GdR Complexe Annual Workshop

Reflectionless states in complex media

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Acknowledgment


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Outline

1) General introduction

2) The reflectionless scattering operator

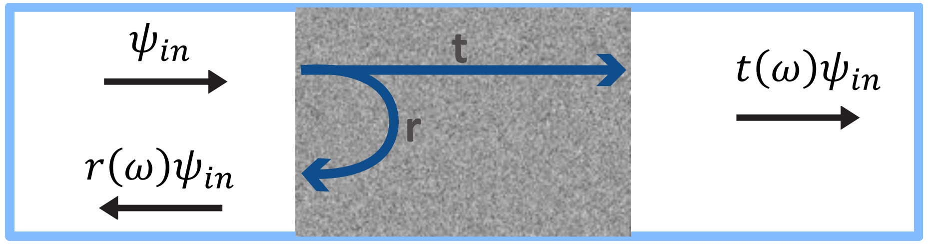
3) Exceptional Points in *PT-symmetric* systems

4) Enhanced broadband transmission through barriers in symmetric systems

5) Anti-reflection structures for perfect transmission through complex media

Scattering matrix

Wave equation
$\Delta\phi + k^2 n^2(\boldsymbol{x})\phi = 0$



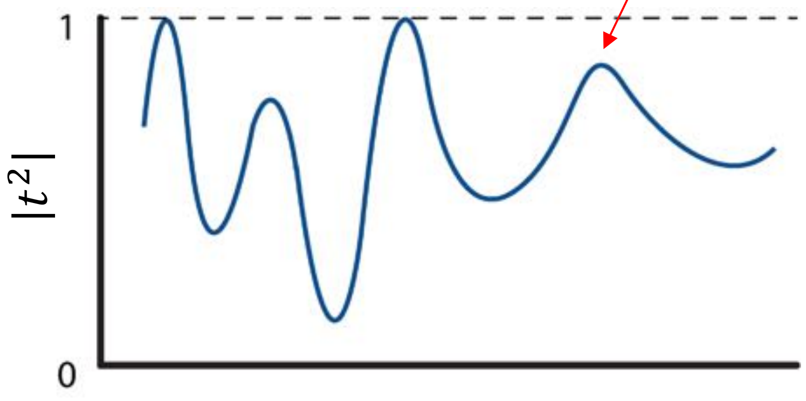
$2N{\times}2N$ Scattering matrix
N: Number of coupled channels

$$S(\omega) = \begin{pmatrix} r(\omega) & t'(\omega) \\ t(\omega) & r'(\omega) \end{pmatrix}$$

ReflectionLess states → Perfect transmission

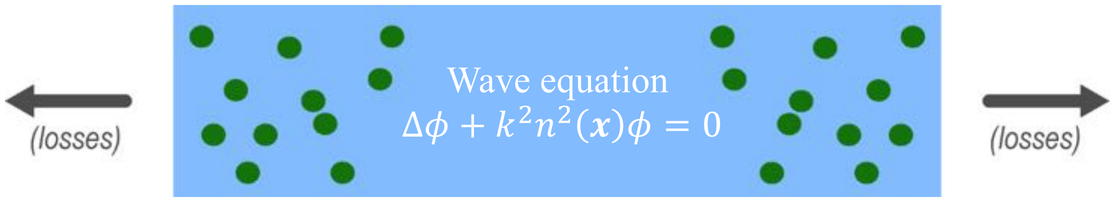
$$T = ||\psi_{out}||^2 = 1 \text{ and } R = ||r\psi_{in}||^2 = 0$$

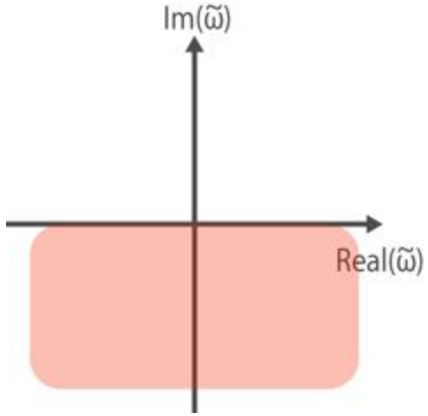
Transmission spectrum for $N = 1$



Quasi-Normal Modes & ReflectionLess Modes

- Quasi-Normal Modes

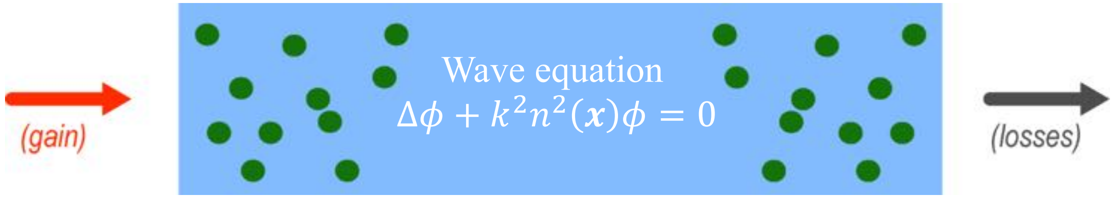




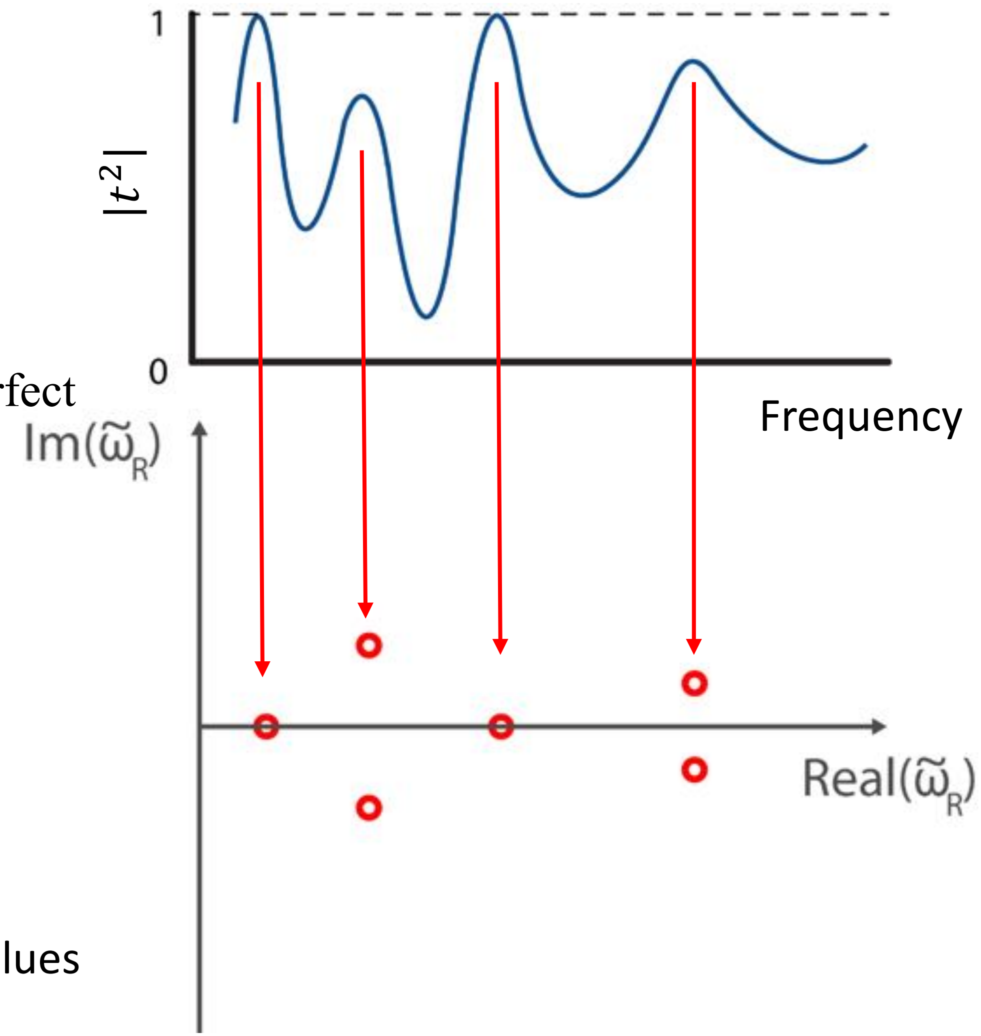
$\tilde{\omega}$: eigenvalues

  - Non-Hermitian problem
  - Very used tools to study the transmission behavior
    - Lack of information on perfect transmission

Correspondence with the transmission



Direct relation between reflectionless-eigenvalues and perfect transmission peaks

