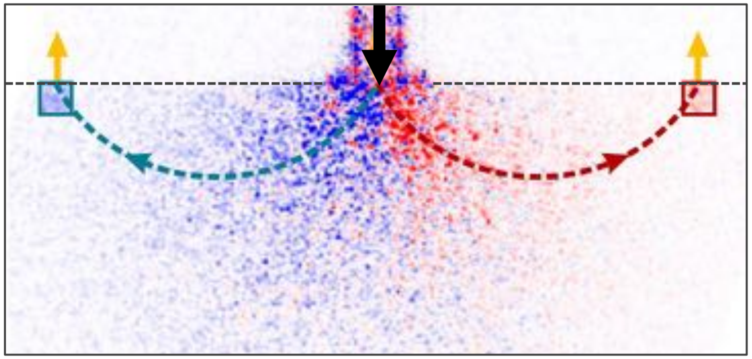
GDR Complexe Workshop – December 2022

In-depth interaction with opaque scattering media

ESPCI PARIS

Arthur Goetschy

Institut Langevin ONDES ET IMAGES



Collaborators

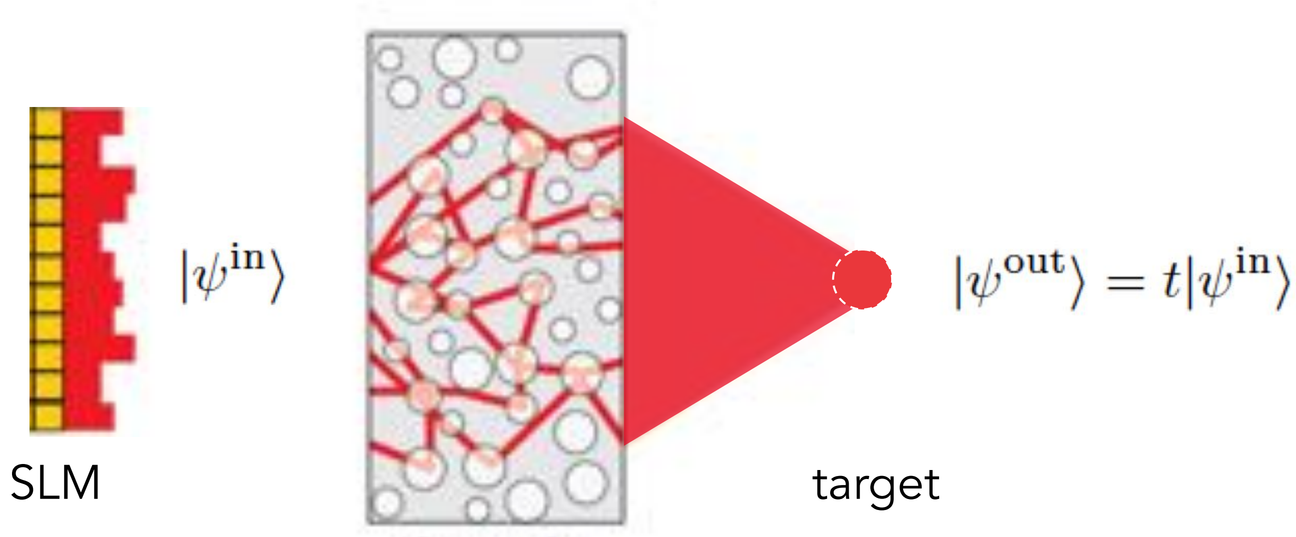
Yale: H. Cao's group

Missouri: A. Yamilov

Ankara: H. Yilmaz

Los Angeles: H.Hsu

Context: wavefront-shaping in complex systems



Mosk *et al.*, Nature Photon. (2012)

Objective Maximize $I = \langle\psi^{\text{out}}|\psi^{\text{out}}\rangle = \langle\psi^{\text{in}}|t^\dagger t|\psi^{\text{in}}\rangle$

Similar problems dwell-time, absorption, energy in embedded target, information, remission, etc

