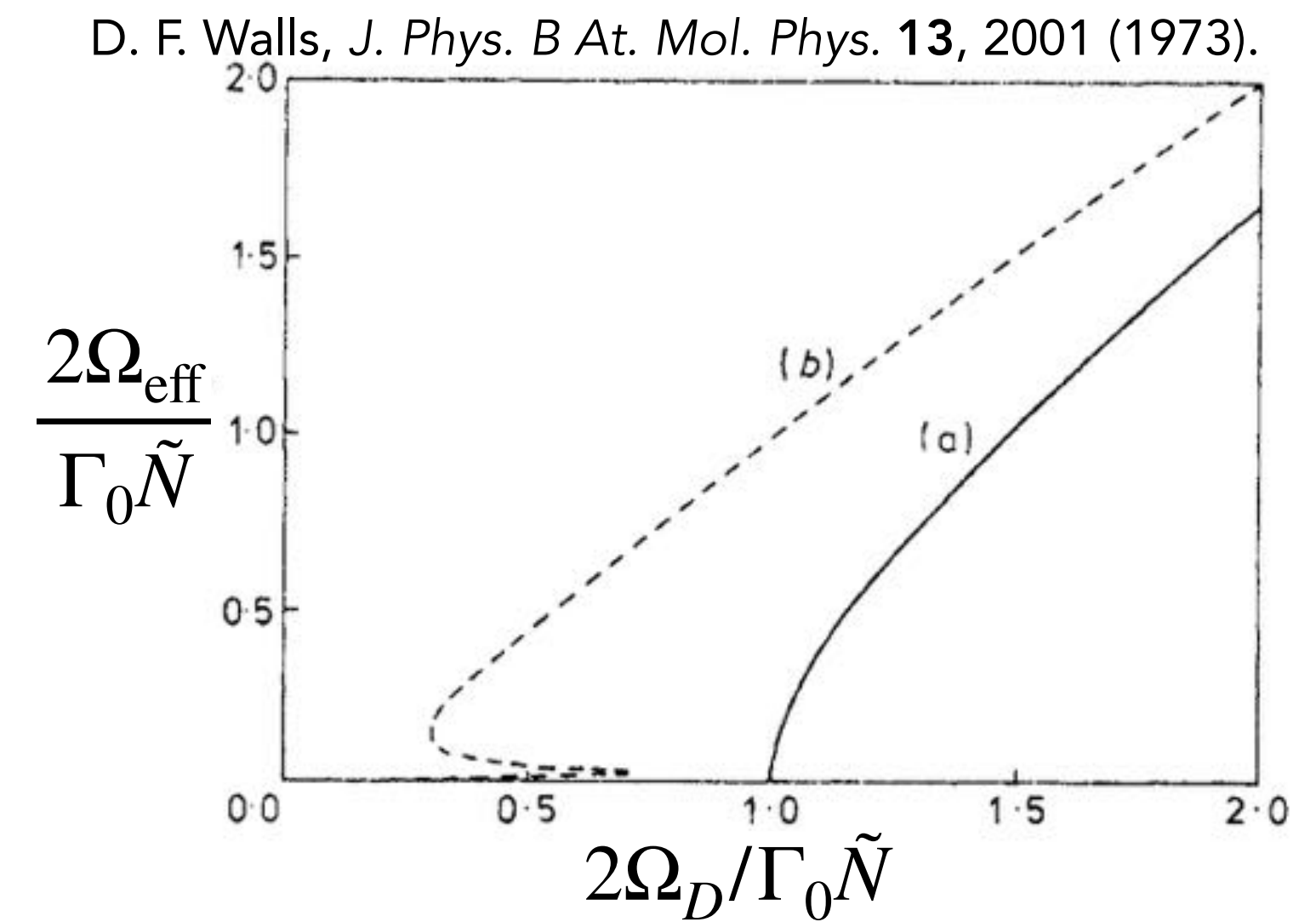
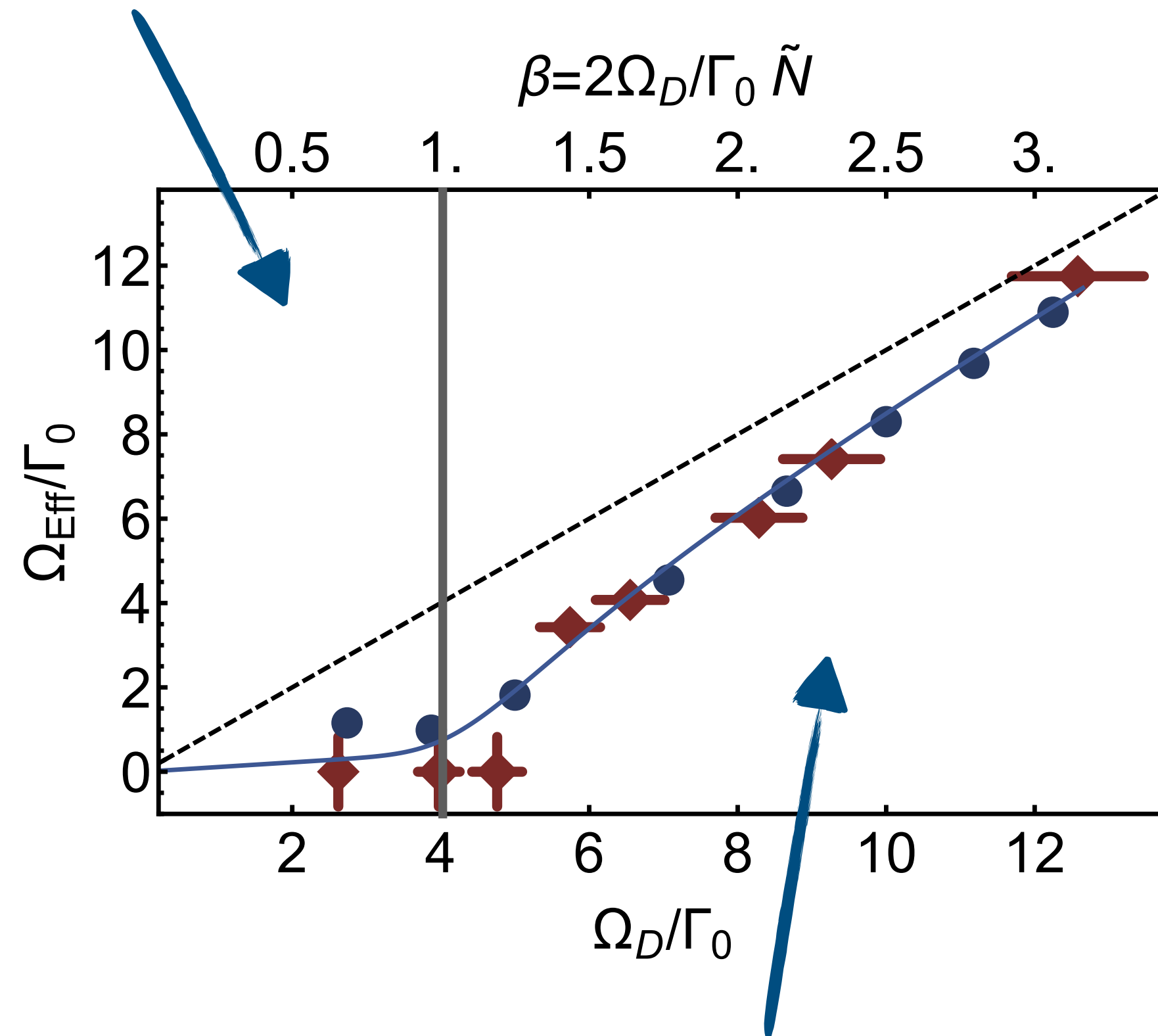
Effective Rabi frequency extracted by fit

Data rescaled to  $\tilde{N} = \mu N$  with  $\mu = 0.005$



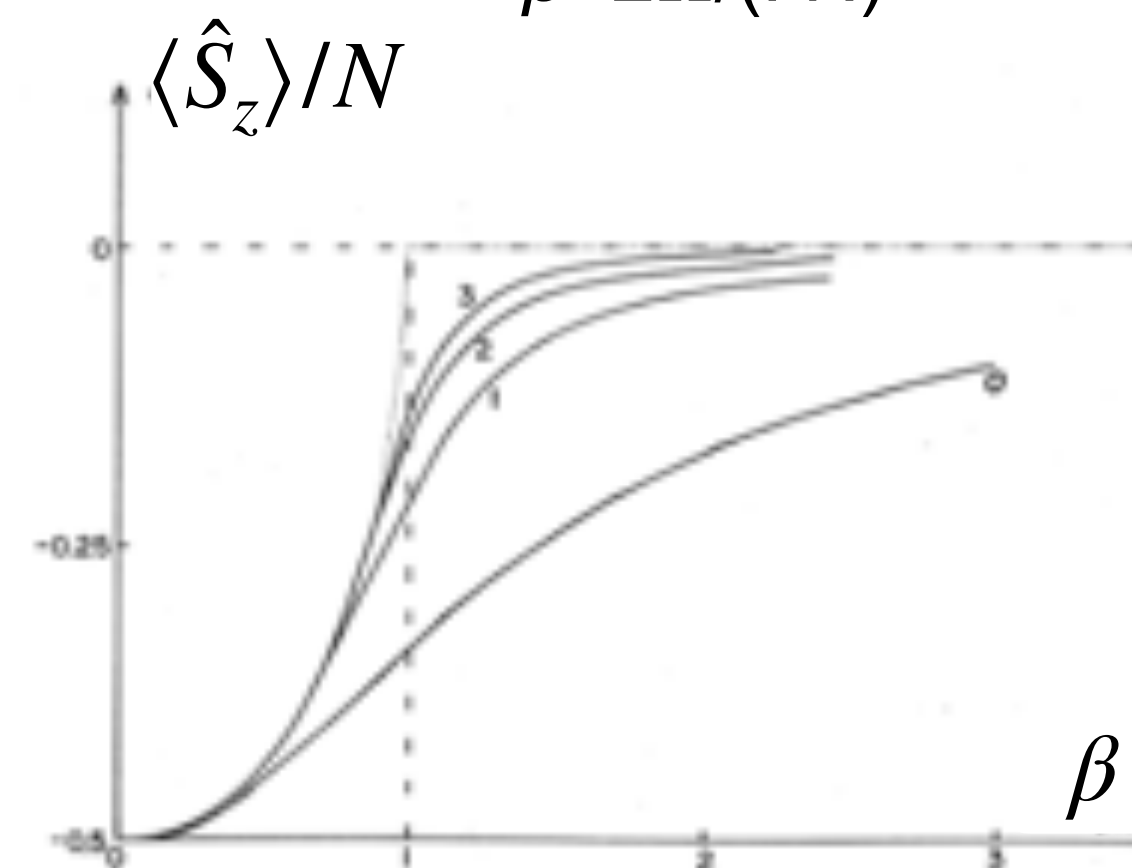
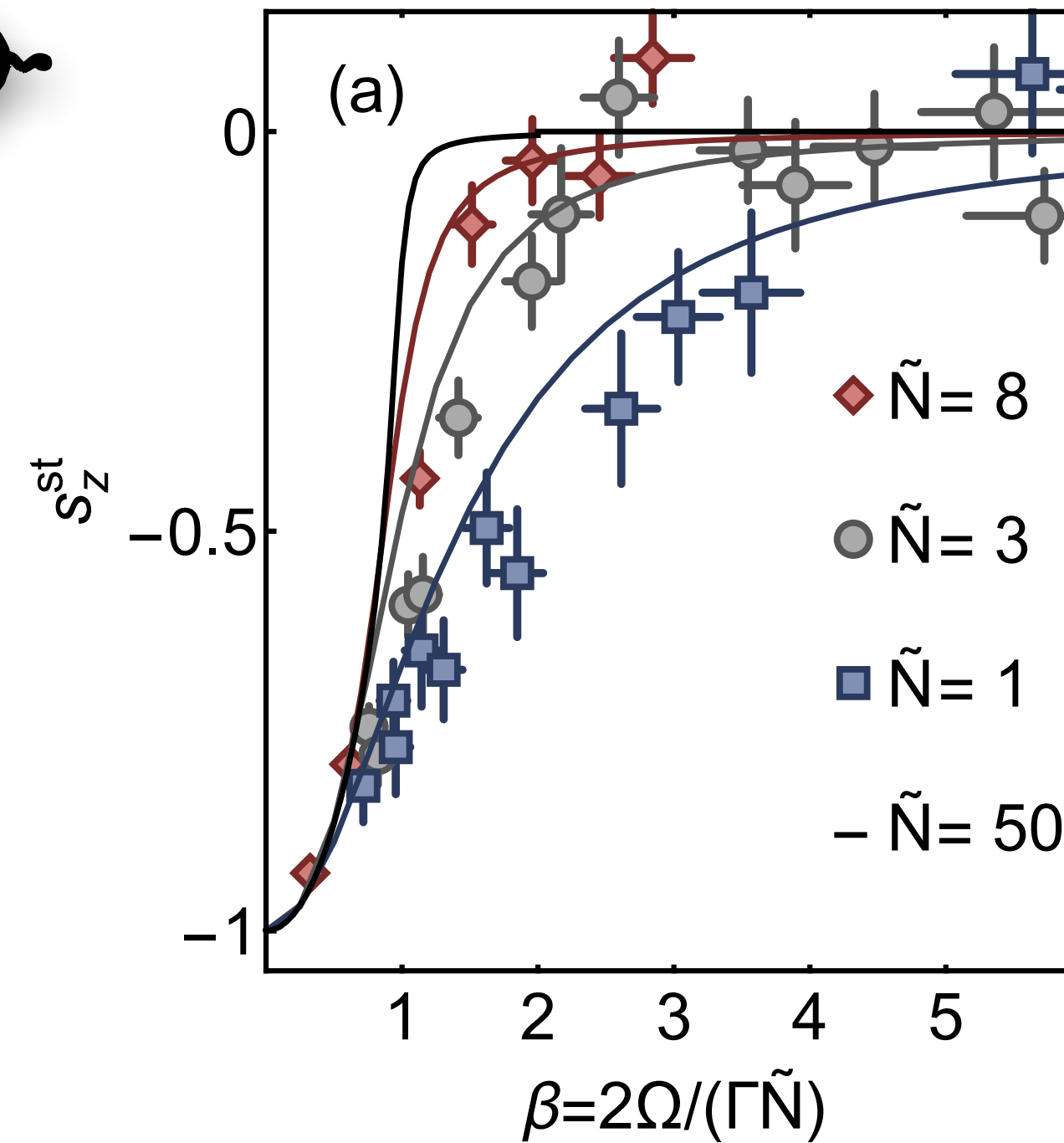
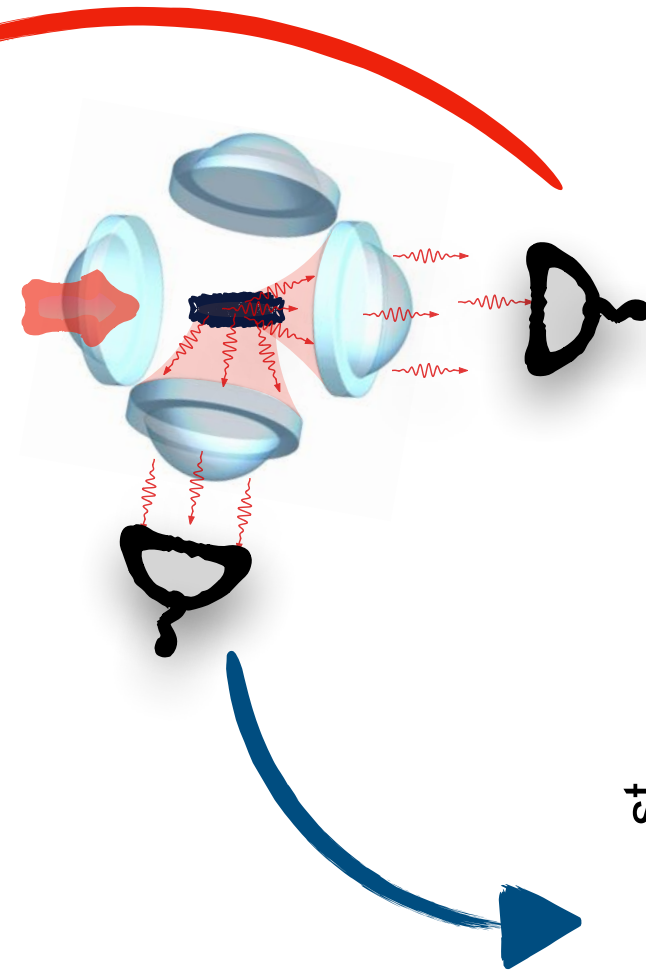
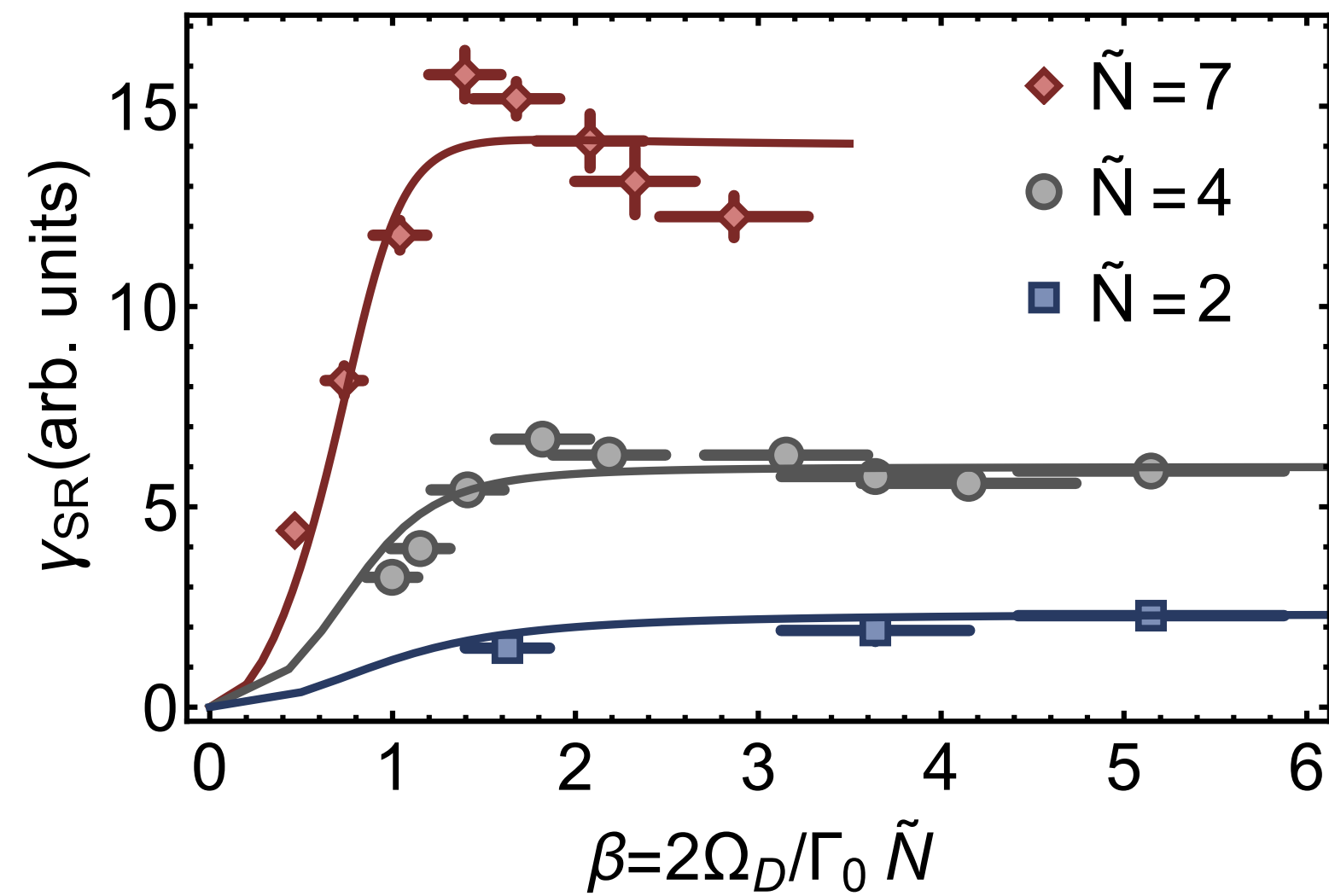
# Phase transition