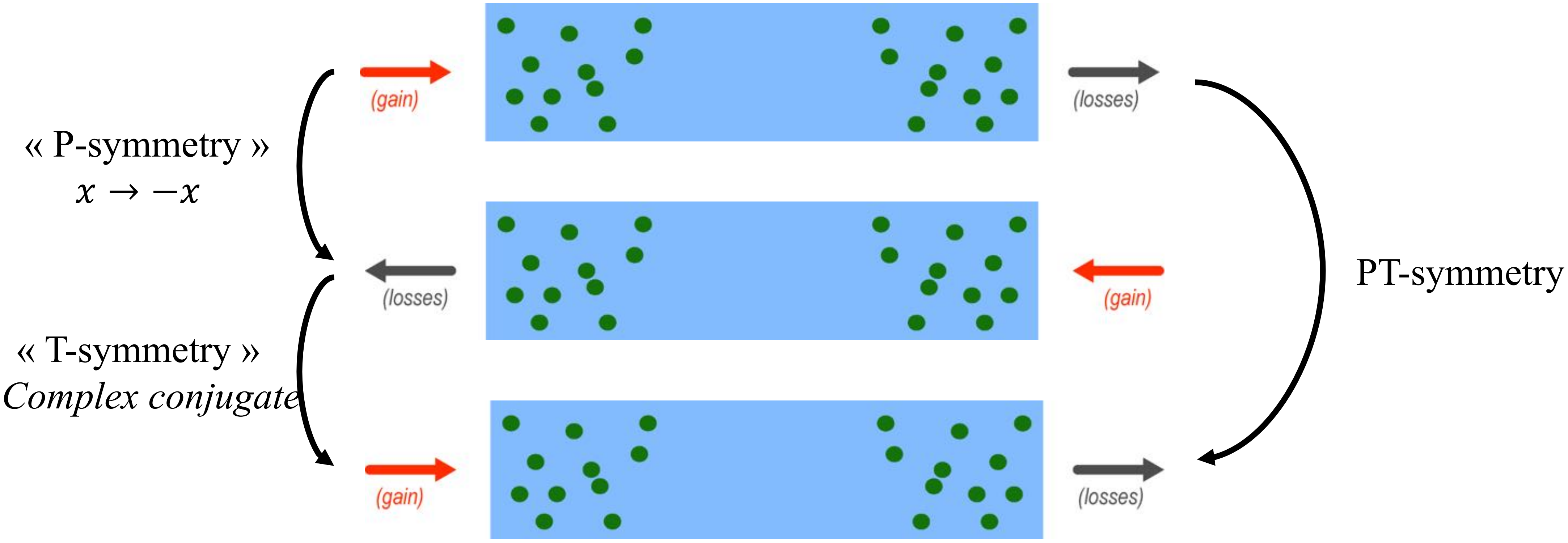


$\boldsymbol{Im(\tilde{\omega}_R) = 0}$ **→ Zero reflection**

$\tilde{\omega}_R$: eigenvalues

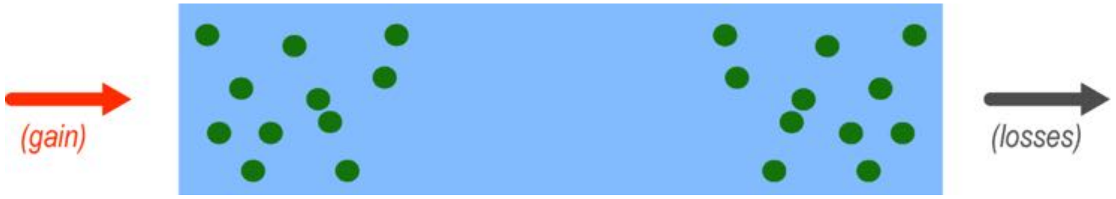
Parity-Time Symmetry

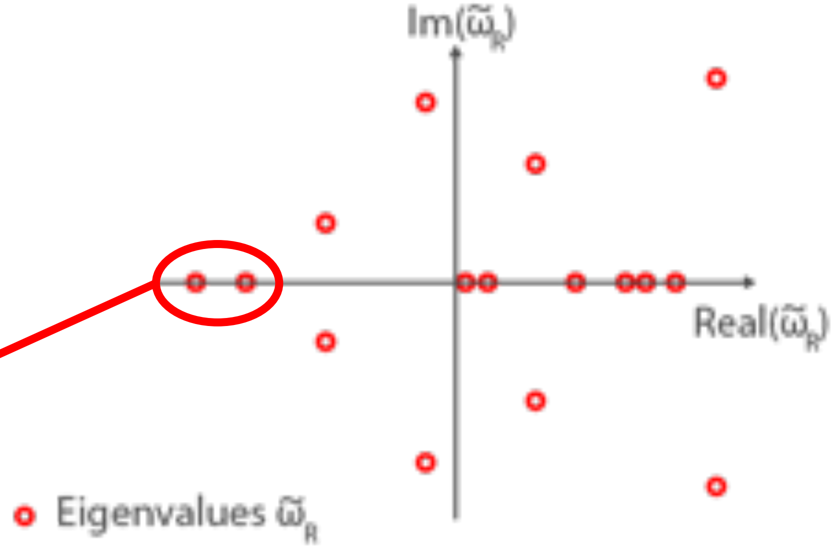
ReflectionLess Modes : PT-symmetry



Symmetrical system → ReflectionLess Mode with PT-symmetry

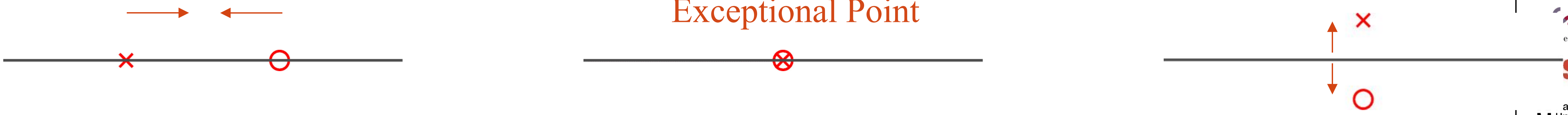
Reflectionless eigenvalues in *PT*-symmetric systems





PT-symmetric operator :

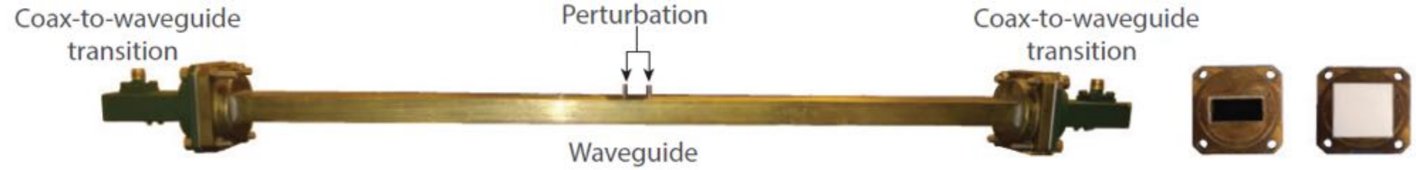
- Eingenvalues are either real or come as complex conjugate pair on the real axis.