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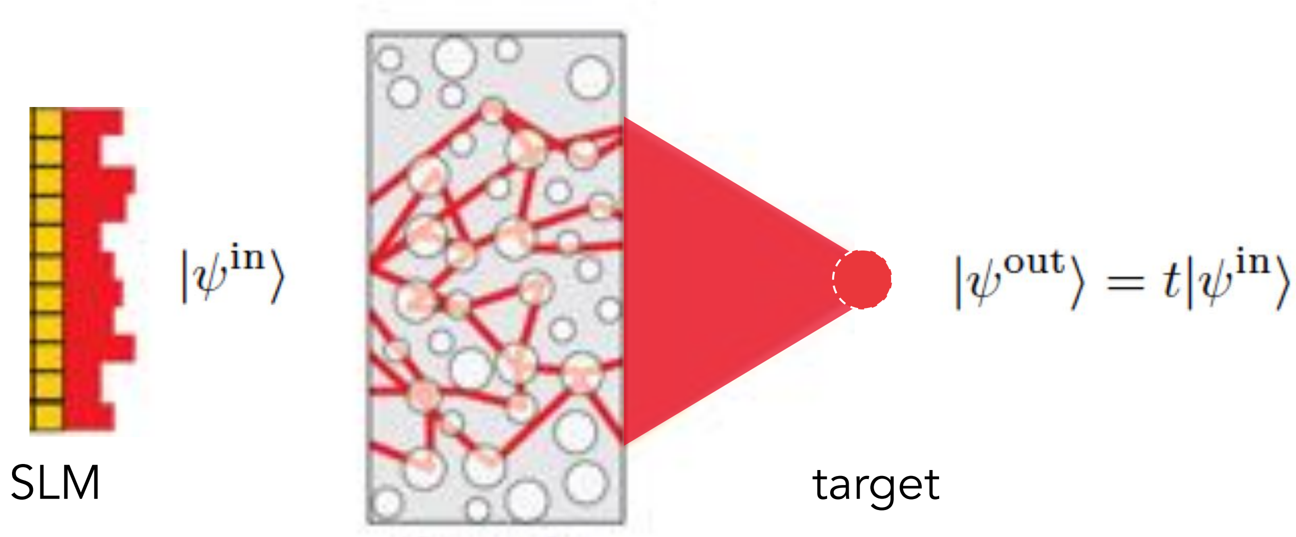

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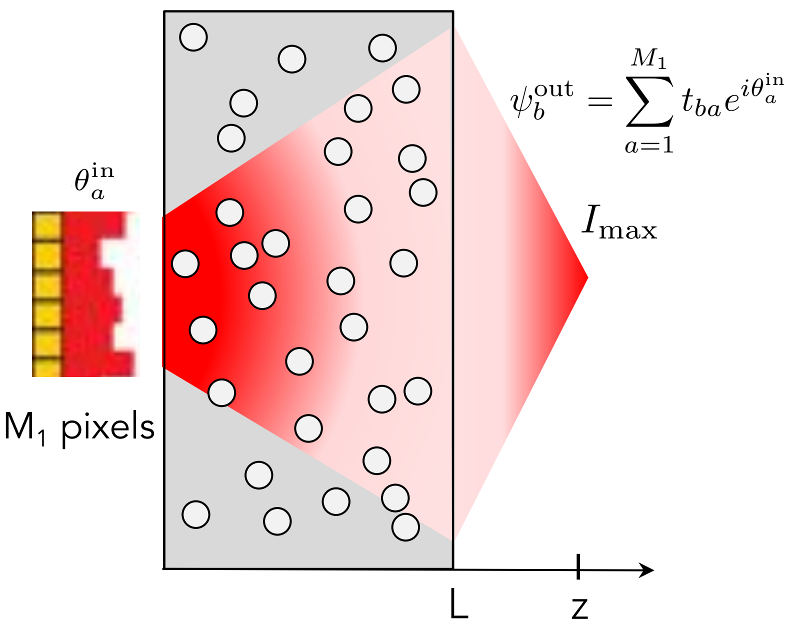
H. Cao, A. Yamilov, H. Yilmaz, H. Hsu