Magnetic phase, collective dipole screening  $\Omega_{\text{eff}} = \Omega_D - i\Gamma_0 \langle \hat{S}^- \rangle \rightarrow 0$



Superradiant phase for strong driving  $\beta > 1$

# Superradiant transition



Clear scaling with  $\beta = 2\Omega_D / \Gamma_0 \tilde{N}$

Convergence towards universal curve for  $\tilde{N} \rightarrow \infty$

# Superradiant transition

Two collective steady-states

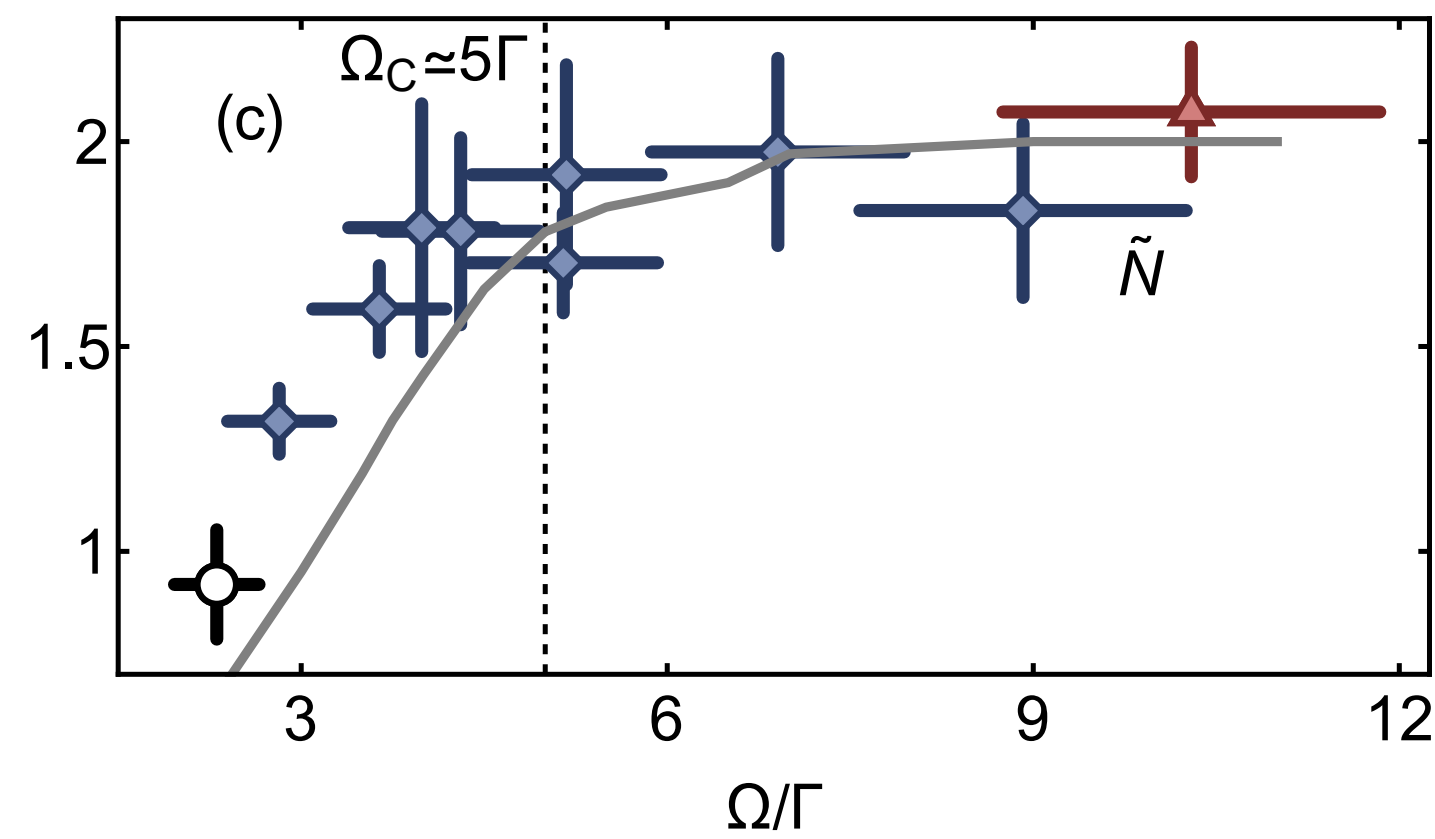
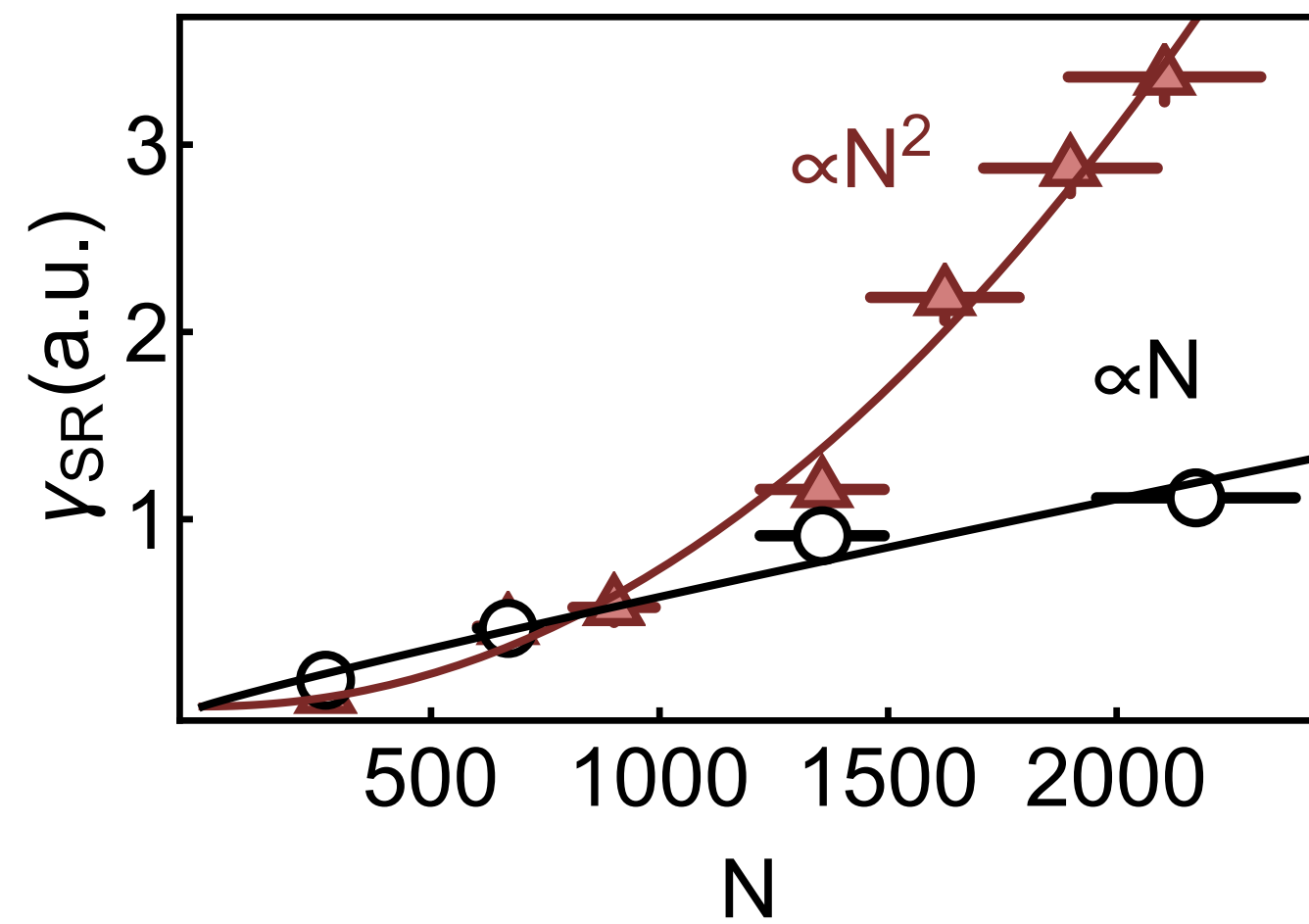
$$\beta \ll 1$$

Magnetic phase

$$\beta \geq 1$$

Superradiant phase

Dicke state with  $\Gamma_{\tilde{N}} \propto \tilde{N}^2$





# Superradiant transition

Two collective steady-states

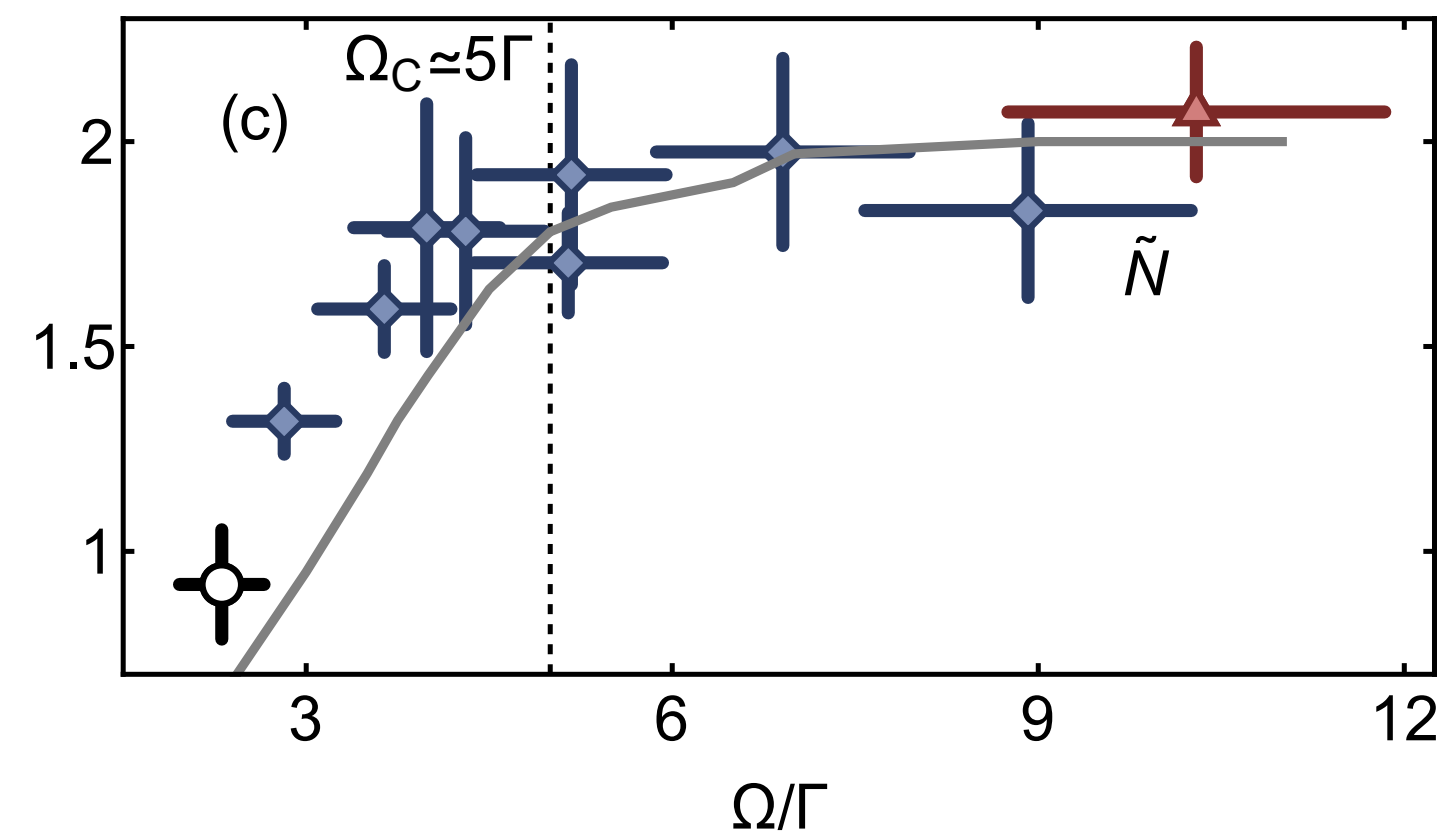
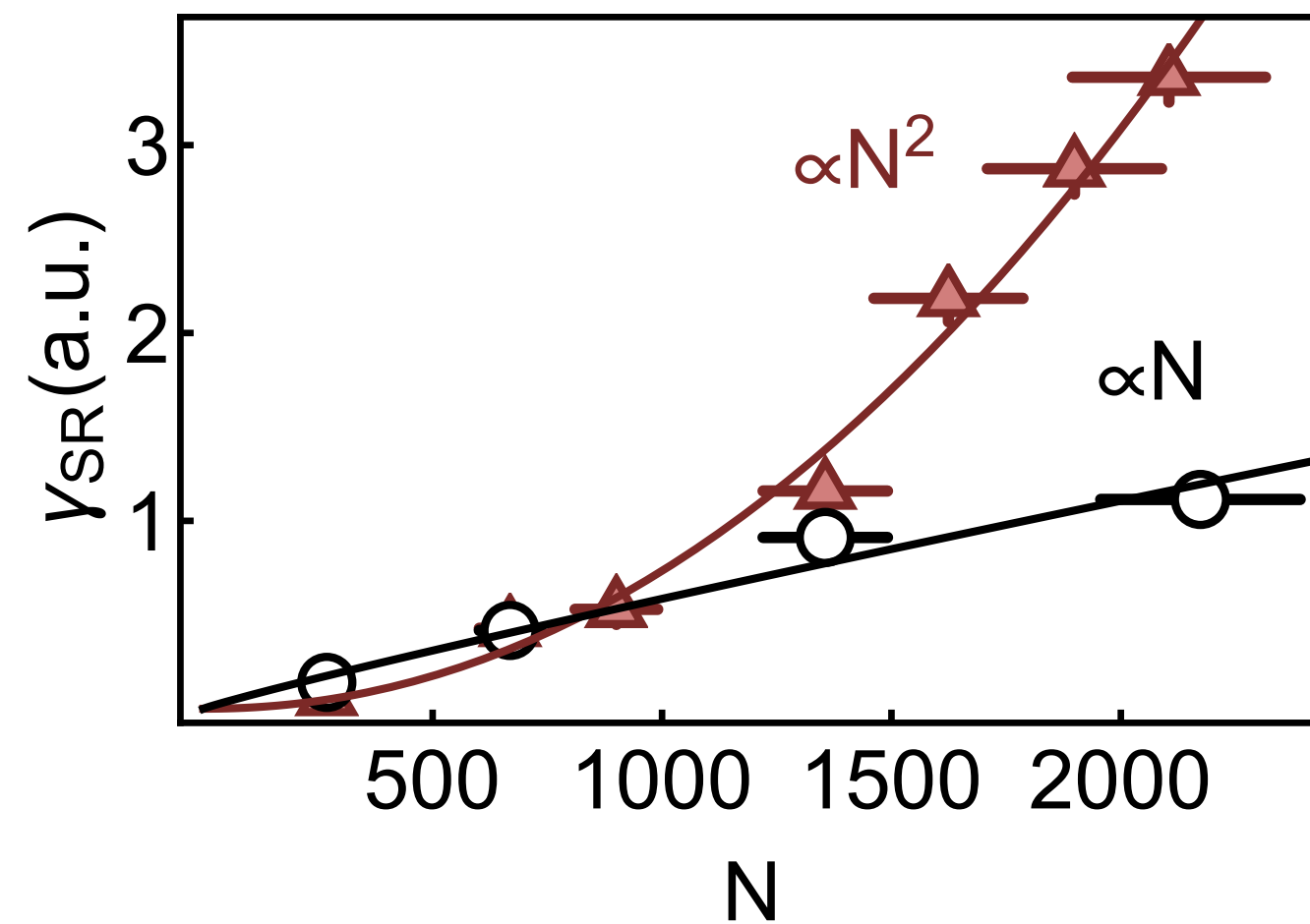
$\beta \ll 1$