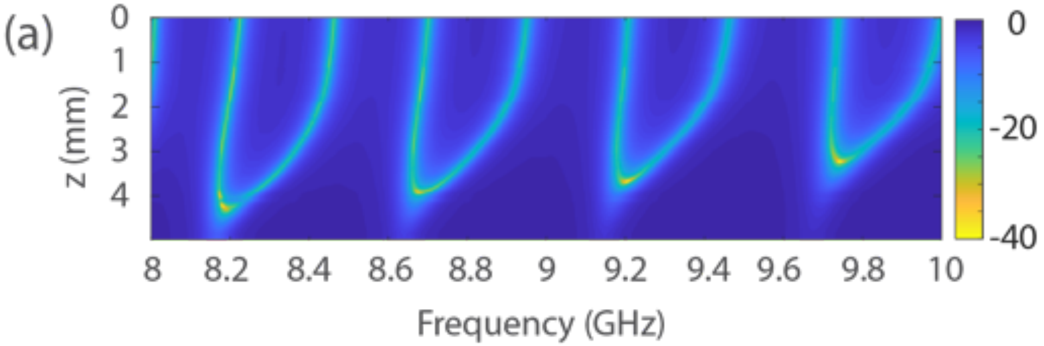
Perfect transmission until coalescence

Perturbed Fabry-Perot system

$$(L = 400mm\ ; W = 22{,}86mm\ ; h = 10{,}16mm)$$

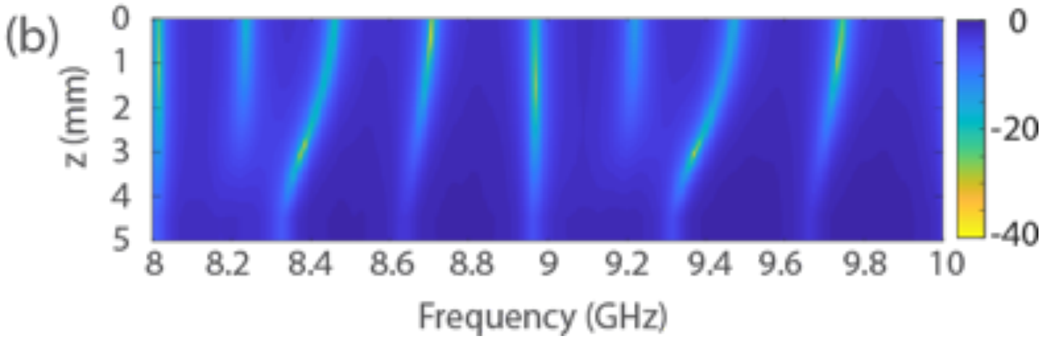


Symmetrical perturbation *z*



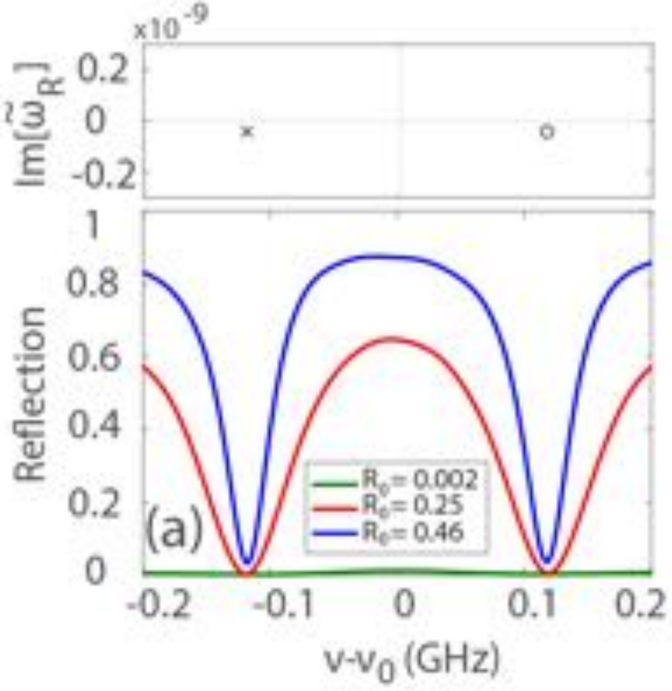
Breaking the symmetry:

Asymmetrical perturbation *z*



C. Ferise, P. Del Hougne, S. Félix, V. Pagneux, and M. Davy, "Exceptional Points of P T-Symmetric Reflectionless States in Complex Scattering Systems," Physical Review Letters, vol. 128, p. 203904, 2022.

Exceptional Point feature



Flattened quartic line shape

$$\boldsymbol{R \sim \frac{\Delta\omega^4}{\gamma^2}}$$

Coupled mode theory

Effective Hamiltonian:

2 channels, 2 resonances

Gain

$$H_{RL} = H_0 + i\frac{V_0 V_0^T}{2} - i\frac{V_1 V_1^T}{2}$$

Losses

Hamiltonian of the closed system

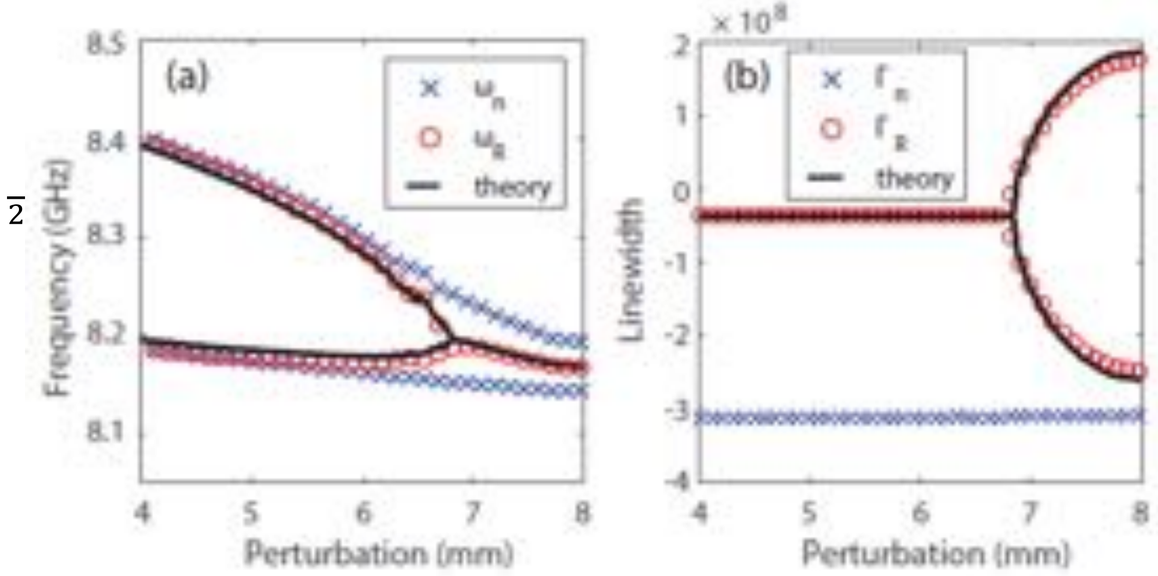
Left coupling vector

Right coupling vector

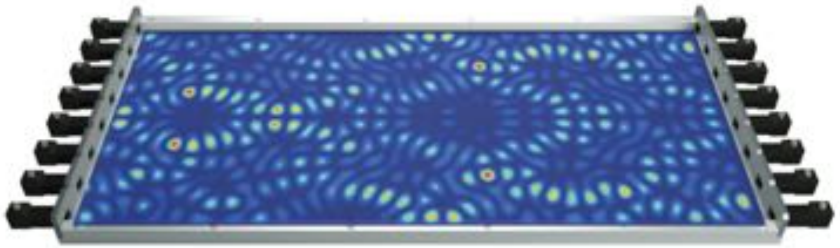
Reflectionless operator $H_{RL} = \omega_0(z)\mathbb{I} + \frac{1}{2}\begin{pmatrix} -\delta\omega_0(z) & i\gamma \\ i\gamma & \delta\omega_0(z) \end{pmatrix}$