Naive picture

Focusing outside



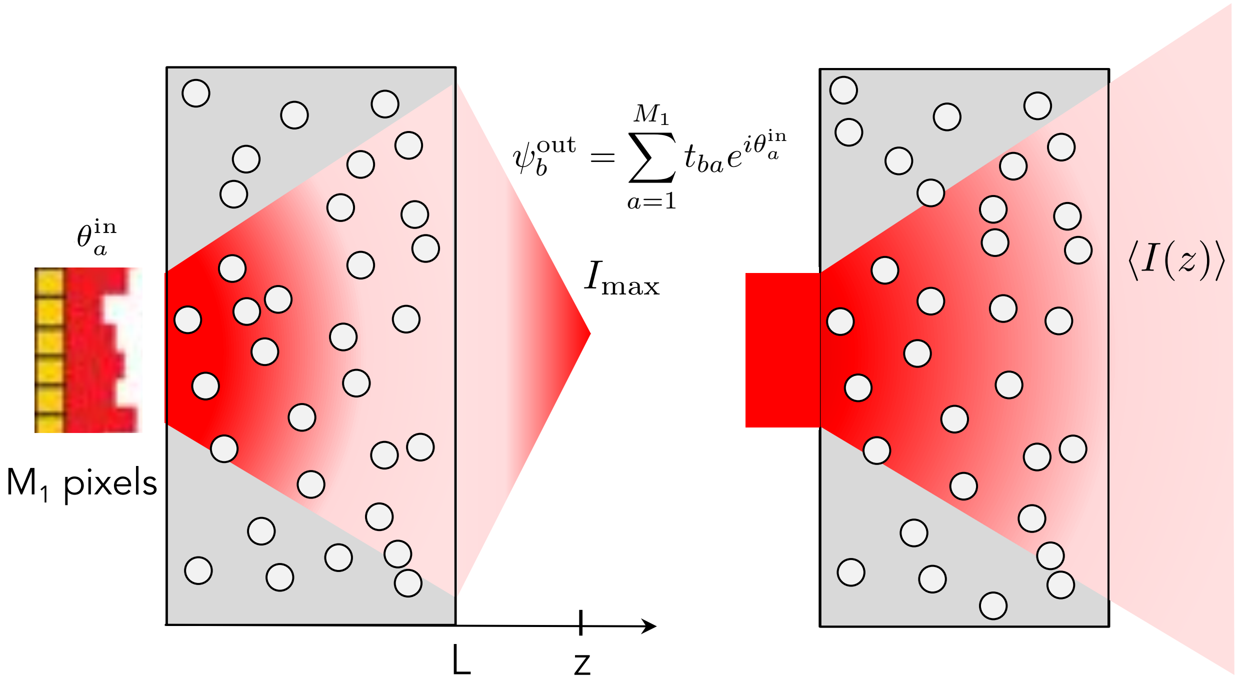
$$I_{\max} \sim M_1 \left\langle I(z) \right\rangle$$

Part I Delivering energy in deep targets

Naive picture

Focusing outside

Diffusion



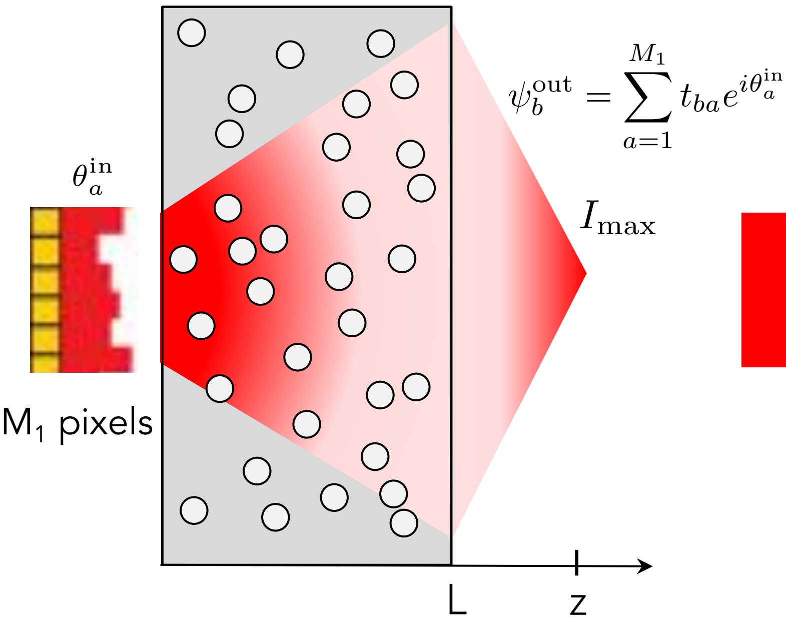
$$I_{\text{max}} \sim M_1 \langle I(z) \rangle \sim M_1 \frac{\ell}{L}$$