Magnetic phase

$\beta \geq 1$

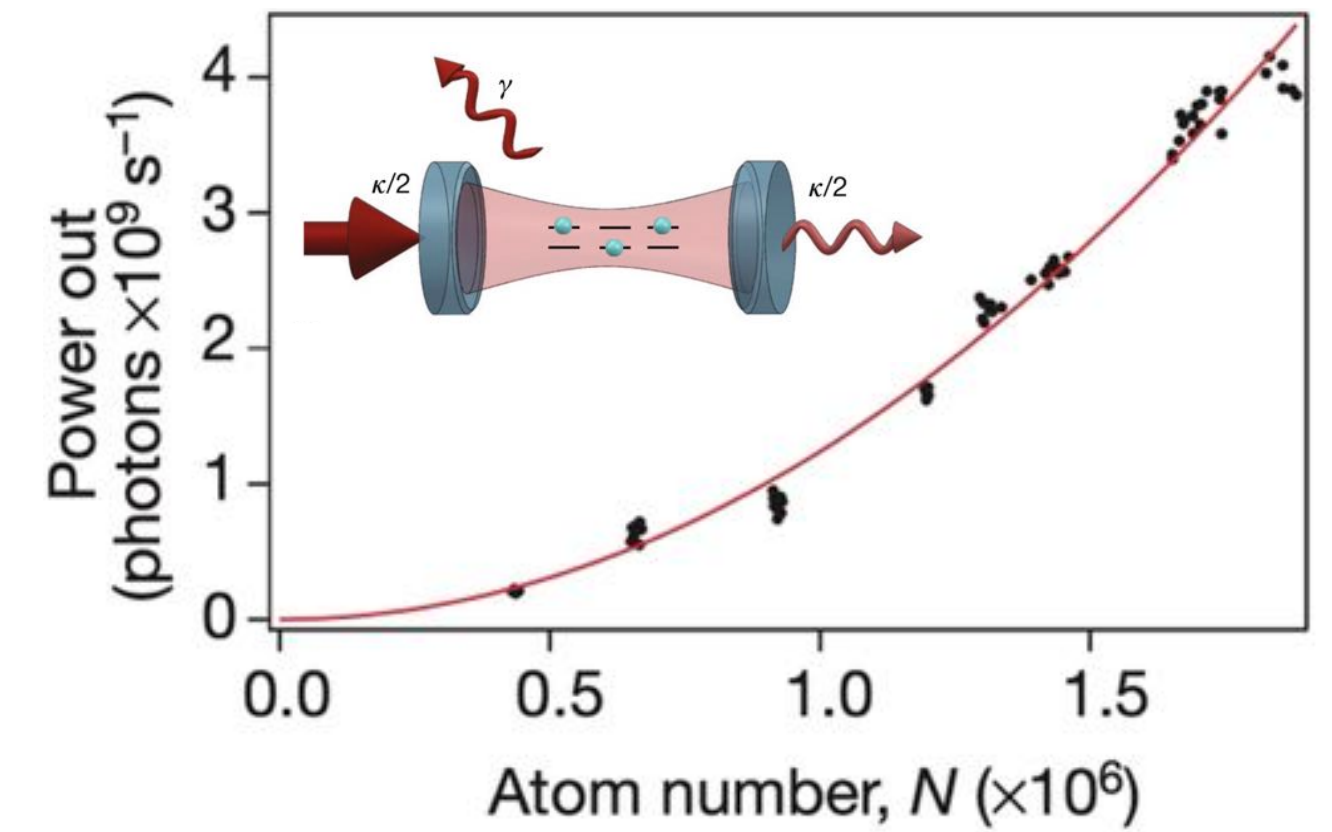
Superradiant phase

Dicke state with  $\Gamma_{\tilde{N}} \propto \tilde{N}^2$

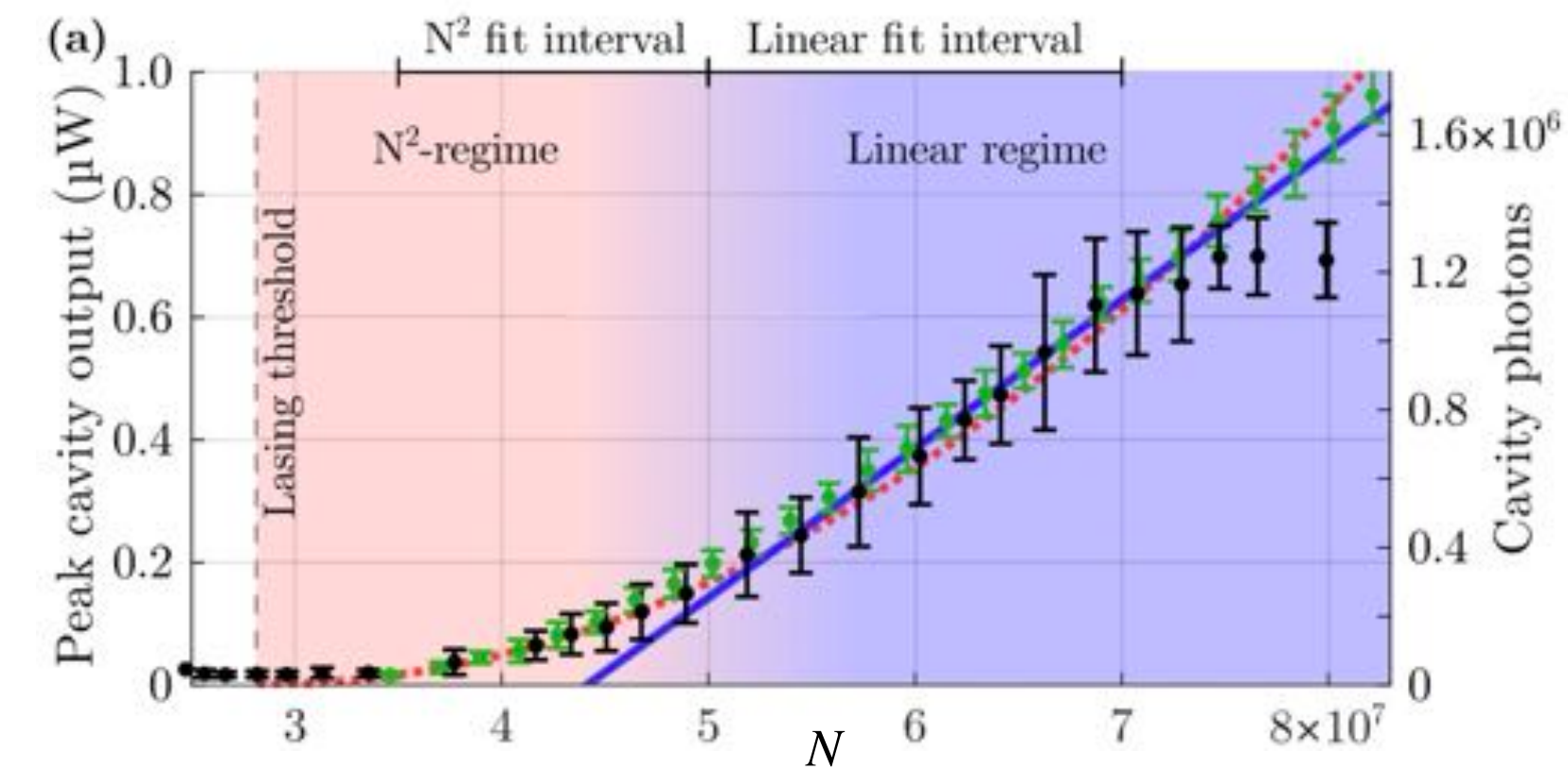


## Superradiant lasers

J. G. Bohnet et al., *Nature* **484**, 78–81 (2012).



S. A. Schäffer et al., *Phys Rev A* **101**, 013819 (2020).



# Superradiant transition

Two collective steady-states

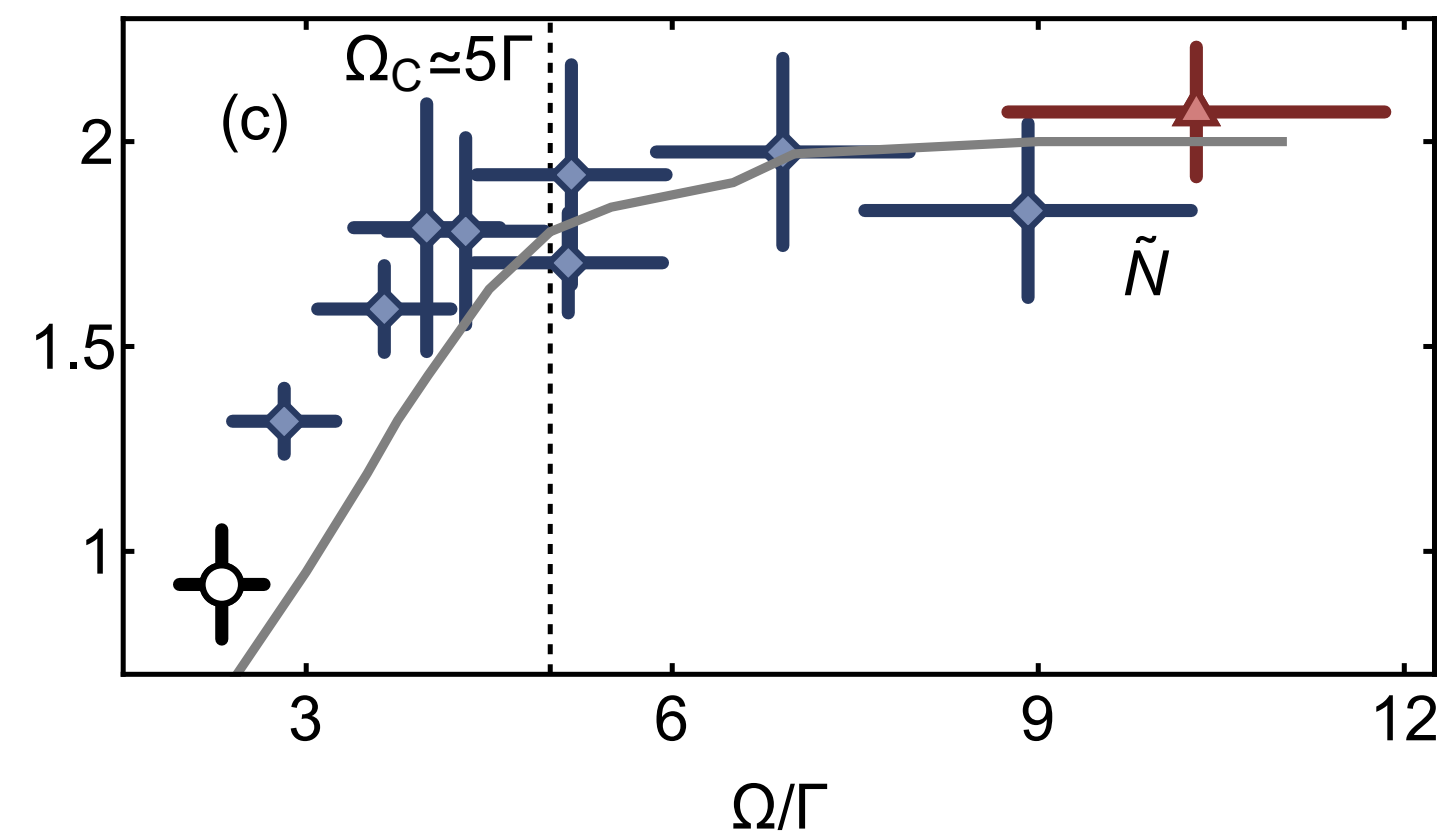
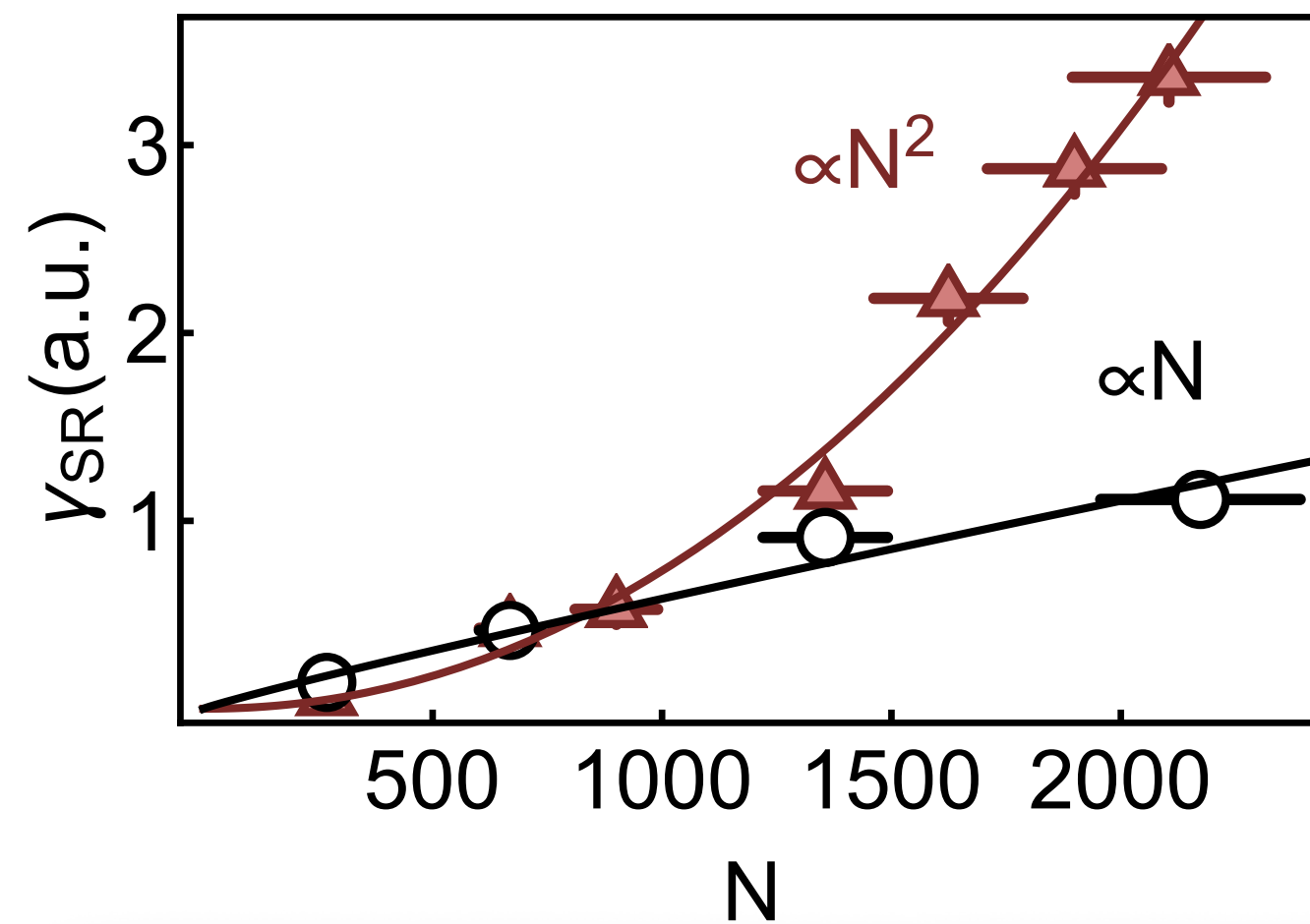
$$\beta \ll 1$$