$\omega_0(z) \pm \delta\omega_0(z)$: resonances of the closed system

γ: coupling coefficient

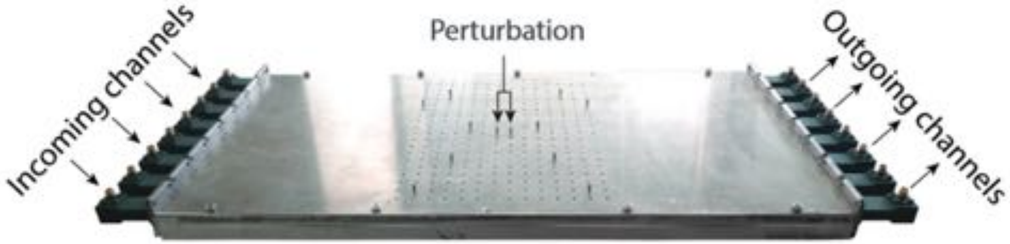


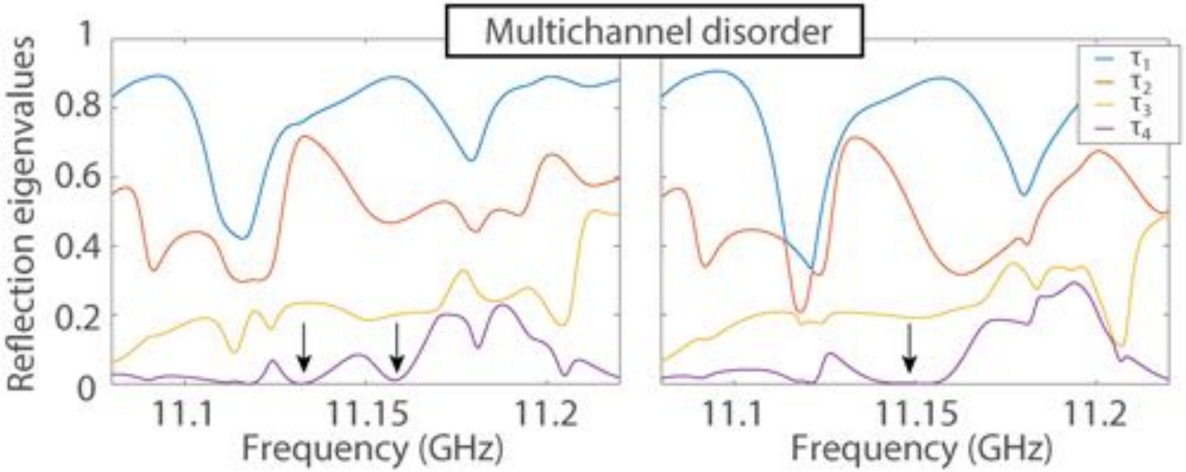
Multichannel Disorder



2D Cavity: $L = 500\text{mm}$, $W = 250\text{mm}$ and $h = 8\text{mm}$

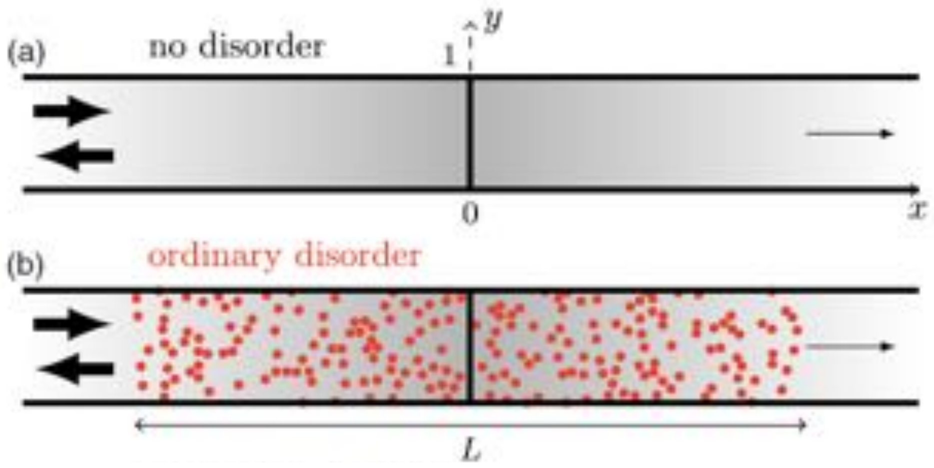
Reflectionless exceptional point identify on the last eigenvalue $\tau_4(\nu, z)$ of $r^\dagger r$.

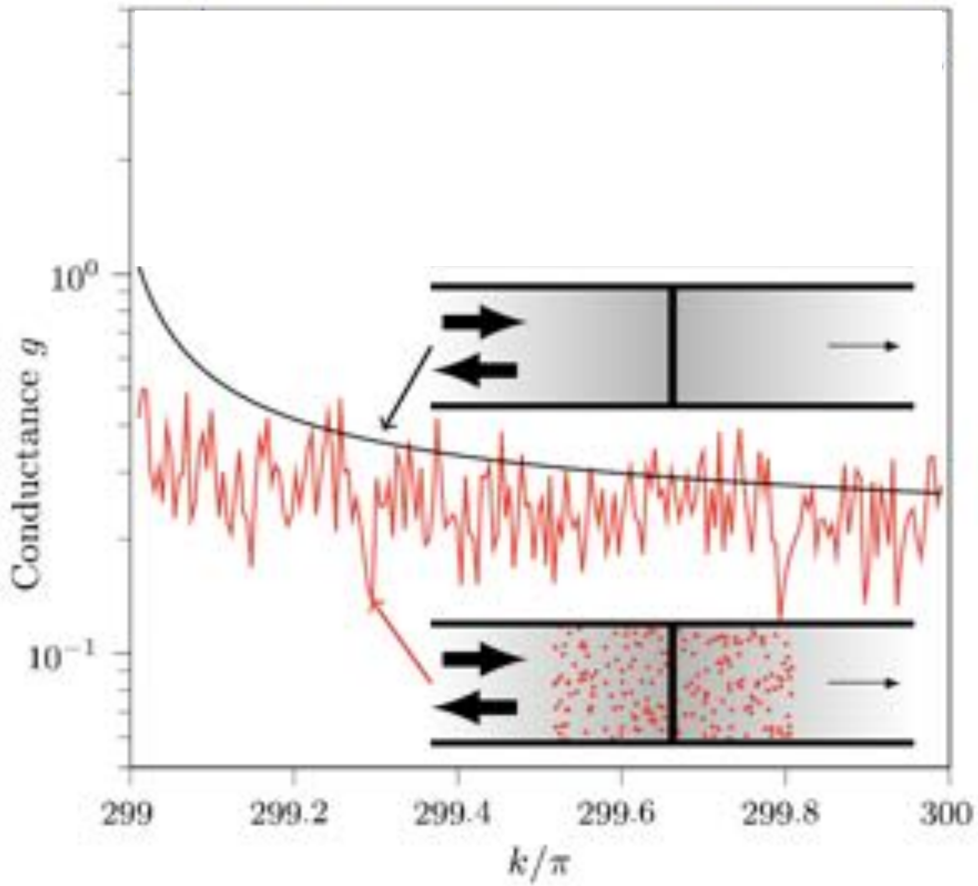




C. Ferise, P. Del Hougne, S. Félix, V. Pagneux, and M. Davy, "Exceptional Points of P T-Symmetric Reflectionless States in Complex Scattering Systems," Physical Review Letters, vol. 128, p. 203904, 2022.

Broadband enhancement of transmission

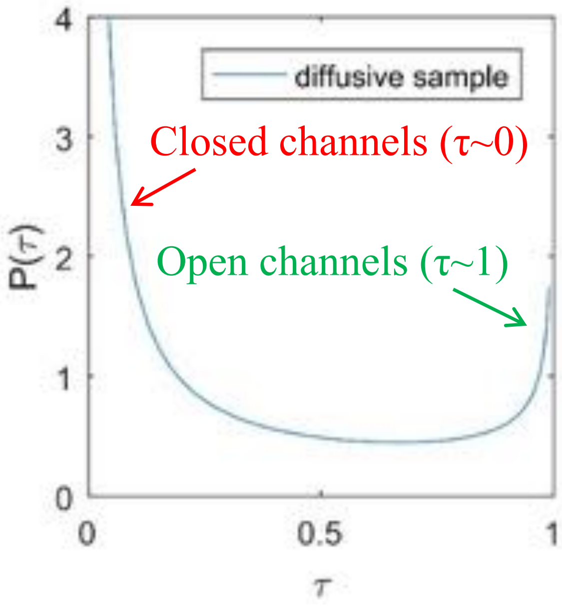




É. Chéron, S. Félix, and V. Pagneux, "Broadband-Enhanced Transmission through Symmetric Diffusive Slabs," Phys. Rev. Lett., vol. 122, p. 125501, 2019.

Bimodal distribution

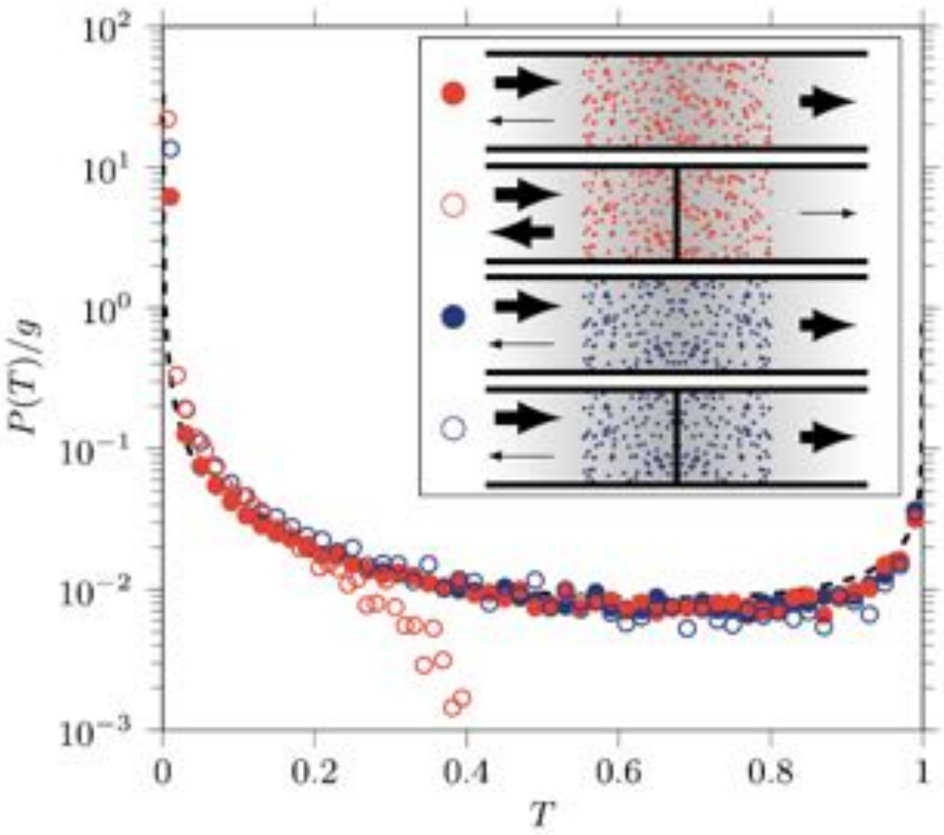
Distribution of the transmission eigenvalues