$$\langle I(z) \rangle \begin{cases} \sim \ell / L & \text{outside} \\ \text{indep of } \ell & \text{inside} \end{cases}$$

Part I Delivering energy in deep targets

Naive picture

Focusing outside



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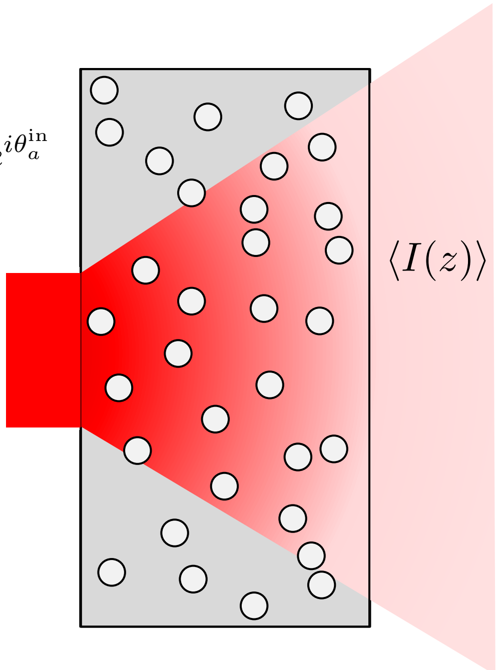

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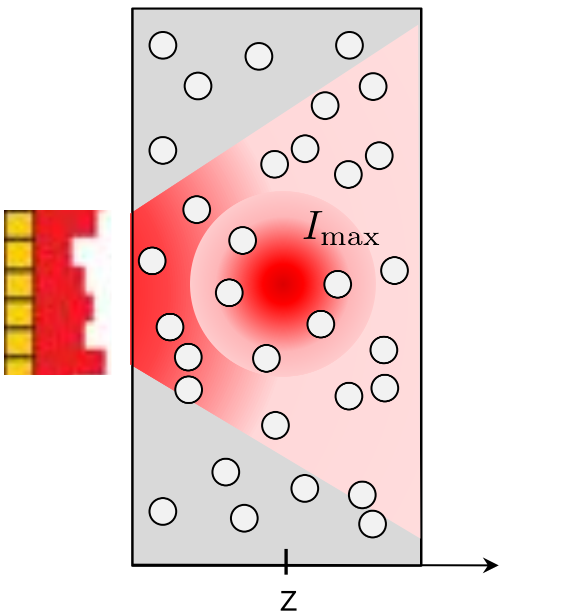