Magnetic phase

$$\beta \geq 1$$

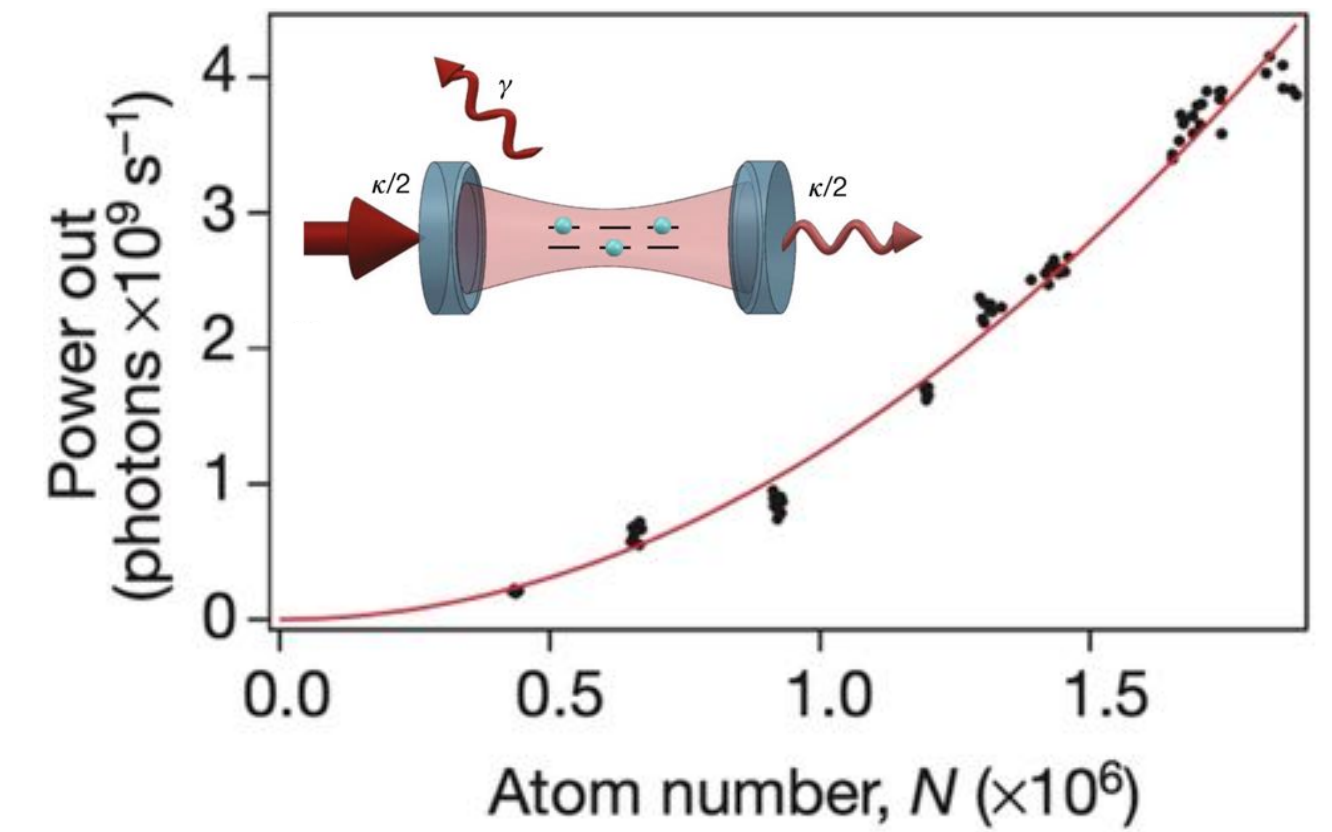
Superradiant phase

Dicke state with  $\Gamma_{\tilde{N}} \propto \tilde{N}^2$

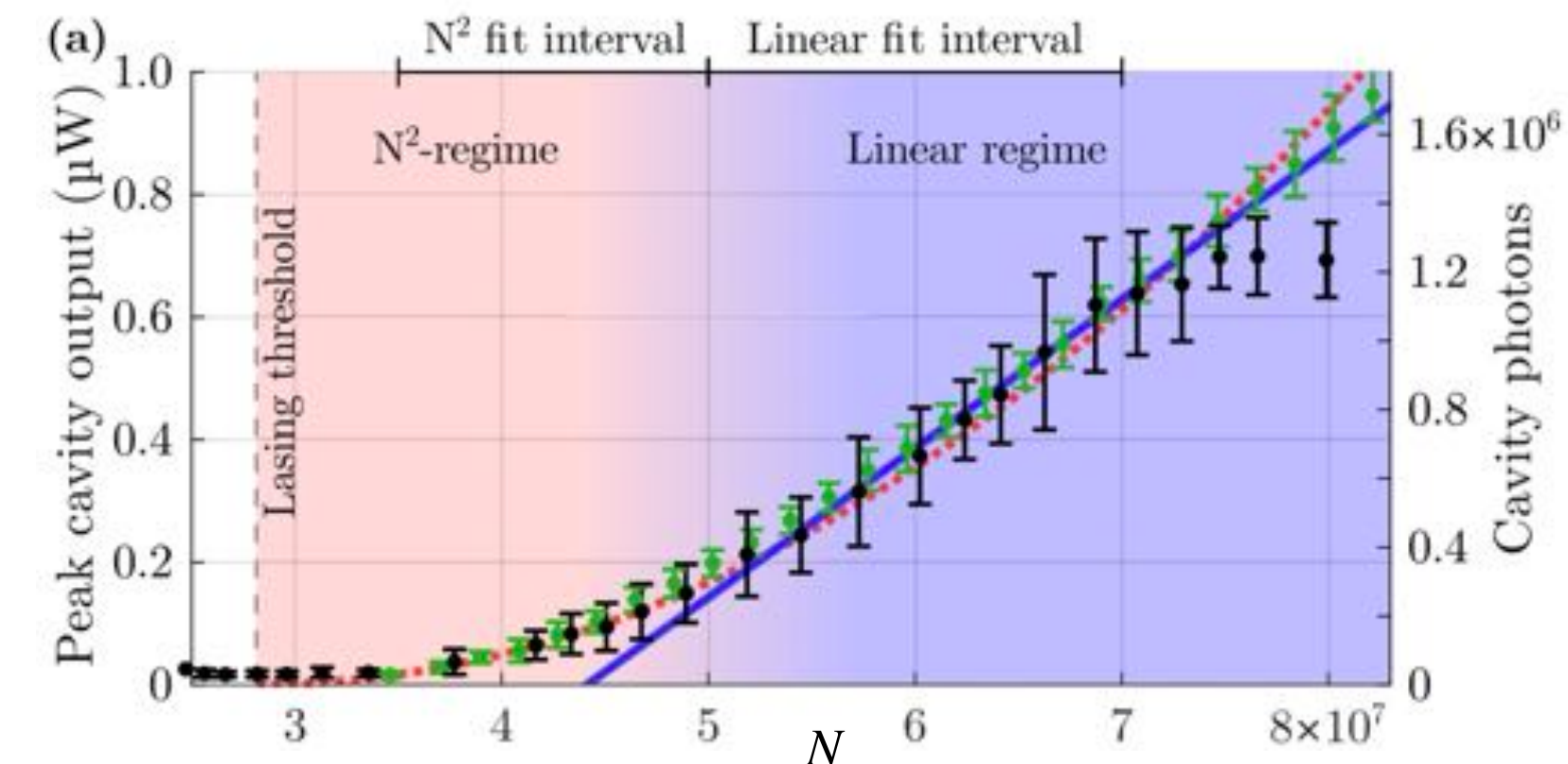


## Superradiant lasers

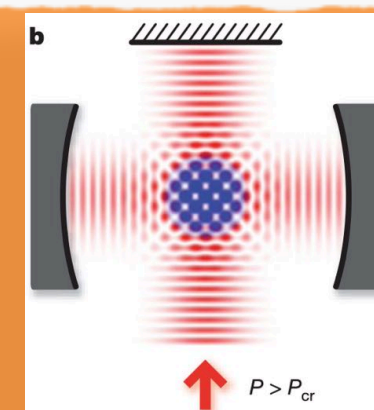
J. G. Bohnet *et al.*, *Nature* **484**, 78–81 (2012).



S. A. Schäffer *et al.*, *Phys Rev A* **101**, 013819 (2020).



Mapping with Dicke transition in cavity



K. Baumann *et al.*, *Nature* **464**, 1301 (2010).



# Thank you

Antoine  
Browaeys



Sara  
Pancaldi



Giovanni  
Ferioli



Antoine  
Glicenstein



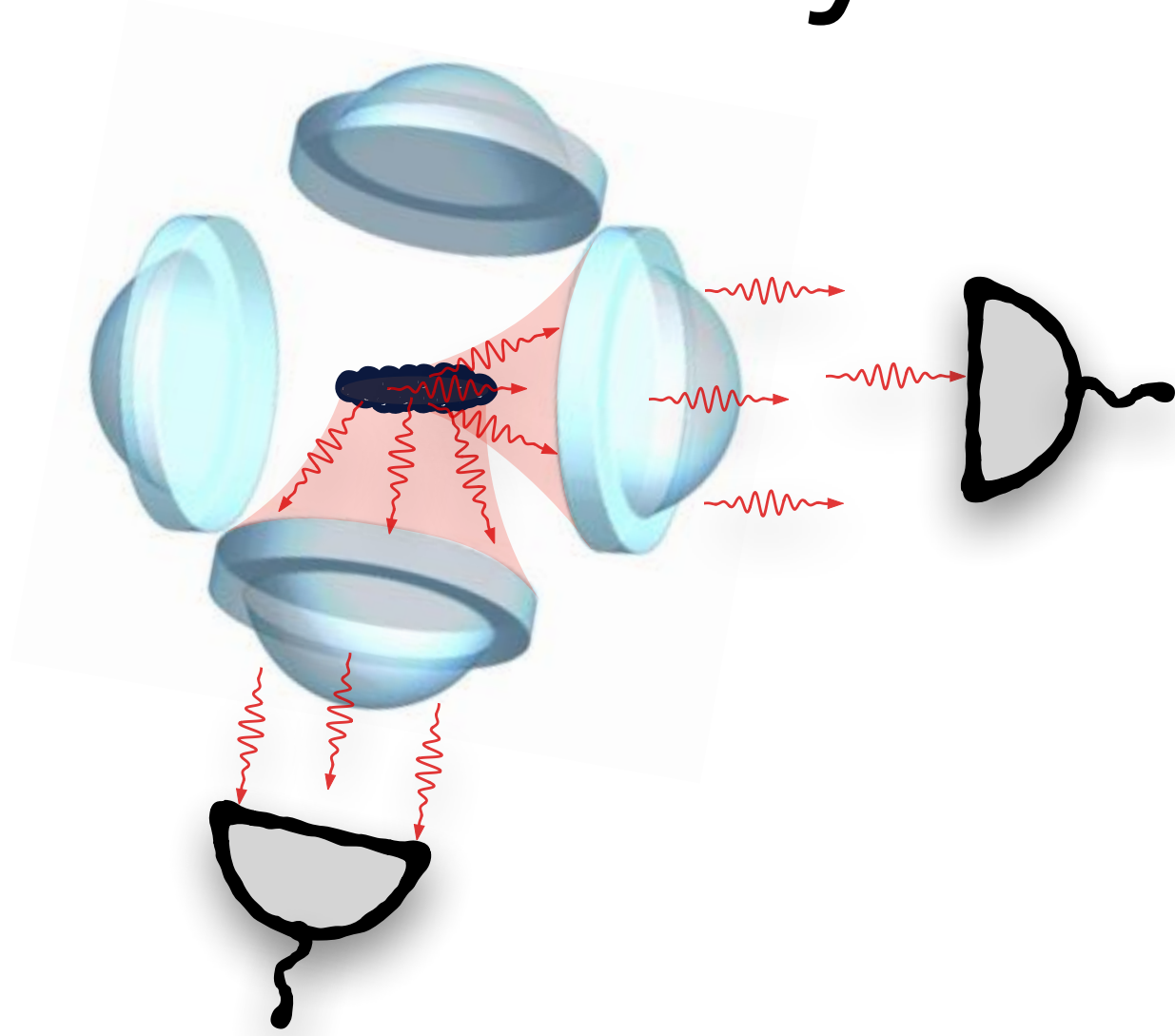
Francis  
Robicheaux



Collab.