Decomposition of the transmission matrix into eigenchannels

$$t = \Sigma_{n=1}^{N} u_n \sqrt{\tau_n} v_n^{\dagger}$$

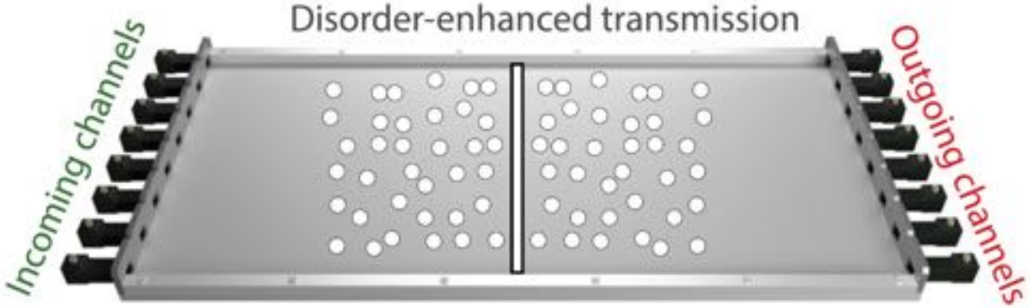
O.N.Dorokhov, JETP Lett, 1982.
P. A. Mello, P. Pereyra, and N. Kumar, Ann. Phys. (N.Y.), 1988.

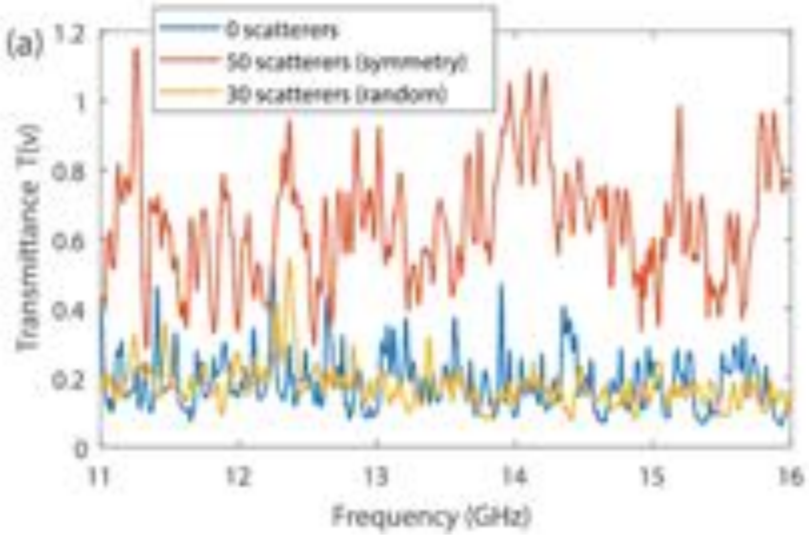


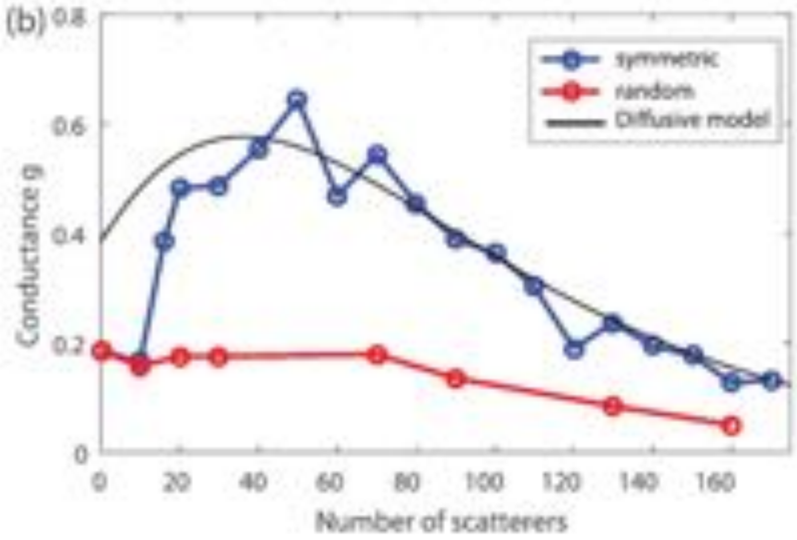
Without barrier the symmetry has no significant effect

É. Chéron, S. Félix, and V. Pagneux, "Broadband-Enhanced Transmission through Symmetric Diffusive Slabs," Phys. Rev. Lett., vol. 122, p. 125501, 2019.

Microwave experiment







Experimental validation

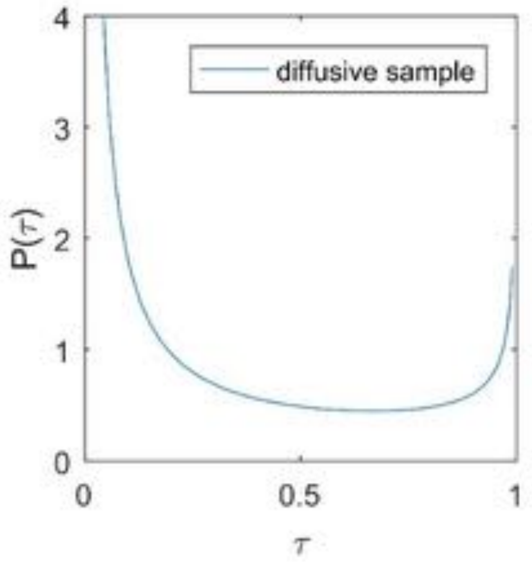
Agreement with a diffusive model for a large number of scatterers

Including scatterers can enhance transmission

M. Davy, C. Ferise, É. Chéron, S. Félix, and V. Pagneux, "Experimental evidence of enhanced broadband transmission in disordered systems with mirror symmetry," Appl. Phys. Lett., vol. 119, p. 141104, 2021.

Perfect transmission for any incident wavefront

Diffusive transport regime



Proportion of open channels : $\frac{\ell}{L}$

Average transmission : $T \sim \frac{\ell}{L}$

ℓ: mean free path
L: length of the scattering system

Wavefront shaping

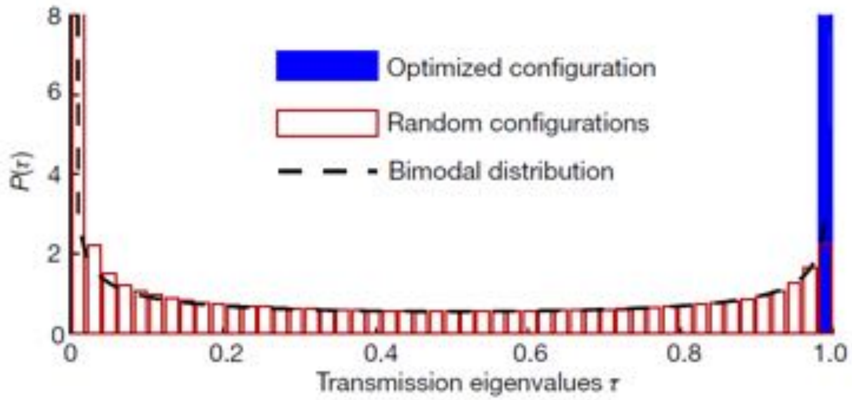
Controlling the outgoing wave $\psi_{out} = t(\omega)\psi_{in}$ by engineering the incoming wave

Enables perfect transmission by sending suitable wavefronts ($\tau = 1$)

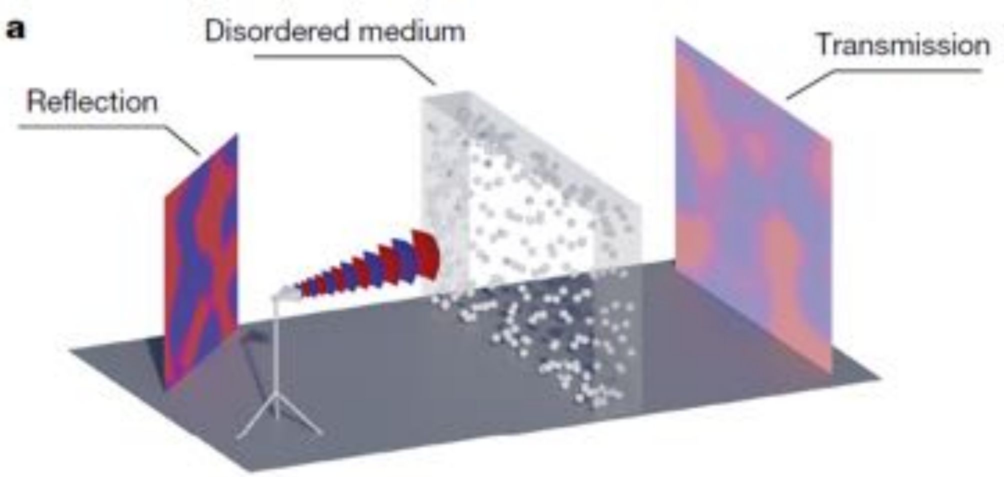
- Only a fraction of transmission channels are open

Q: Is it possible to have a fully transmitting system for any incident wavefront ?