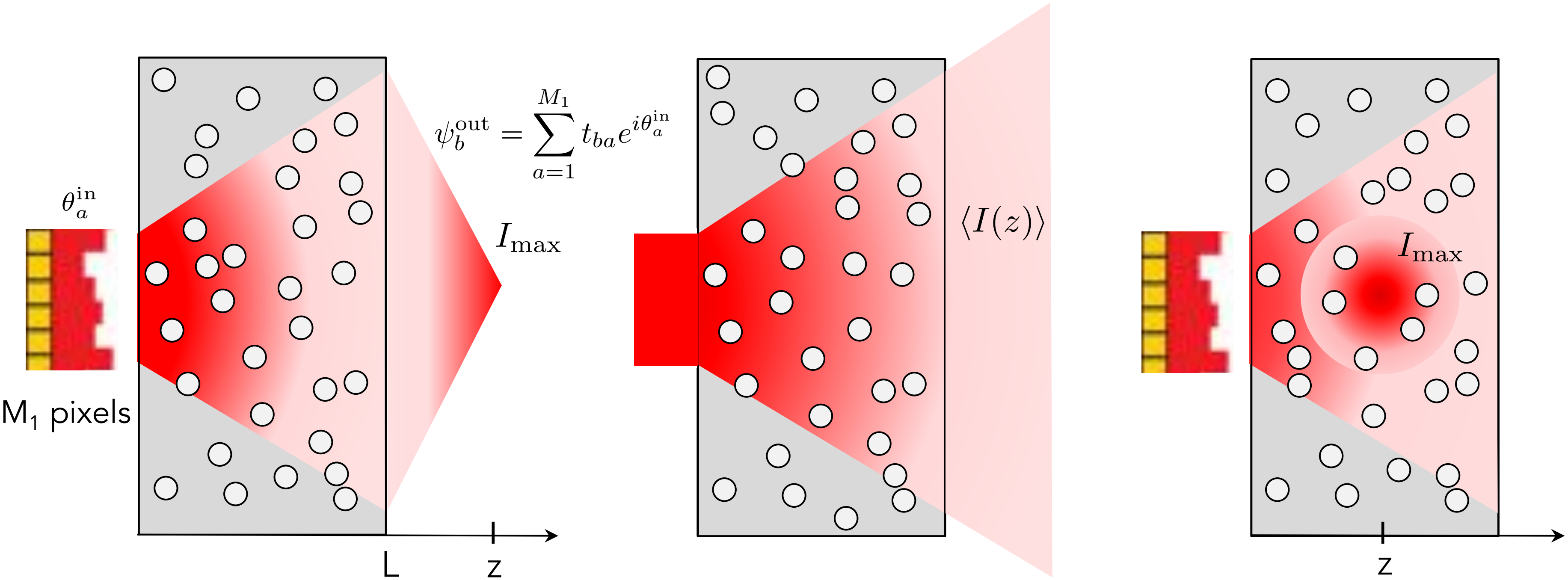
$I_{\max} \sim M_1 \langle I(z) \rangle$?

- indep of ℓ
- decreases with depth z

Delivering energy in deep targets

Naive picture

Focusing outside | Diffusion | Focusing inside



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$I_{\text{max}} \sim M_1 \langle I(z) \rangle$?

- indep of ℓ
- decreases with depth z

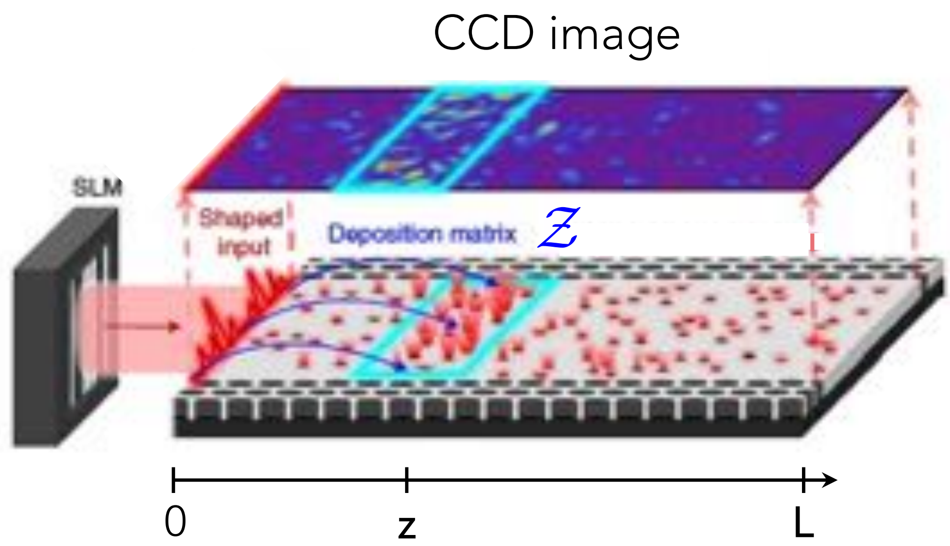
In the following

I_{max}
- **increases** with depth z
- $\sim L/\ell$

Part I Delivering energy in deep targets

Deposition matrix

$$I = \int_{\mathcal{V}} \mathrm{d}\mathbf{r}\, |\, \psi(\mathbf{r}, \omega)|^2$$
$$= \langle \psi^{\mathrm{in}} | \mathcal{Z}^\dagger \mathcal{Z} | \psi^{\mathrm{in}} \rangle$$