# Beyond intensity: light correlations

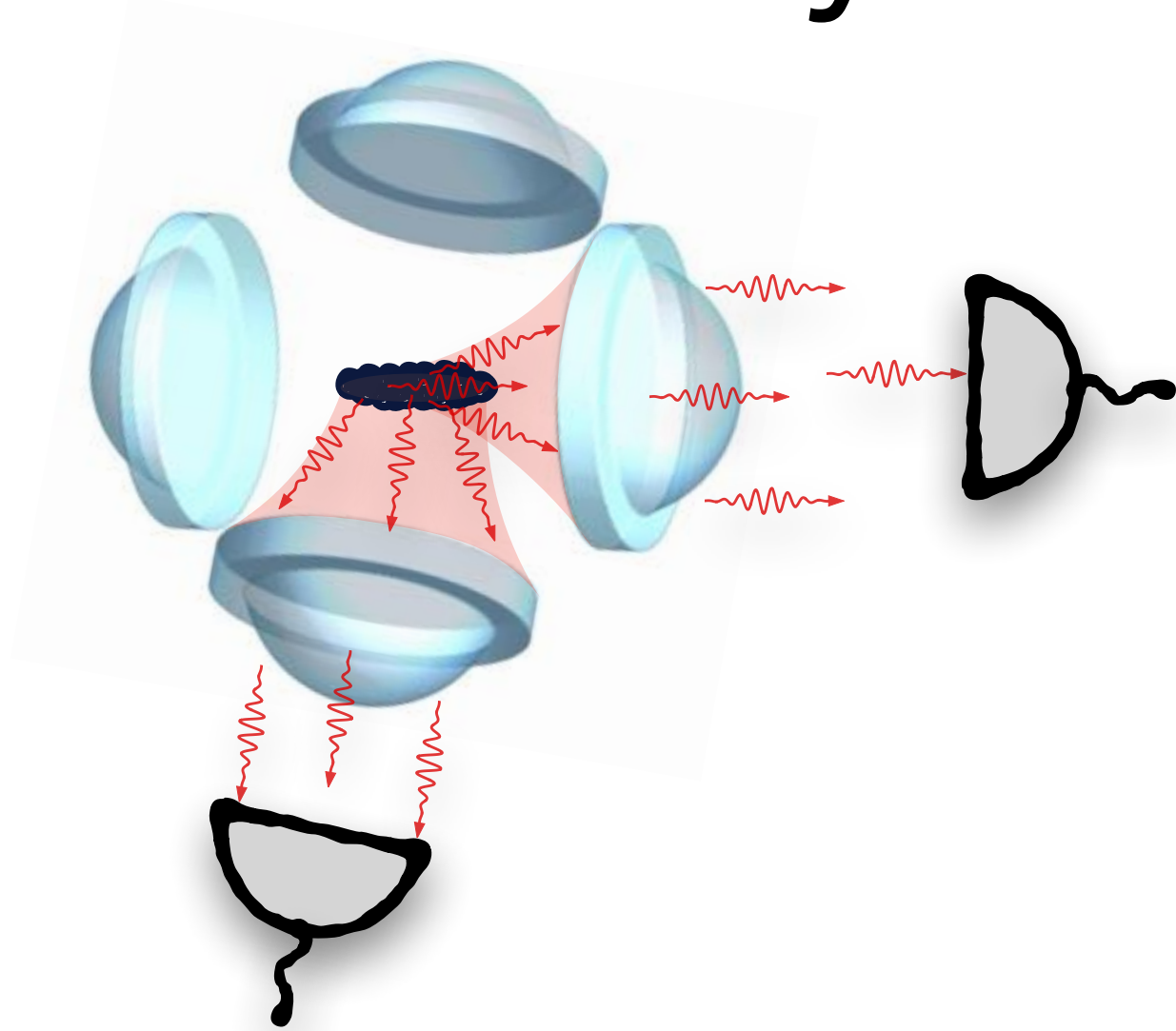


Two-photon correlations

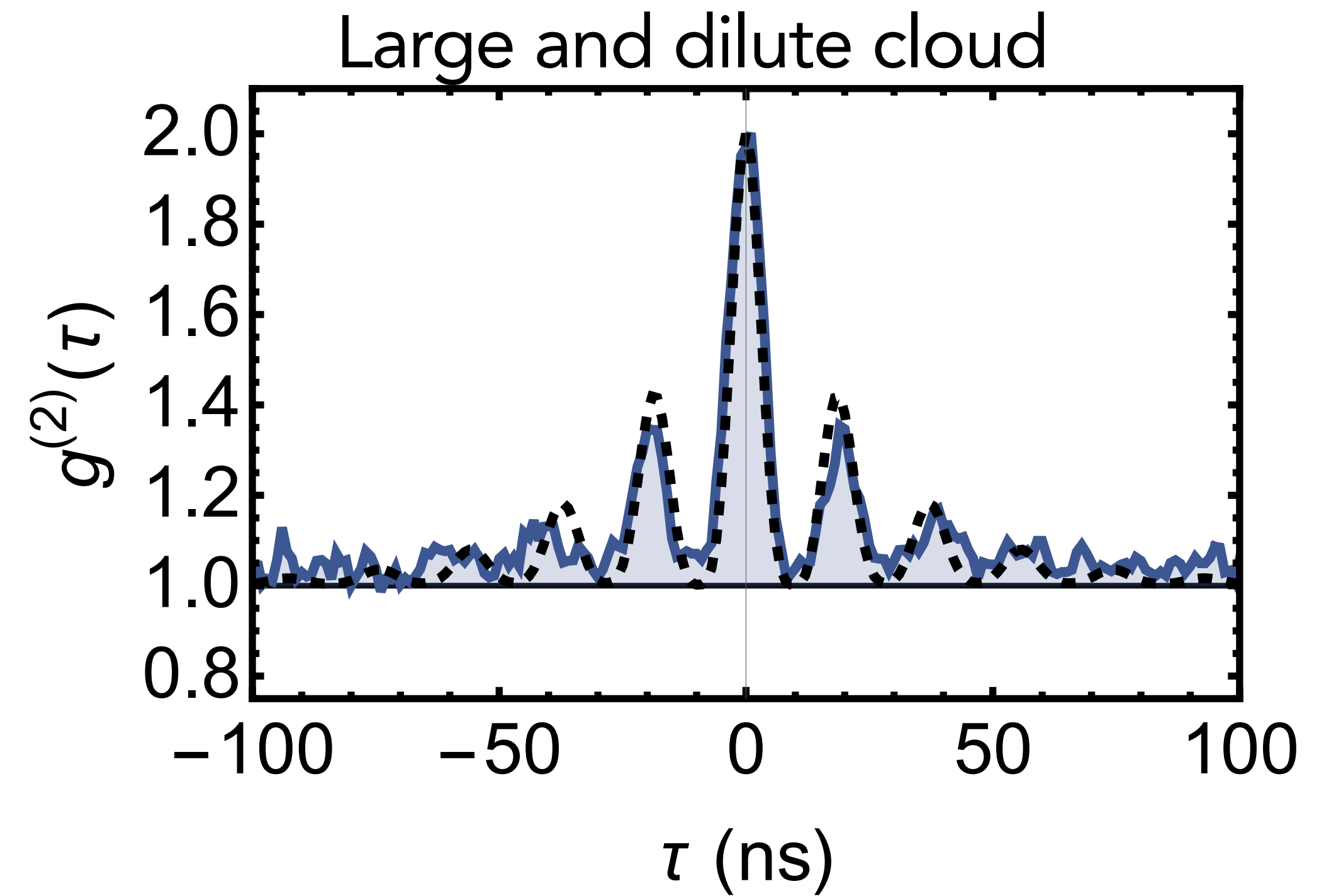
$$g_2(\tau) = \frac{\langle n(t)n(t + \tau) \rangle}{\langle n(t) \rangle \langle n(t + \tau) \rangle}$$



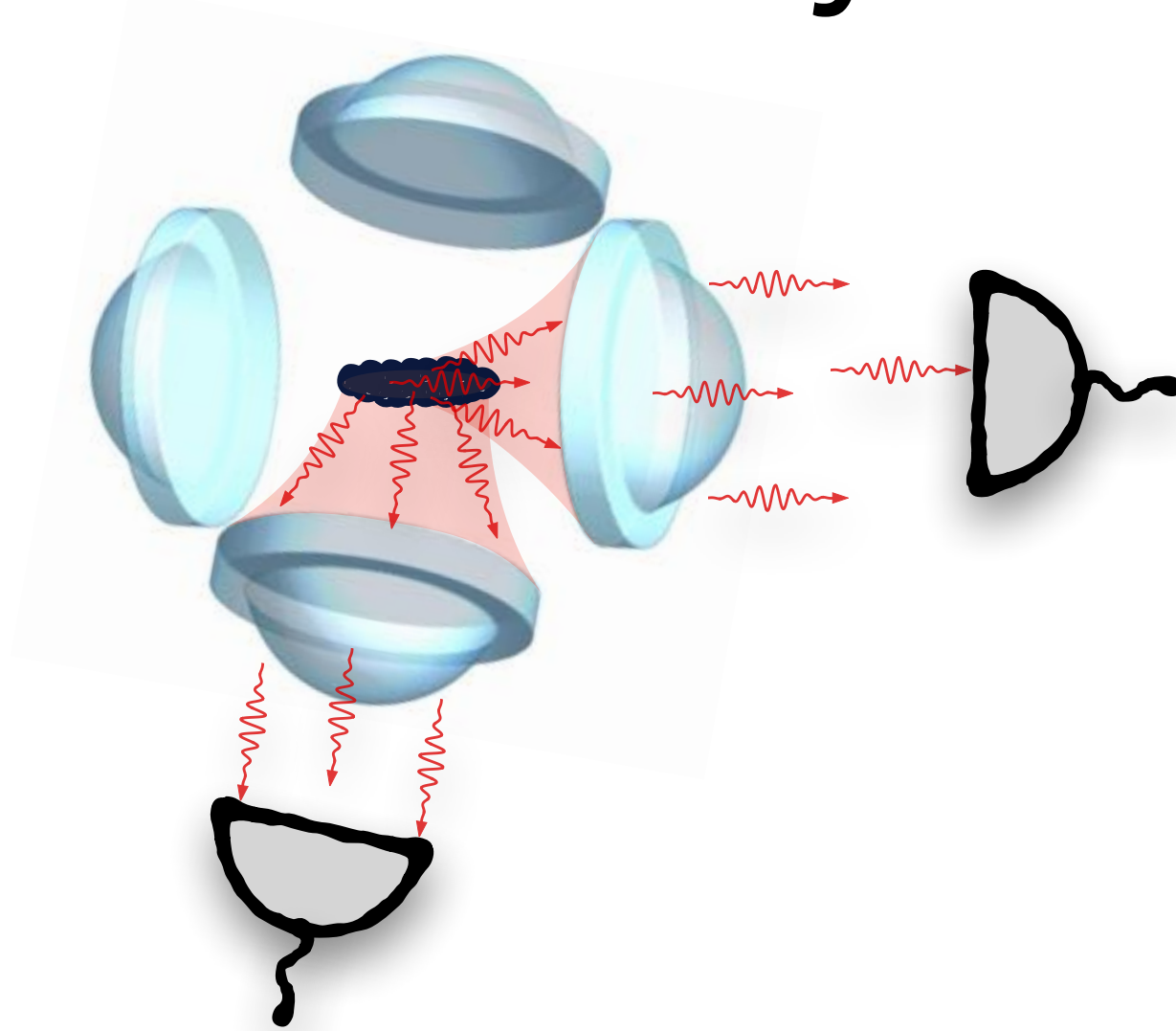
# Beyond intensity: light correlations



Two-photon correlations  $g_2(\tau) = \frac{\langle n(t)n(t + \tau) \rangle}{\langle n(t) \rangle \langle n(t + \tau) \rangle}$



# Beyond intensity: light correlations

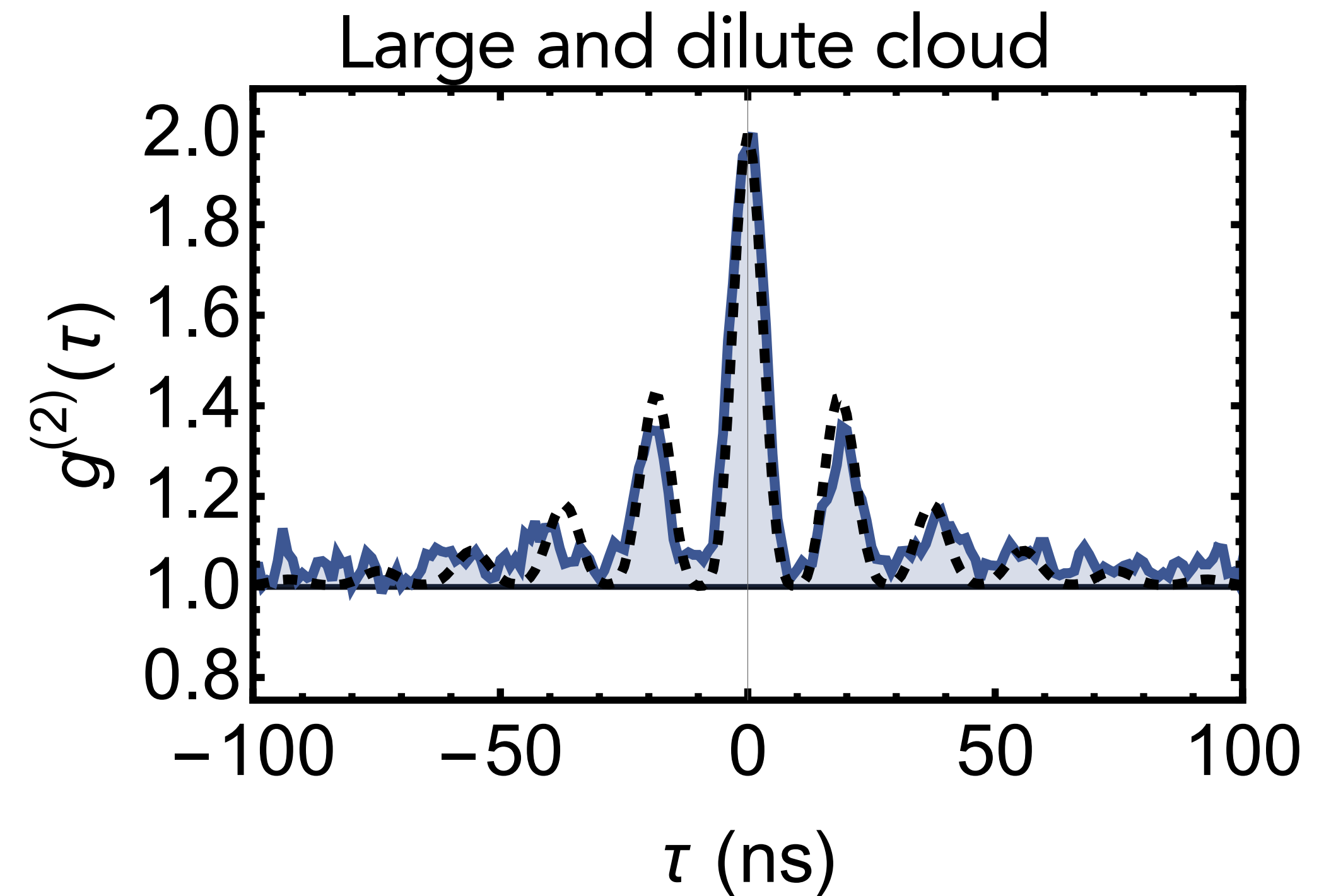


Two-photon correlations  $g_2(\tau) = \frac{\langle n(t)n(t + \tau) \rangle}{\langle n(t) \rangle \langle n(t + \tau) \rangle}$

Independent emitters: Siegert relation

$$g_2^{(N)}(\tau) \underset{N \rightarrow \infty}{=} 1 + \left| g_1^{(1)}(\tau) \right|^2 + \frac{1}{N} g_2^{(1)}(\tau)$$

Large clouds of atoms verify Siegert



See e.g. Ferreira *et al.*, *Am J Phys* **88**, 831–837 (2020).

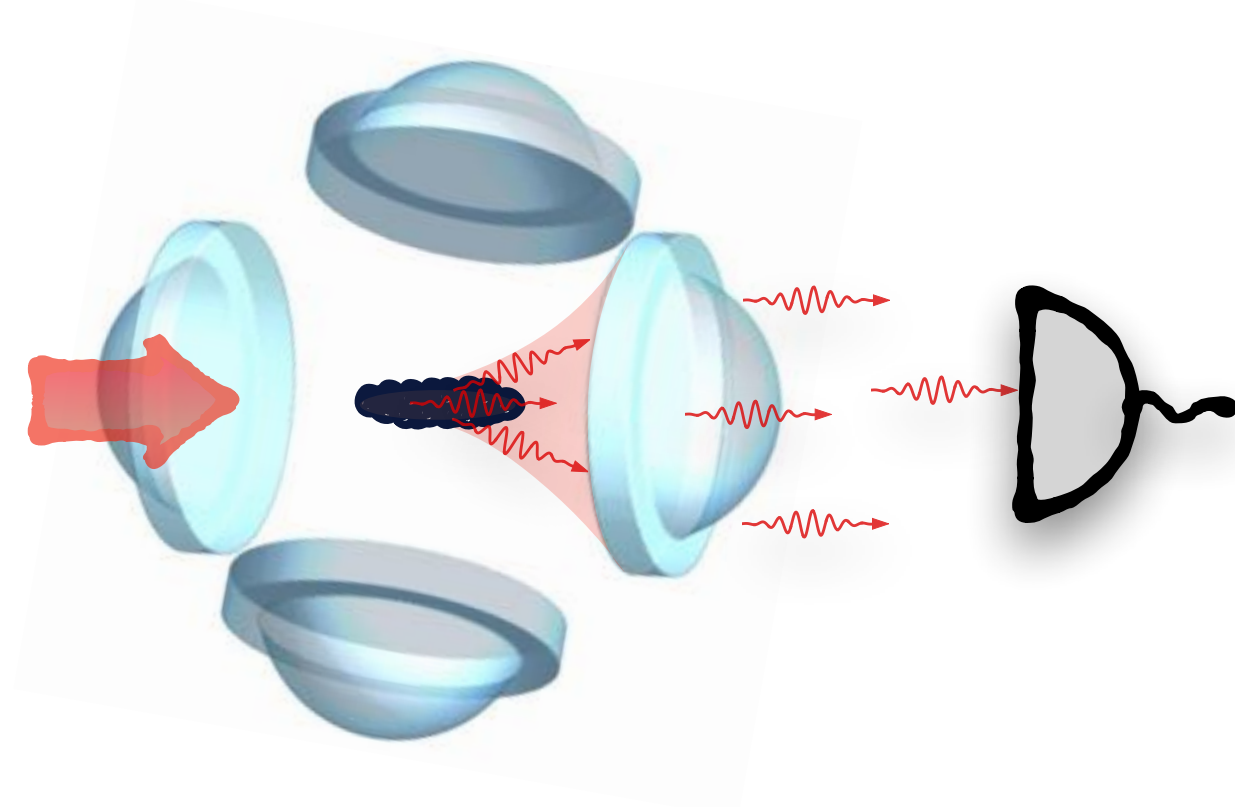
Nice group (Hugbart, Kaiser...)