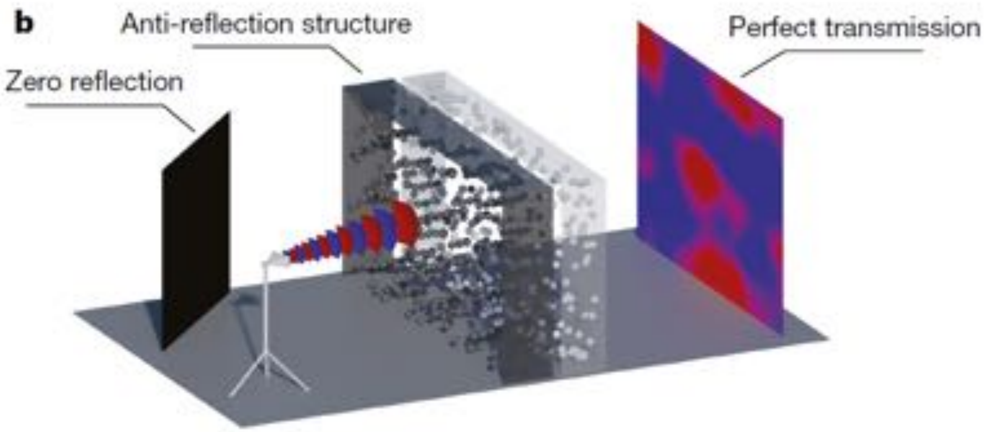
- Wavefront shaping techniques will not suffice



Anti-reflection structure: concept



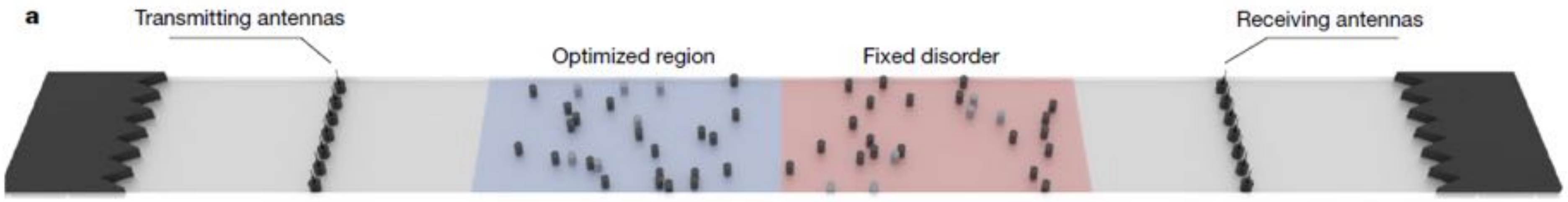
$$\langle T \rangle \sim \frac{\ell}{L}$$

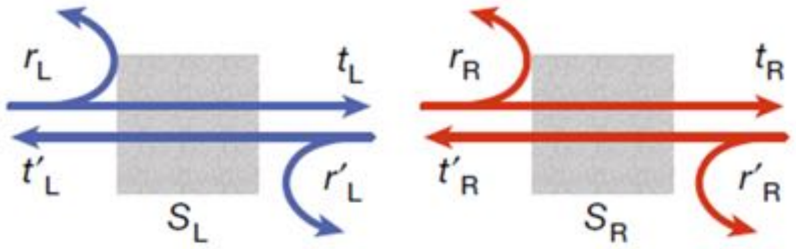


$$\langle T \rangle \sim 1$$

M. Horodynski, M. Kühmayer, C. Ferise, S. Rotter, and M. Davy, "Anti-reflection structure for perfect transmission through complex media," Nature, **607**, 2022.

Theory





Composite scattering matrix $S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$

$$t = t_R[\mathbb{1} - r'_L r_R]^{-1} t_L.$$

$$r = r_L + t'_L r_R[\mathbb{1} - r'_L r_R]^{-1} t_L.$$

Unitarity of the scattering matrices $S_x S_x^\dagger = \mathbb{1}$

Generalized critical coupling condition $r = 0$

$$r'_L = r_R^\dagger$$

Proof $r = r_L + t'_L r'^\dagger_L \left[\mathbb{1} - r'_L r'^\dagger_L\right]^{-1} t_L = r_L + t'_L r'^\dagger_L t_L^{\dagger^{-1}} t_L^{-1} t_L = 0.$

- The internal structure of the disorder does not need to be known
- Only the left-sided reflection matrix r_R is relevant

CNRS, CentraleSupélec, INSA Rennes, Nantes Université, Université de Rennes 1

Experimental setup

Optimized region

Fixed disorder

Experimental setup :

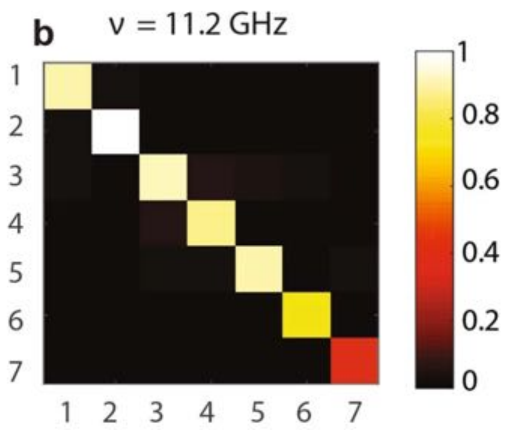
2D waveguide: $L = 800$ mm, $W = 100$mm, $h = 8$mm

Two arrays of 7 wire-antennas

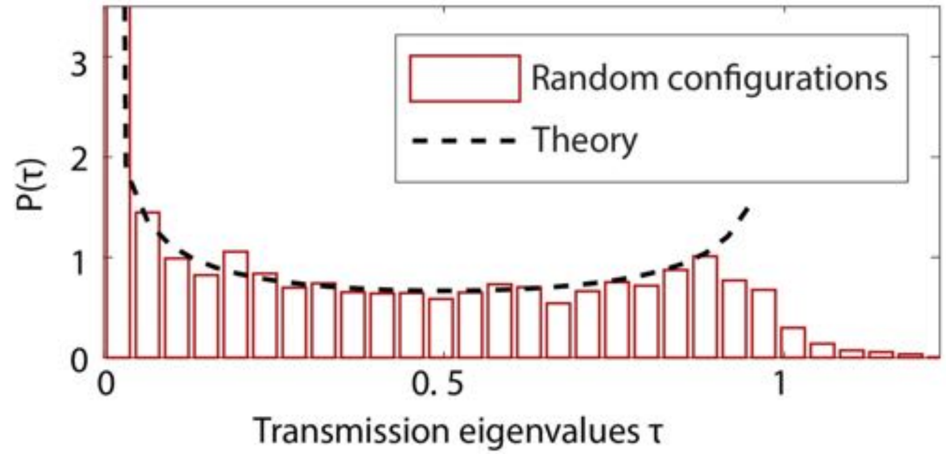
Teflon and Aluminum cylinders

$f_0 = 7$ GHz (4 modes) or $f_0 = 11.2$ GHz (7 modes)