Bender, Yamilov, Goetschy, Yilmaz, Hsu, Cao, Nature Phys. (2022)

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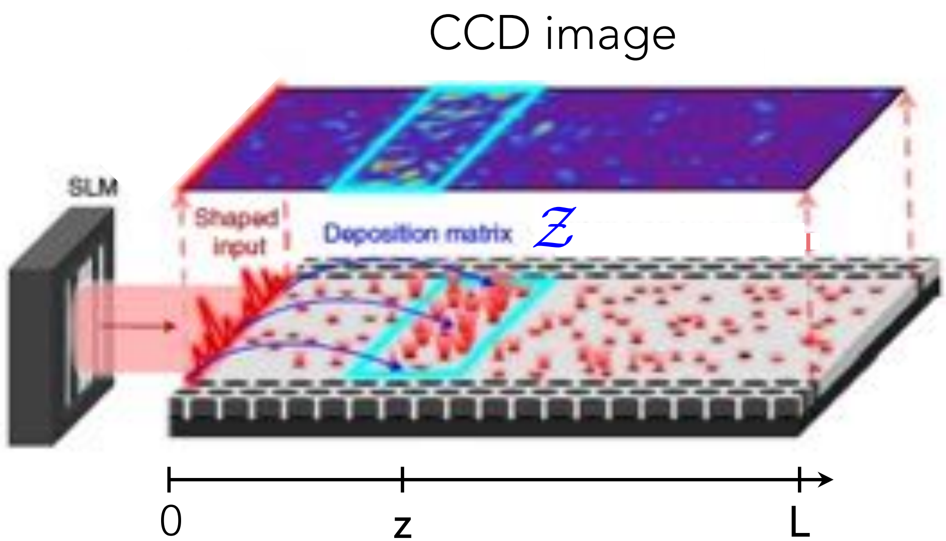
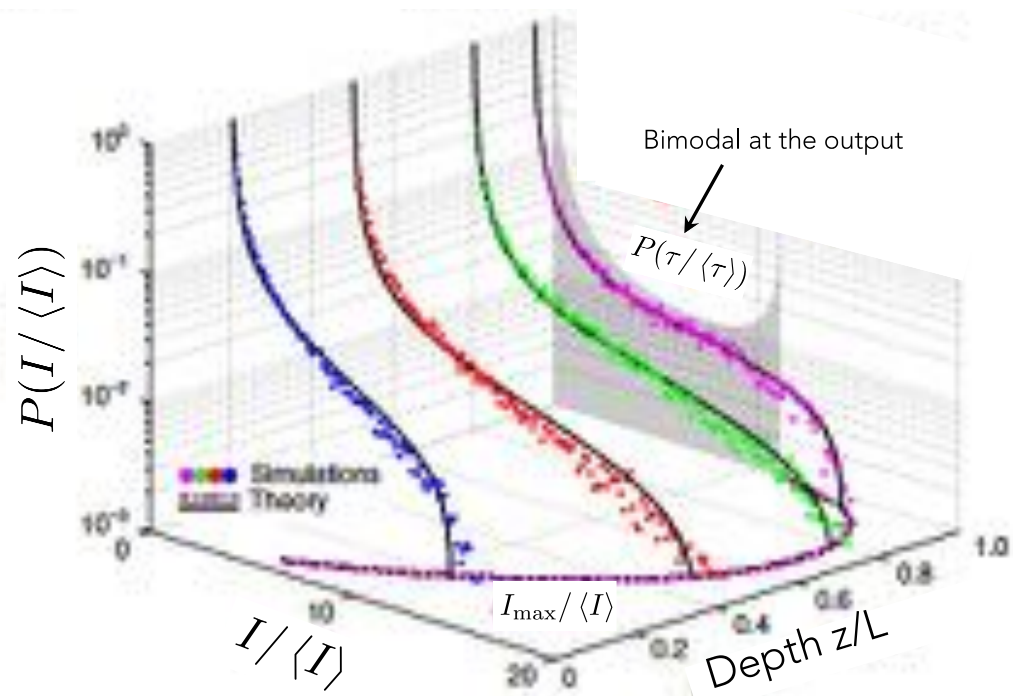
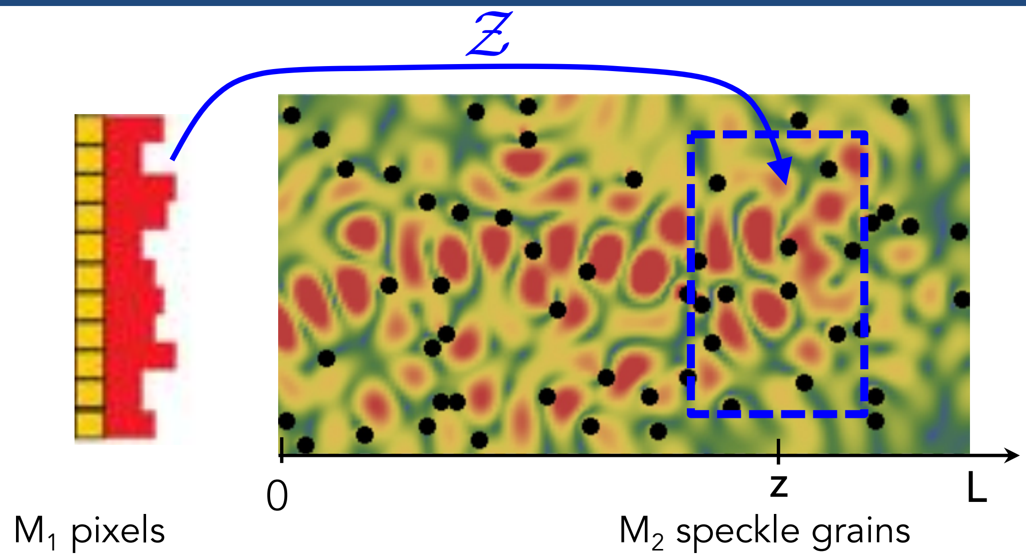