# Beyond intensity: light correlations

General case very challenging

Superradiant ensembles

$$g_2(0, \mathbf{k}) = \frac{\sum_{k,l,m,n} e^{i\mathbf{k}\cdot(\mathbf{r}_k+\mathbf{r}_l-\mathbf{r}_m-\mathbf{r}_n)} \langle \hat{\sigma}_k^+ \hat{\sigma}_l^+ \sigma_m^- \hat{\sigma}_n^- \rangle}{\left[ \sum_{m,n} e^{i\mathbf{k}\cdot(\mathbf{r}_m-\mathbf{r}_n)} \langle \sigma_m^+ \hat{\sigma}_n^- \rangle \right]^2}$$

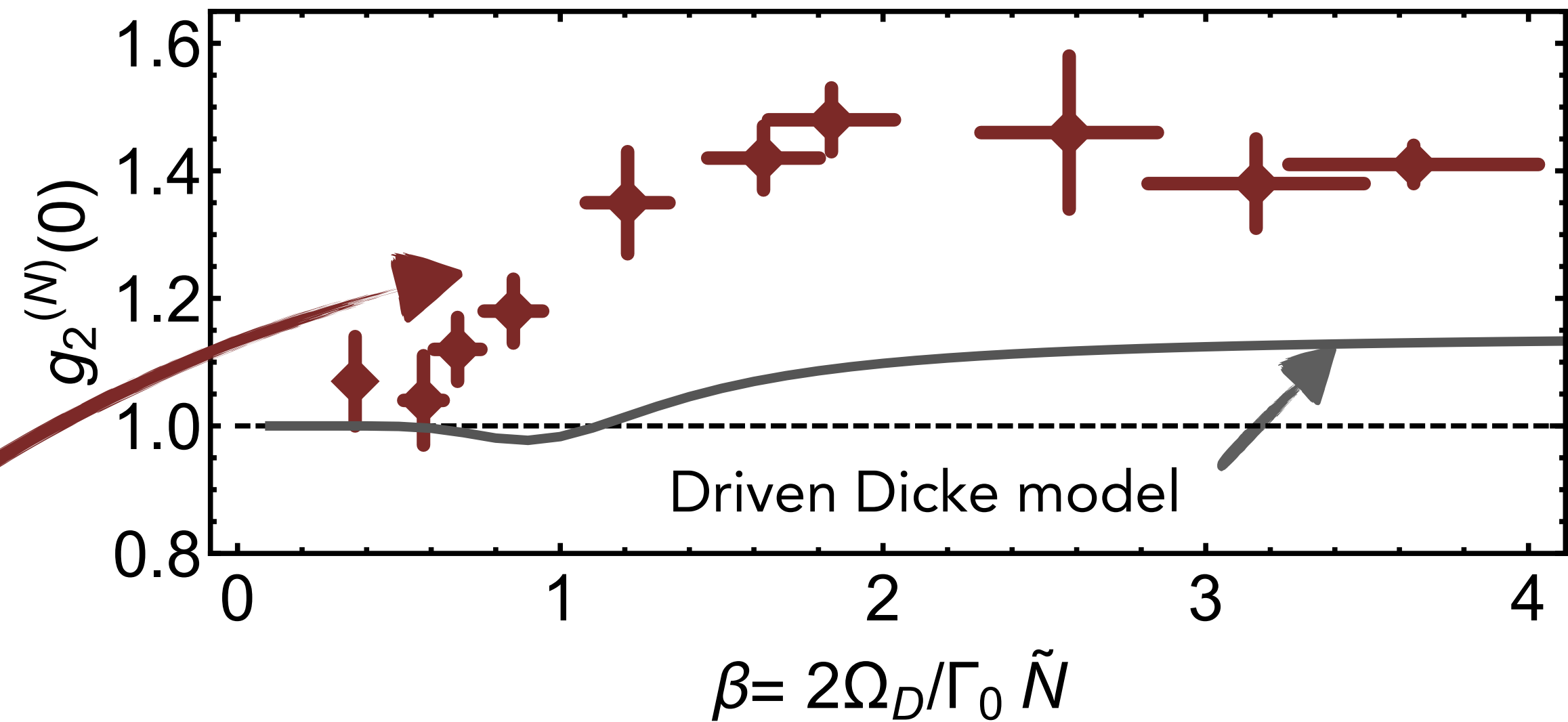
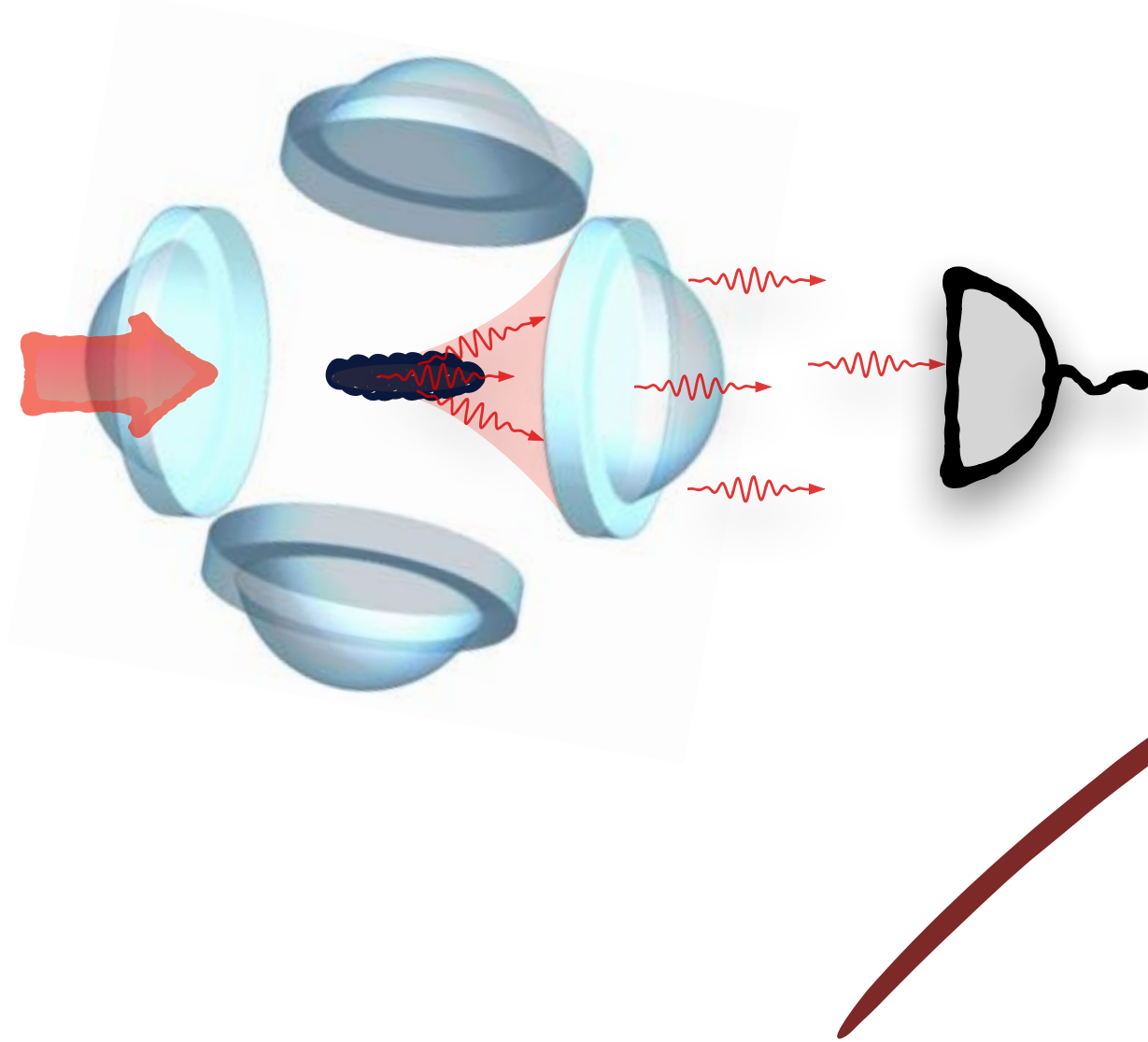


# Beyond intensity: light correlations

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Superradiant ensembles



Strong variation :  $g_2(0) = 1$  (coherent state from large collective dipole)

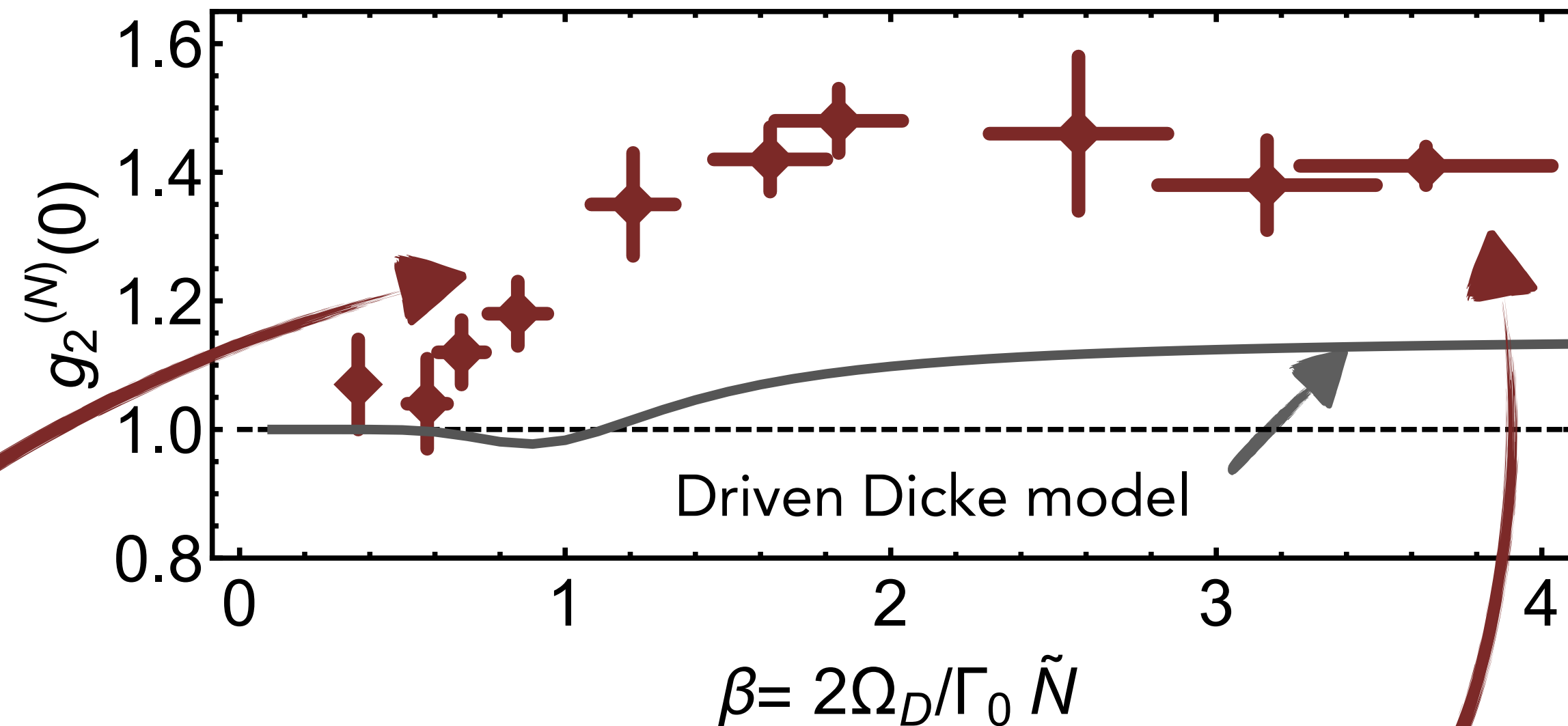
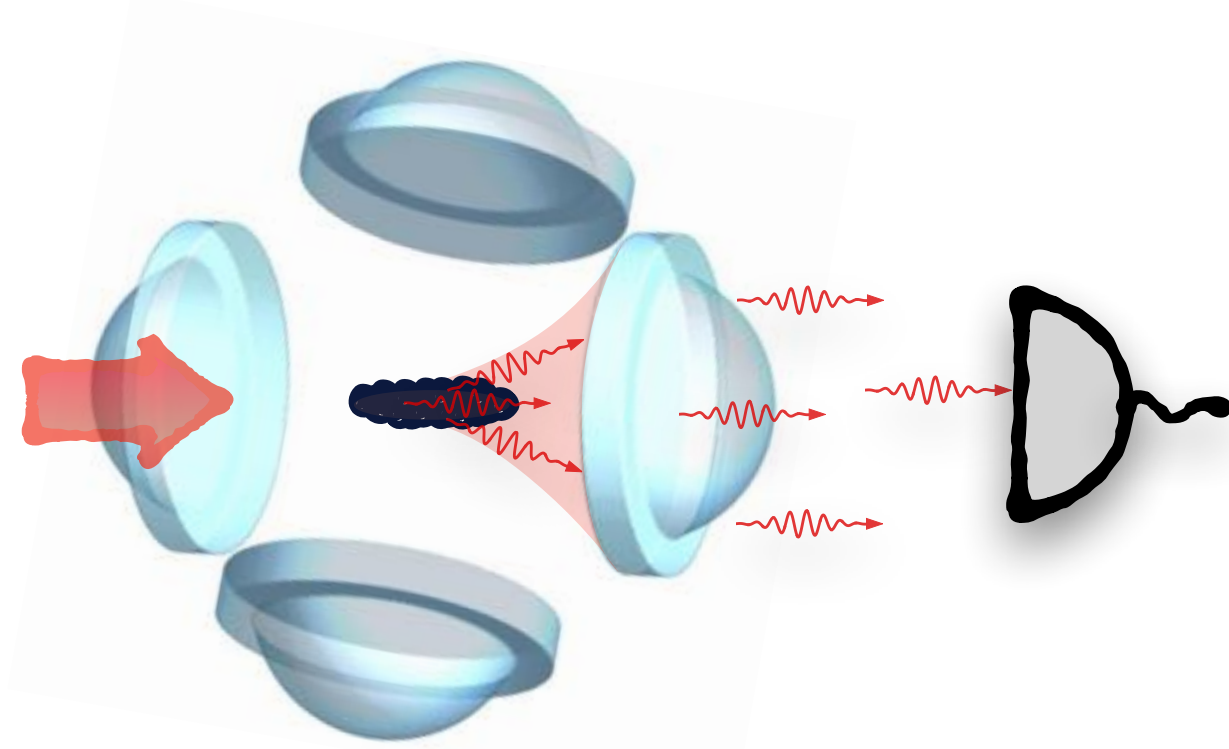


# Beyond intensity: light correlations

General case very challenging

$$g_2(0, \mathbf{k}) = \frac{\sum_{k,l,m,n} e^{i\mathbf{k}\cdot(\mathbf{r}_k+\mathbf{r}_l-\mathbf{r}_m-\mathbf{r}_n)} \langle \hat{\sigma}_k^+ \hat{\sigma}_l^+ \sigma_m^- \hat{\sigma}_n^- \rangle}{\left[ \sum_{m,n} e^{i\mathbf{k}\cdot(\mathbf{r}_m-\mathbf{r}_n)} \langle \sigma_m^+ \hat{\sigma}_n^- \rangle \right]^2}$$