Empty waveguide



Random disorder



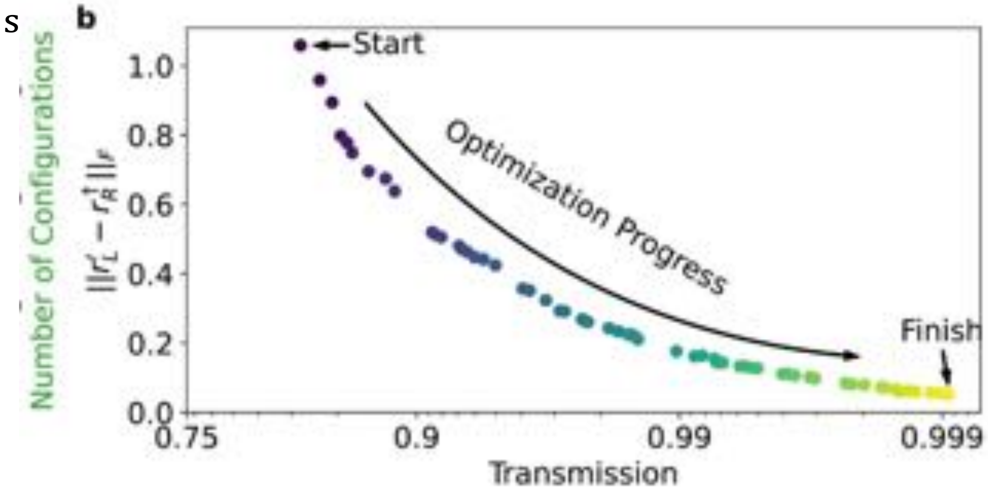
Inverse design process

Optimization technique: iterative procedure based on the gradient of the objective $f = 1 - \frac{\mathrm{Tr}[tt^\dagger]}{N}$

Calculate the gradient: generalized Wigner-Smith operator $Q_\alpha = -iS^{-1}\frac{\partial S}{\partial \alpha}$

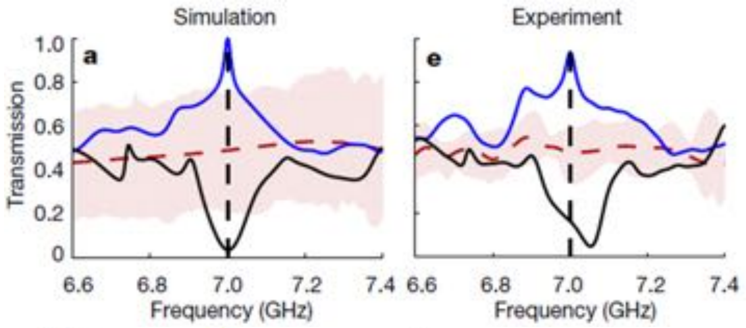
Translation of the nth scatterer: $\langle u \left| -iS^{-1}\frac{\partial S}{\partial \boldsymbol{r}_n} \right| u \rangle \propto$

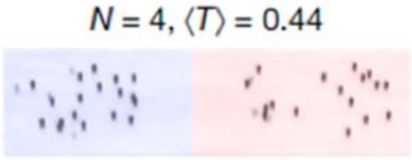
$\int_0^{2\pi} \left(\begin{matrix} \cos\phi \\ \mathrm{s} \end{matrix} \right)$ …



Group of S. Rotter:
P. Ambichl, et al., "Focusing inside Disordered Media with the Generalized Wigner-Smith Operator," Phys. Rev. Lett., 2017.
M. Horodynski, et al., "Optimal wave fields for micromanipulation in complex scattering environments," Nature Photon., 2019.

Experimental results





4 waveguide modes

fixed disorder: 3 metallic and 17 teflon cylinders

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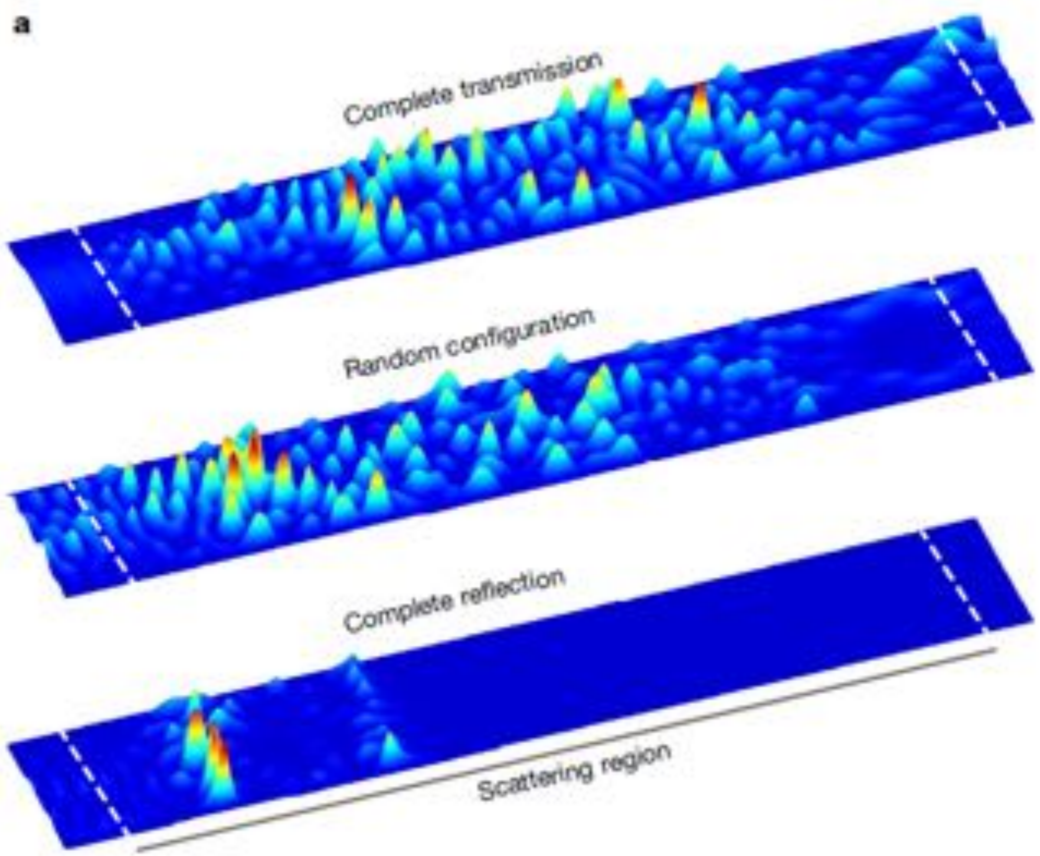
Enhancement of the energy stored

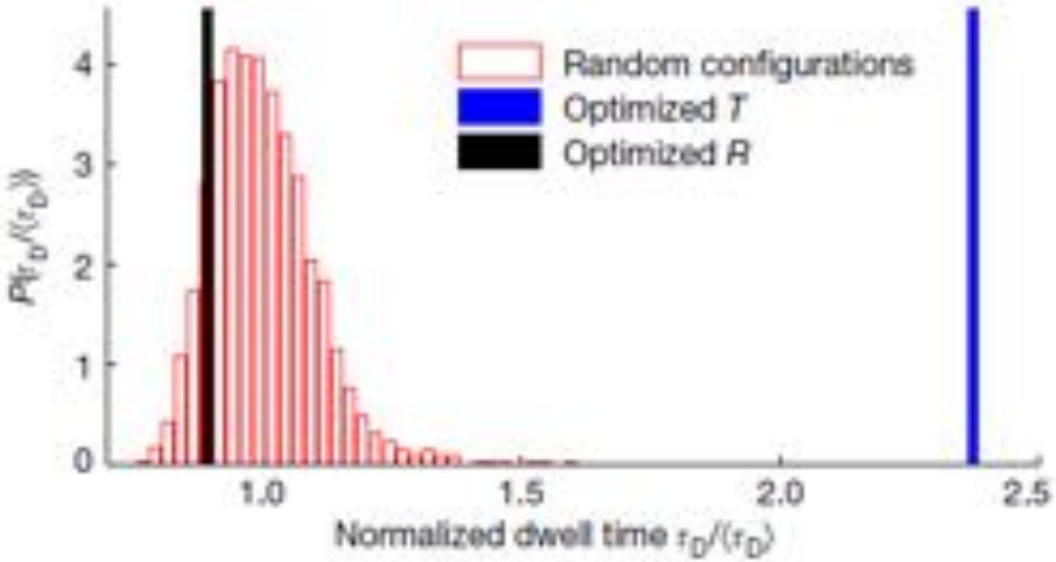


Open channel

Strong enhancement of the energy stored / dwell time

Scattering sample of perfect transmission

Strong enhancement of the average dwell time



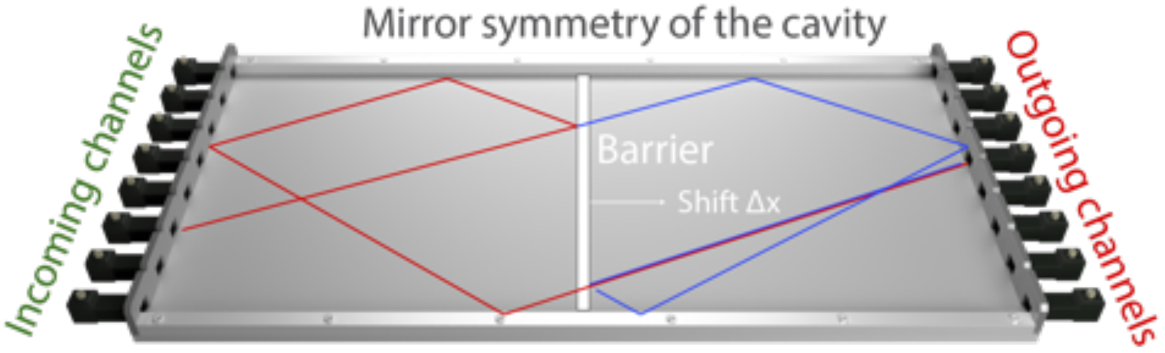
Conclusion

- ReflectionLess states can be identified from eigenvalues of the reflectionless operator