Superradiant ensembles

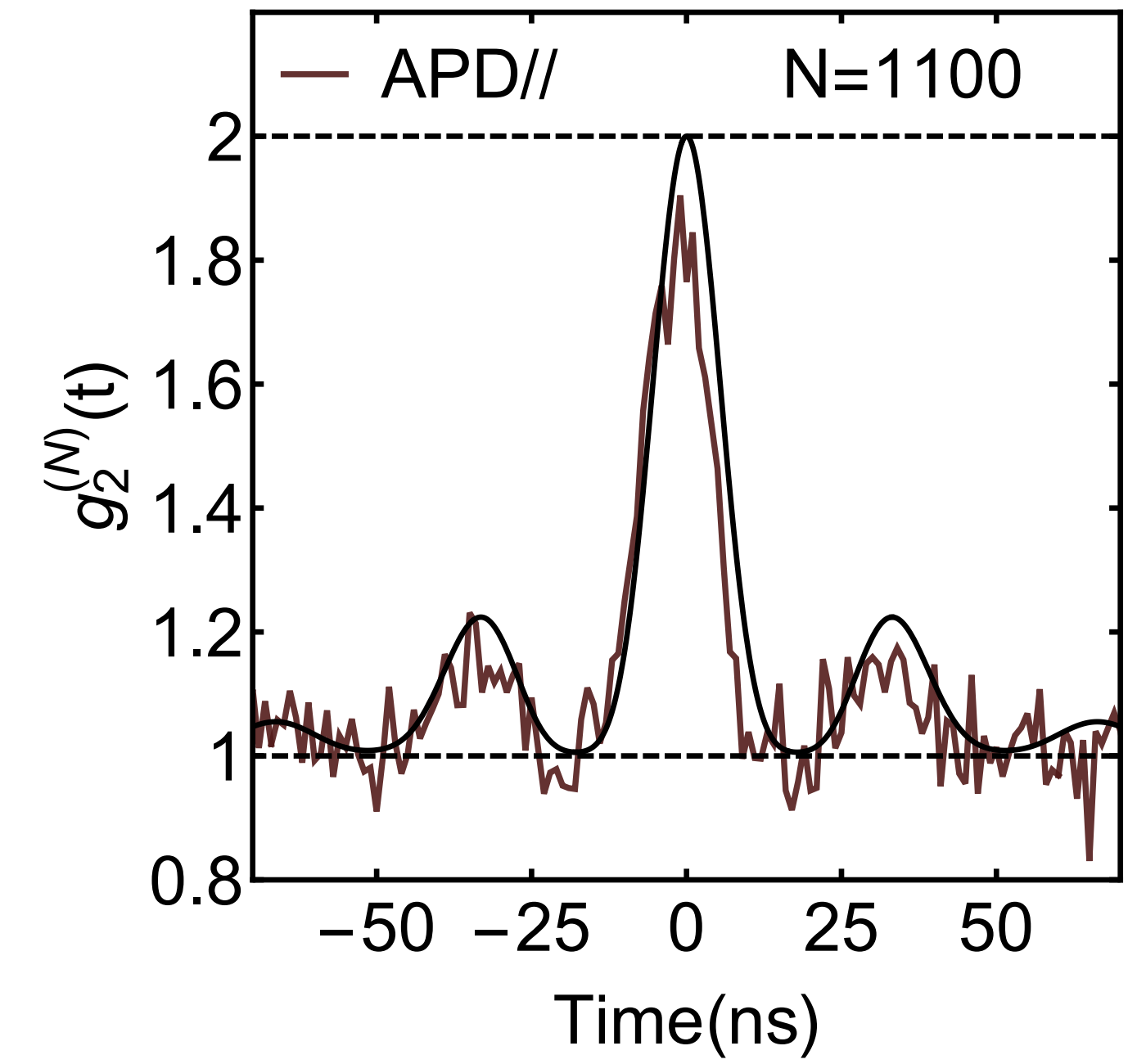
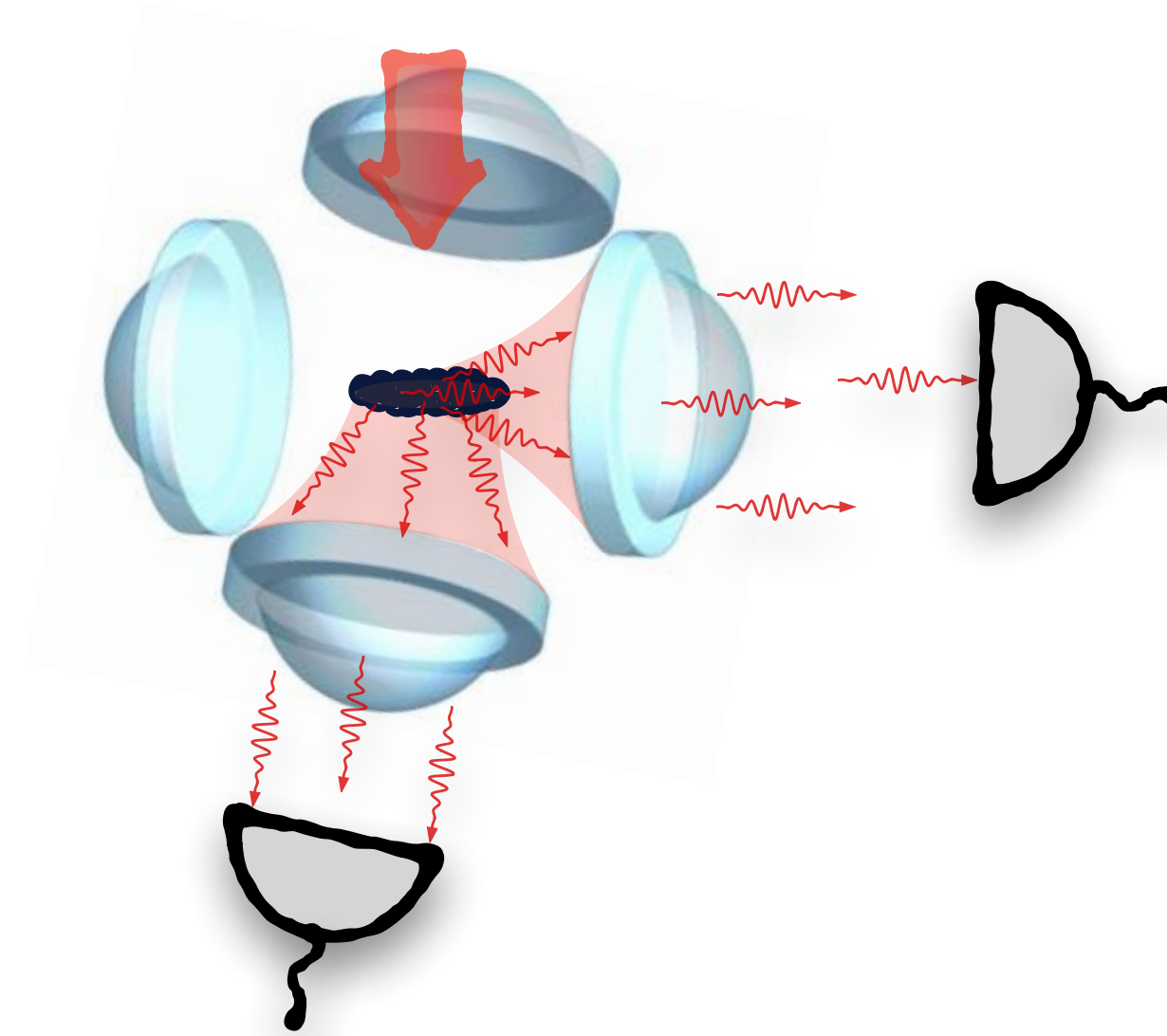


Strong variation :  $g_2(0) = 1$  (coherent state from large collective dipole)

$2 > g_2(0) > 1$  correlated emitters

# Beyond intensity: light correlations

## Directional correlations



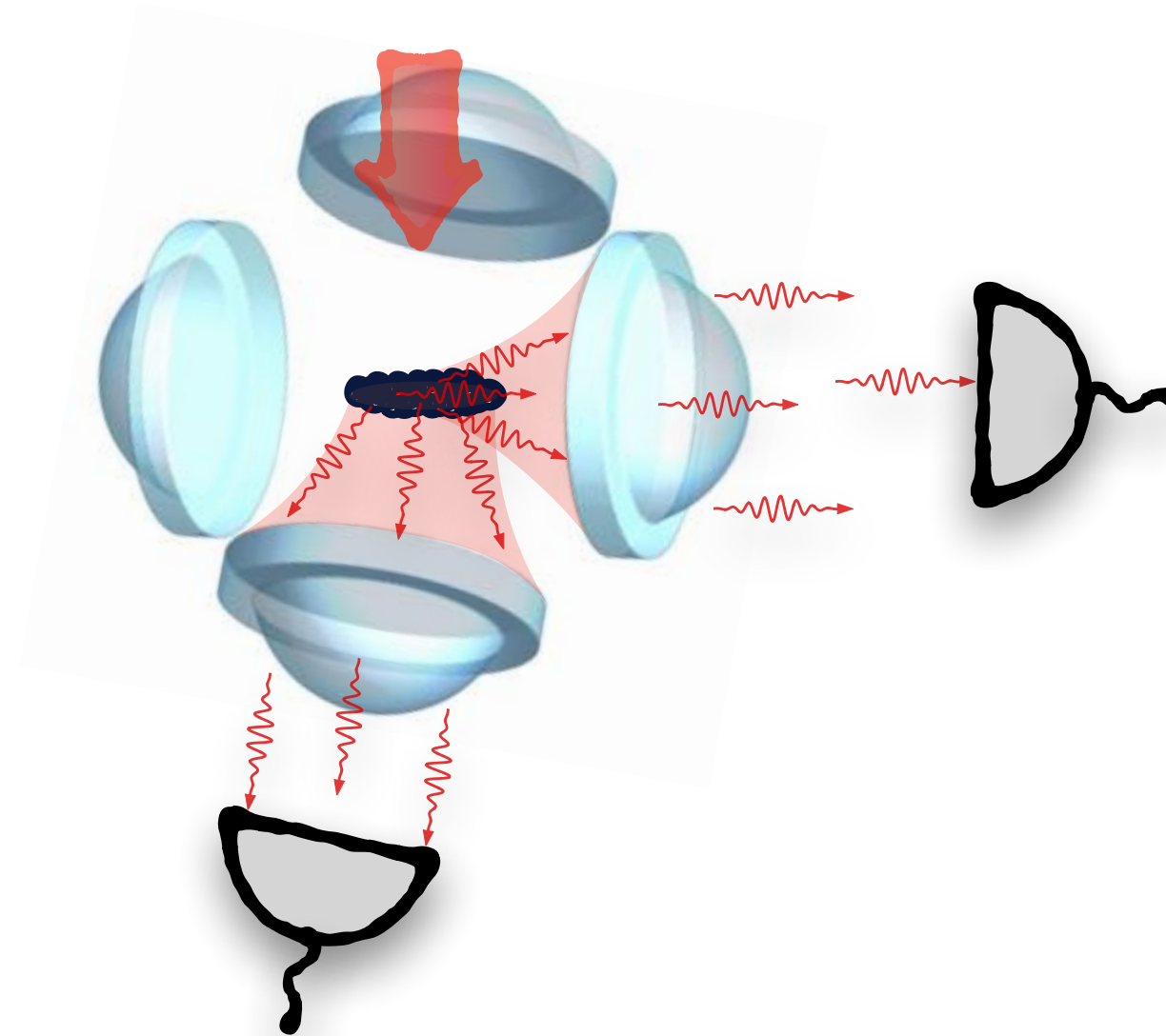
Work in progress

# Beyond intensity: light correlations

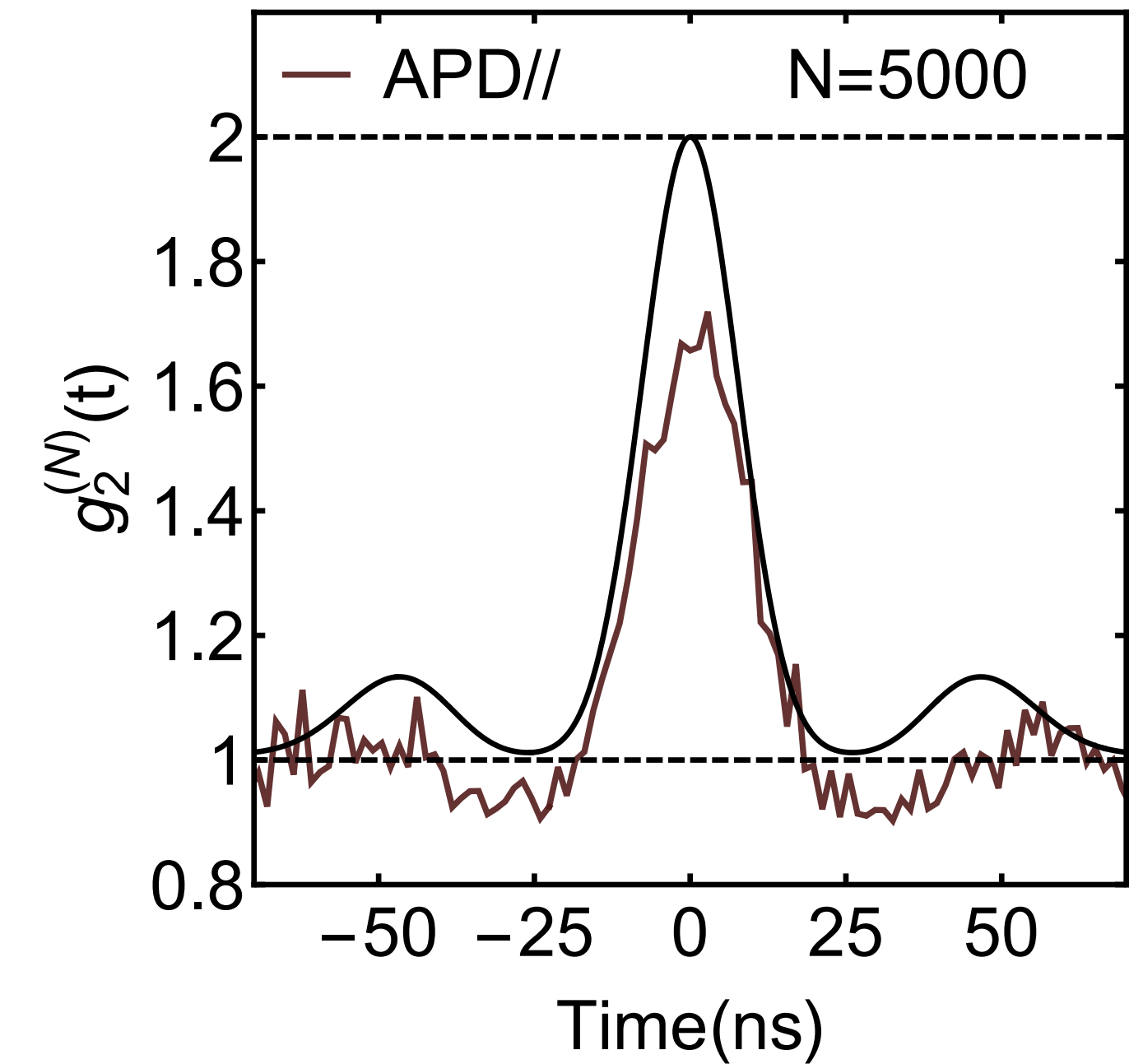
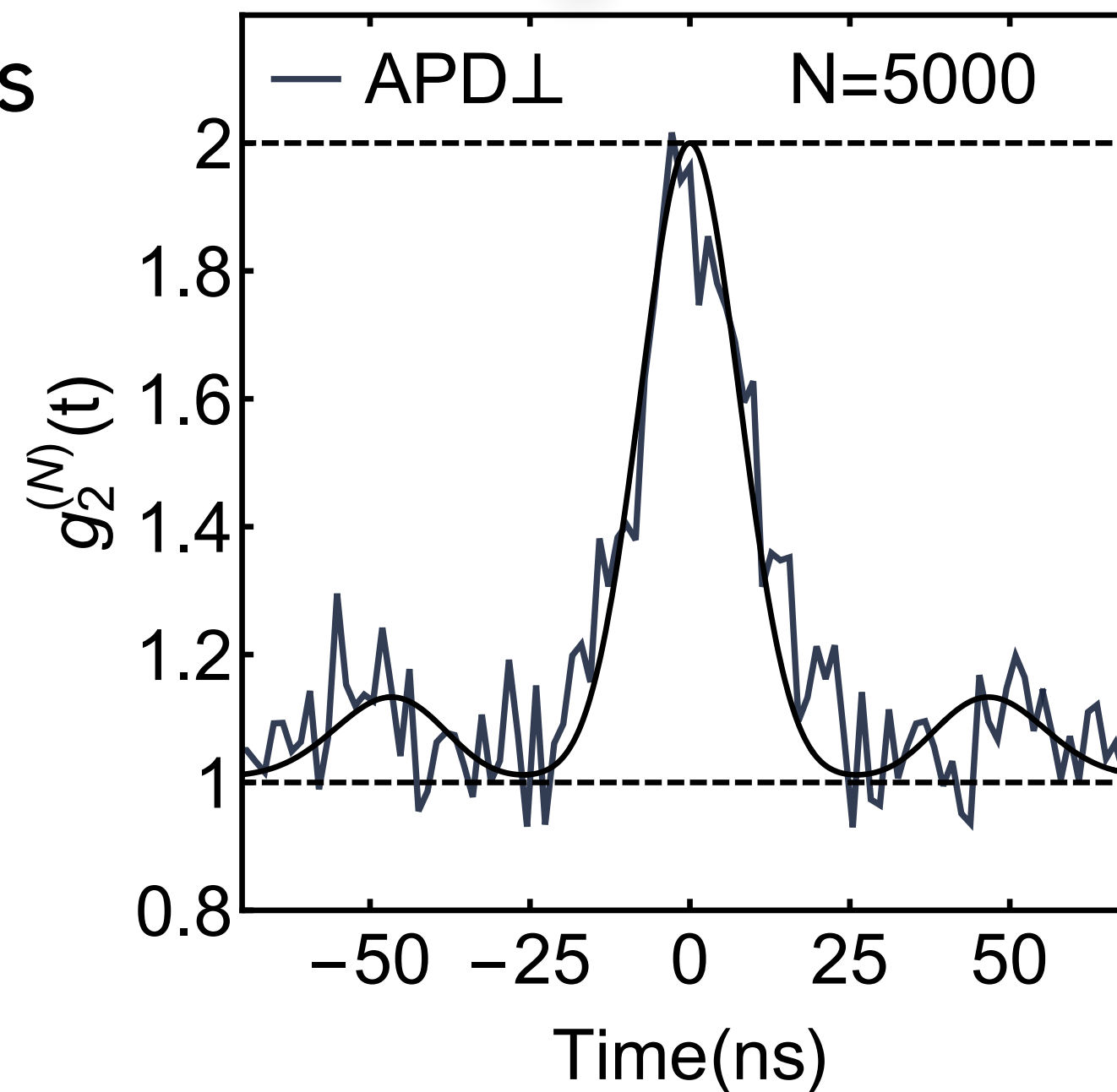
## Directional correlations