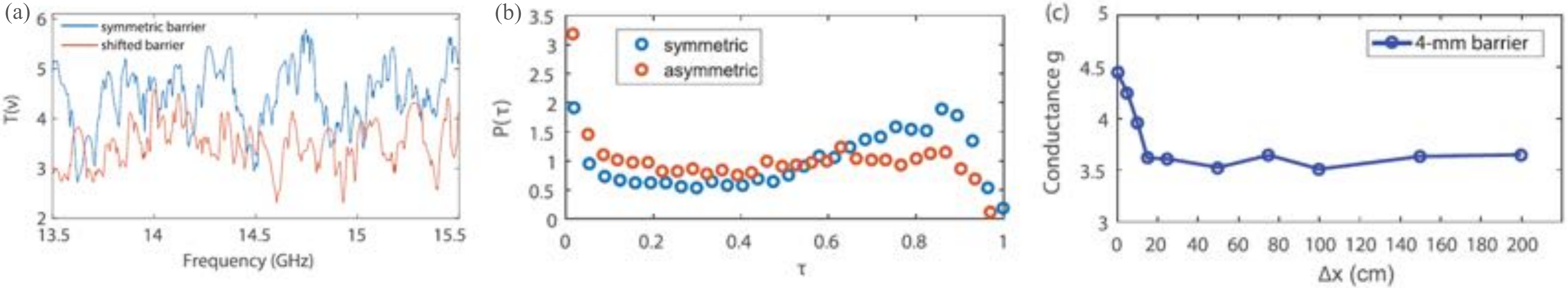
- Mirror-symmetric systems :
  - Reflectionless exceptional points
  - Broadband enhancement of the transmission through barriers

- Anti-reflection structures for perfect transmission open new ways to counteract the impact of scattering

Thank you for your attention



2-dimensional cavity
$(L = 500mm\,;\ W = 250mm\,;\ h = 8mm)$



$g_{\Delta x=0} = 4{,}44$
$g_{\Delta x=50} = 3{,}52$

M. Davy, C. Ferise, É. Chéron, S. Félix, and V. Pagneux, "Experimental evidence of enhanced broadband transmission in disordered systems with mirror symmetry," Appl. Phys. Lett., vol. 119, p. 141104, 2021.

Coupled Mode Theory

Effective Hamiltonian: 2 channels, 2 resonances

Gain → $$H_{RL} = H_0 + i\frac{V_0 V_0^T}{2} - i\frac{V_1 V_1^T}{2}$$ → Losses

Hamiltonian of the closed system $H_0 = \omega_0(z)\mathbb{I} + \frac{1}{2}\begin{pmatrix} -\delta\omega_0(z) & 0 \\ 0 & \delta\omega_0(z) \end{pmatrix}$

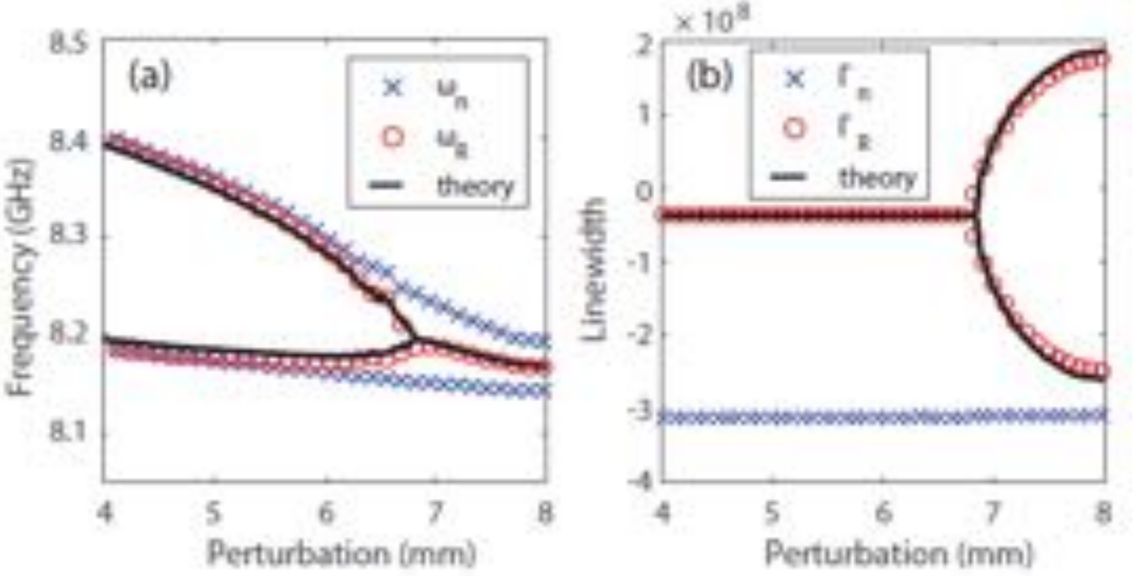
Left coupling vector (symmetric) $V_0 = \frac{\sqrt{\gamma}}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Right coupling vector (anti-symmetric) $V_1 = \frac{\sqrt{\gamma}}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Reflectionless operator $H_{RL} = \omega_0(z)\mathbb{I} + \frac{1}{2}\begin{pmatrix} -\delta\omega_0(z) & i\gamma \\ i\gamma & \delta\omega_0(z) \end{pmatrix}$

Eigenvalues $\tilde{\omega}_{R\pm} = \omega_0(z) \pm \frac{1}{2}\sqrt{\delta\omega_0(z)^2 - \gamma^2}$

Exceptional Point $\boldsymbol{\delta\omega_0(z_{EP}) = \gamma}$

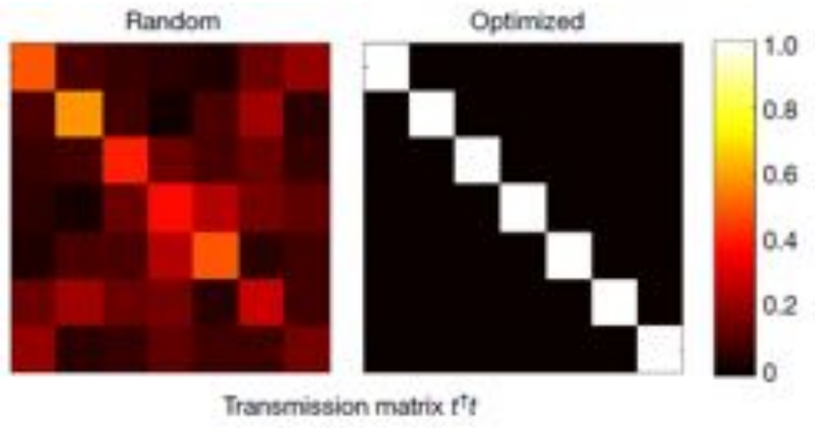


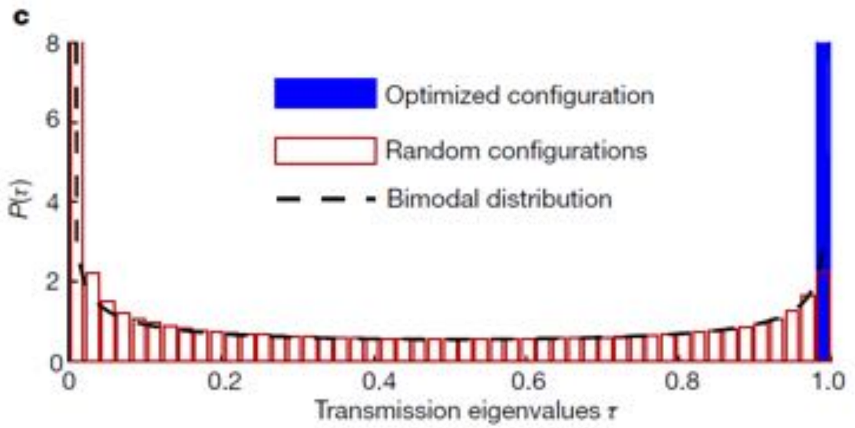
Simulation

Random

Fully transmission

Distribution of transmission eigenvalues





Transmission $T = 0.999$
Reflexion $R = 0.001$