Axially: violation of Siegert

- $g_2(0) < 2$
- $g_2(\tau) < 1$



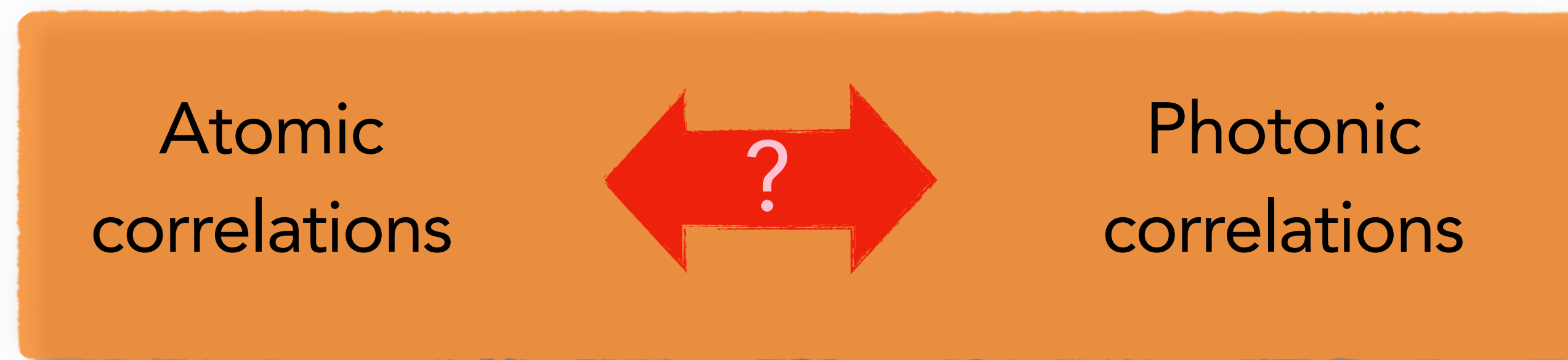
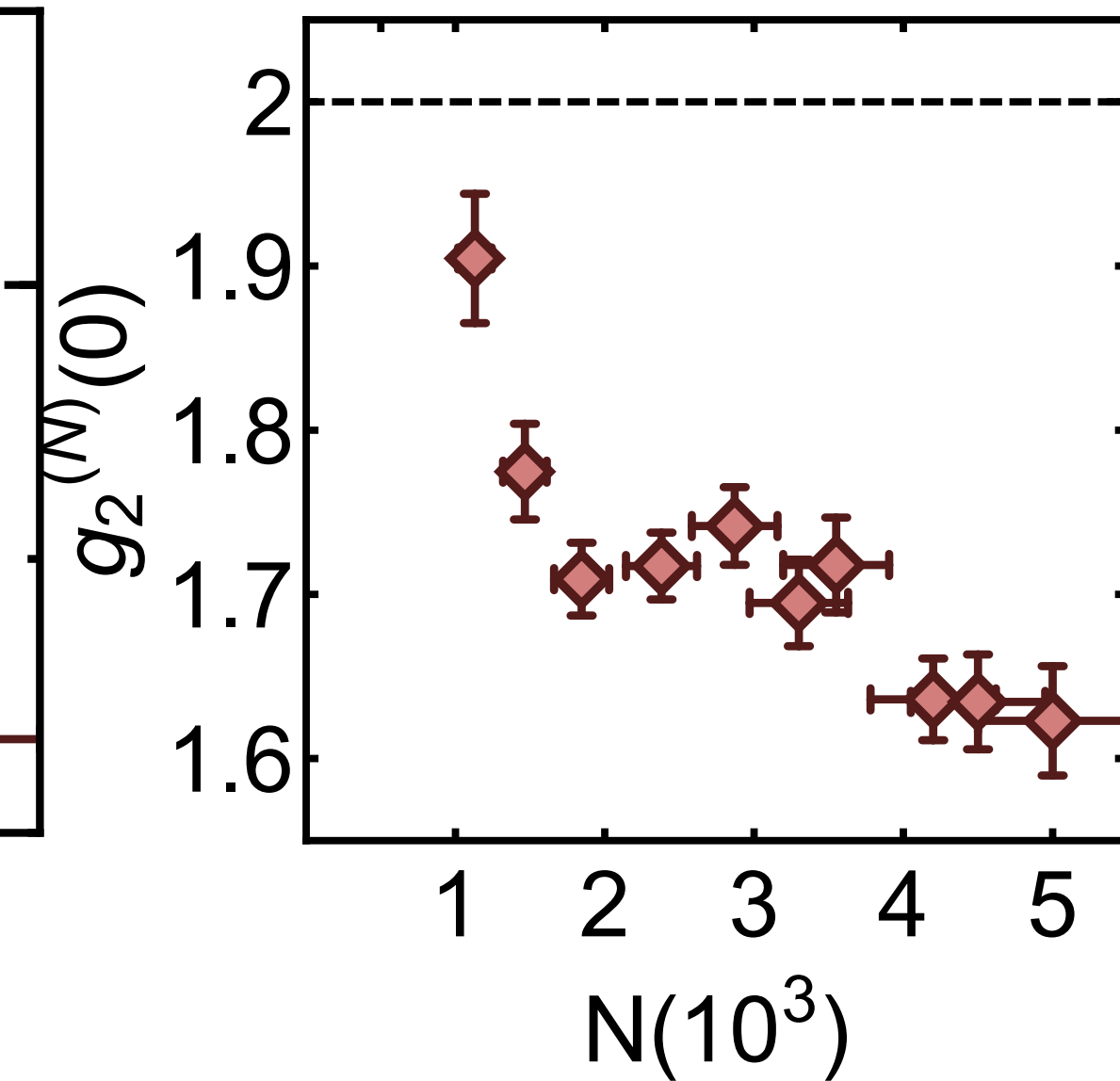
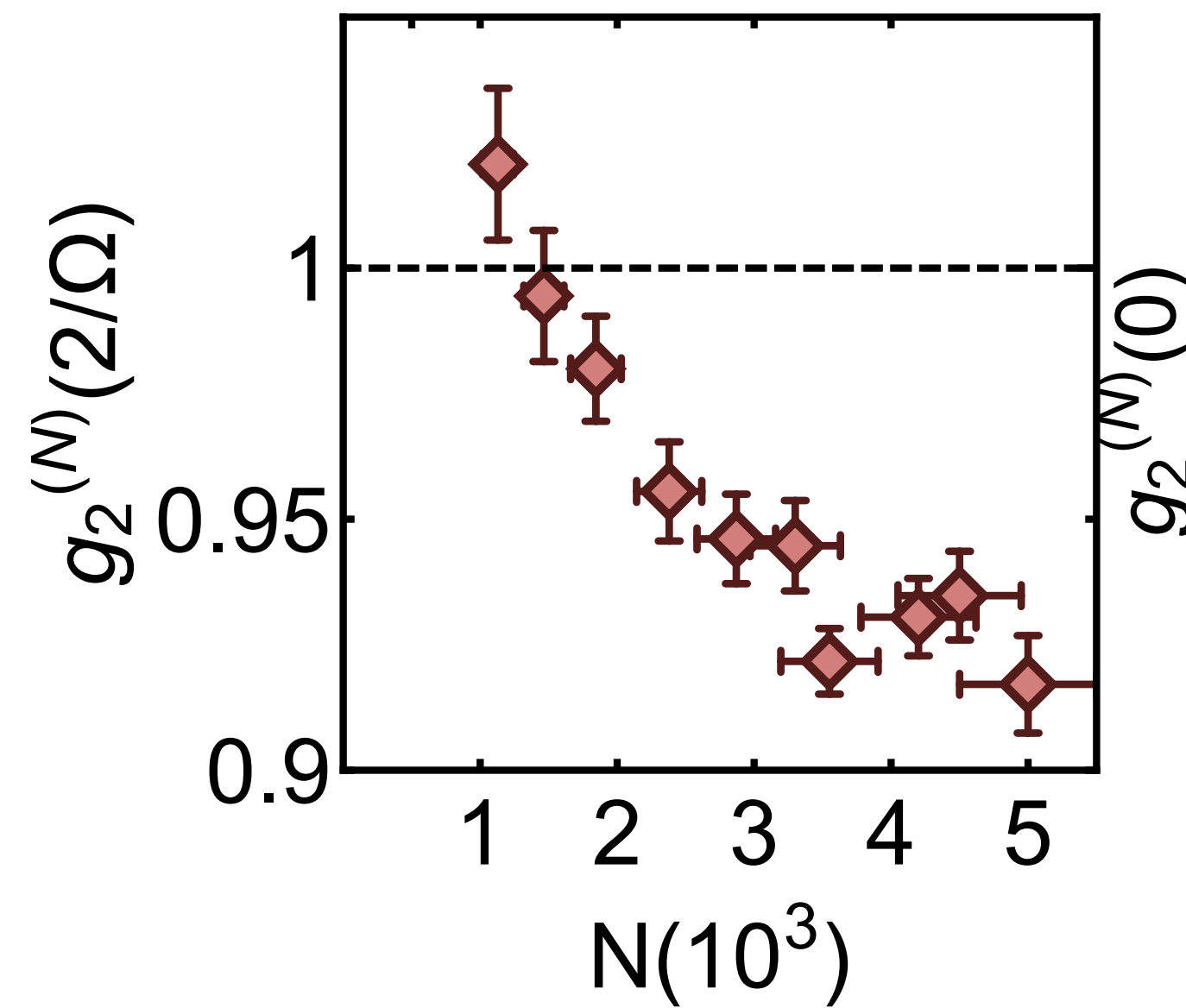
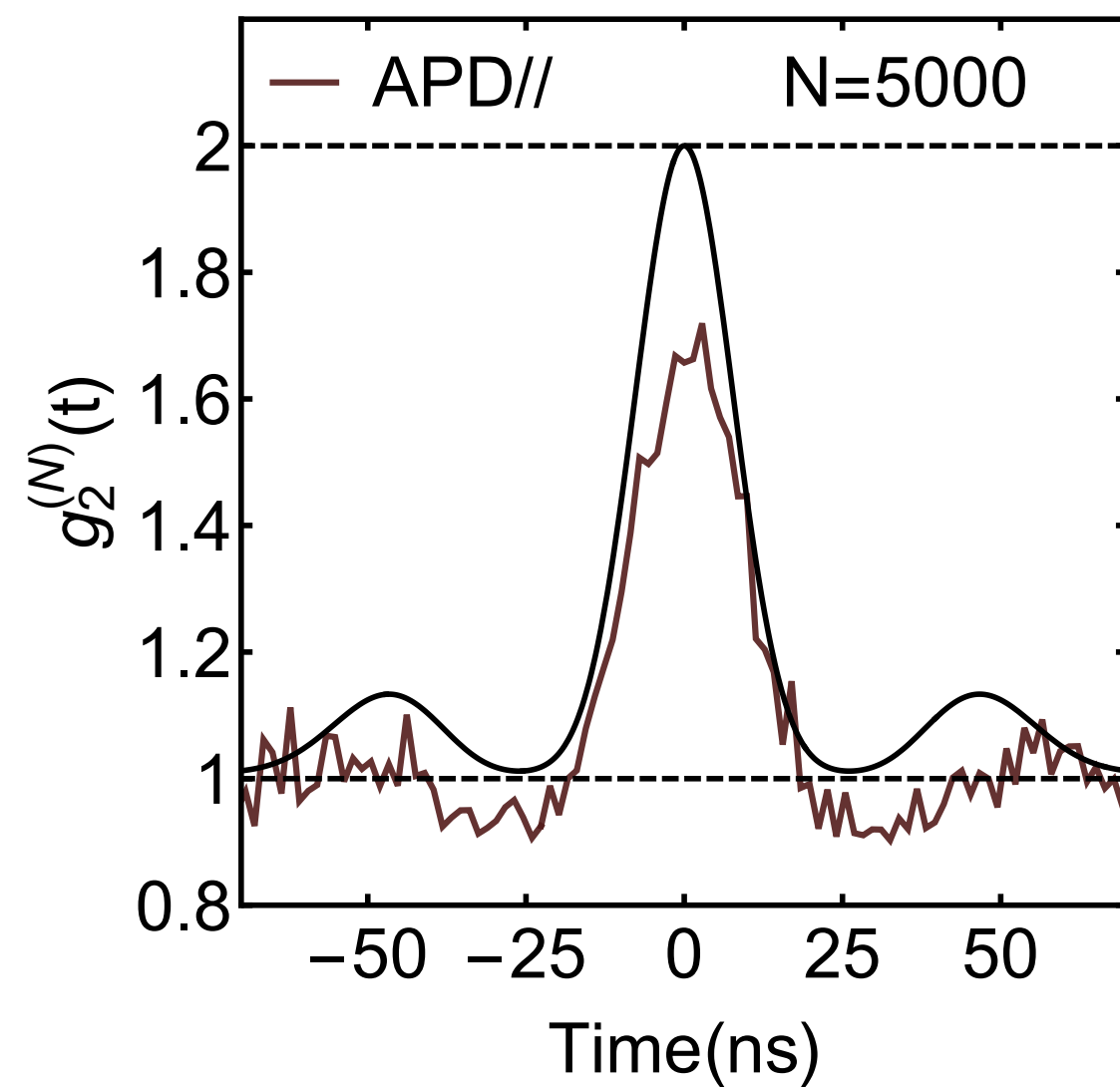
Radially: independent scatterers



Work in progress

# Beyond intensity: light correlations

## Growth of collective effects



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