Eigenvalue distribution at different depths

$$I = \text{ eigenvalue of } \mathcal{Z}^\dagger \mathcal{Z}$$



Delivering energy in deep targets



Maximum energy enhancement

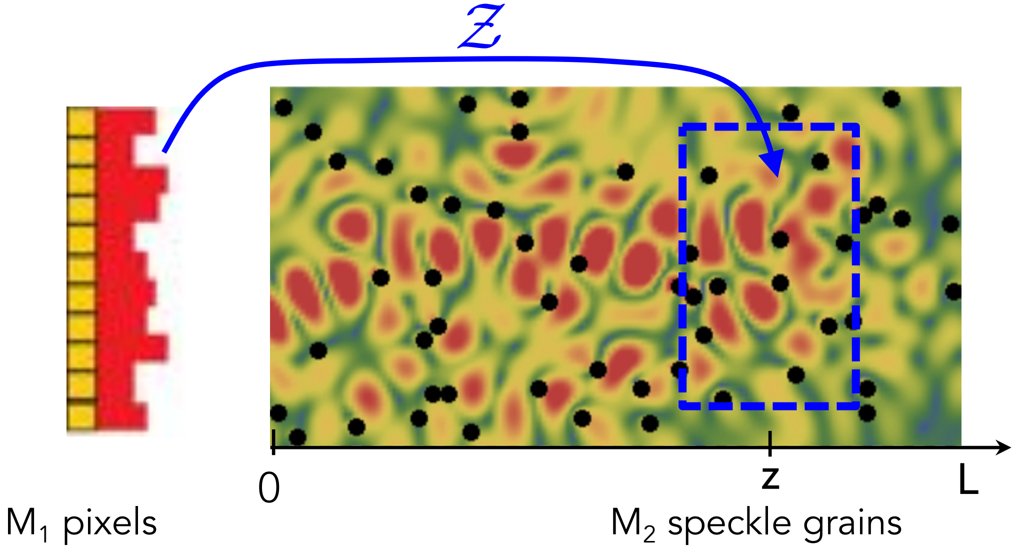
Indep speckle grains

$\mathcal{Z} \sim$ M_2 x M_1 Gaussian random matrix

$$I_{\max} = \max \text{ eigenvalue of } \mathcal{Z}^\dagger \mathcal{Z}$$

Delivering energy in deep targets

Maximum energy enhancement



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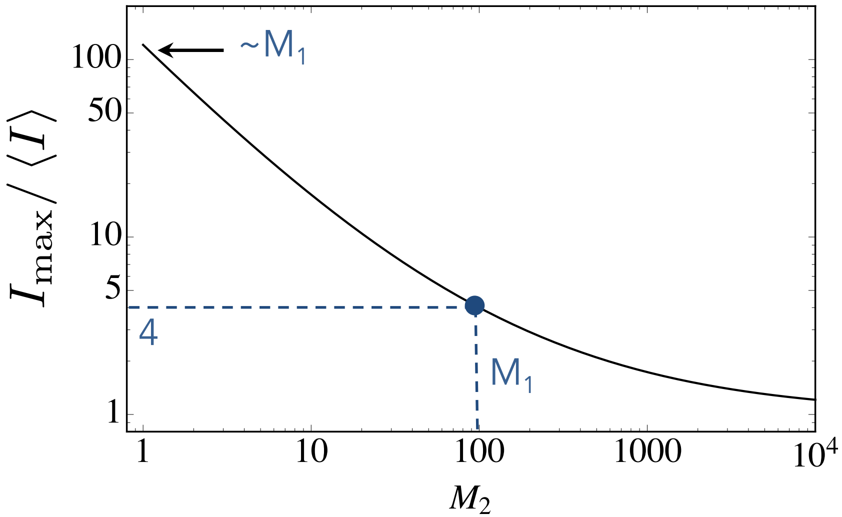
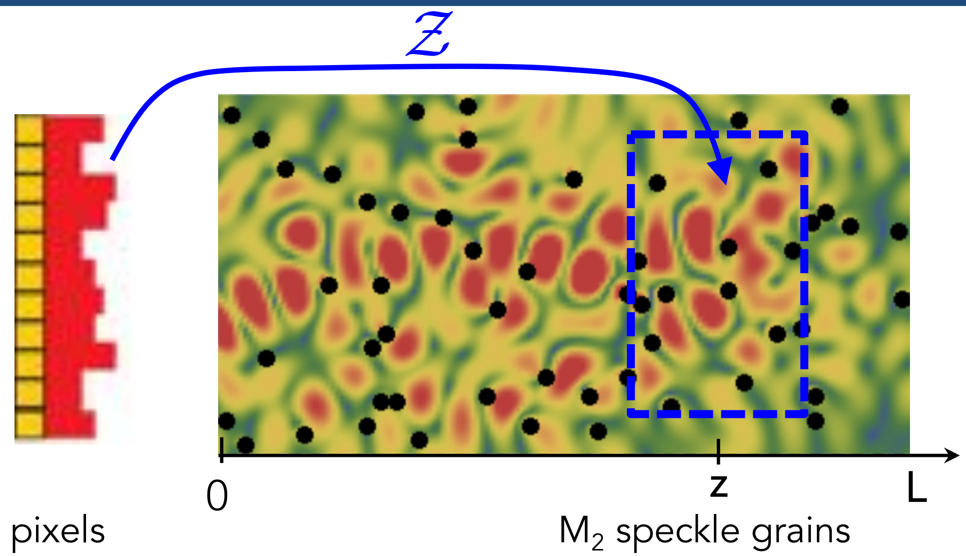


$$\frac{I_{\max}}{\langle I \rangle} = \left(1 + \sqrt{\frac{M_1}{M_2}}\right)^2 \simeq \frac{M_1}{M_2}$$

Delivering energy in deep targets

Maximum energy enhancement



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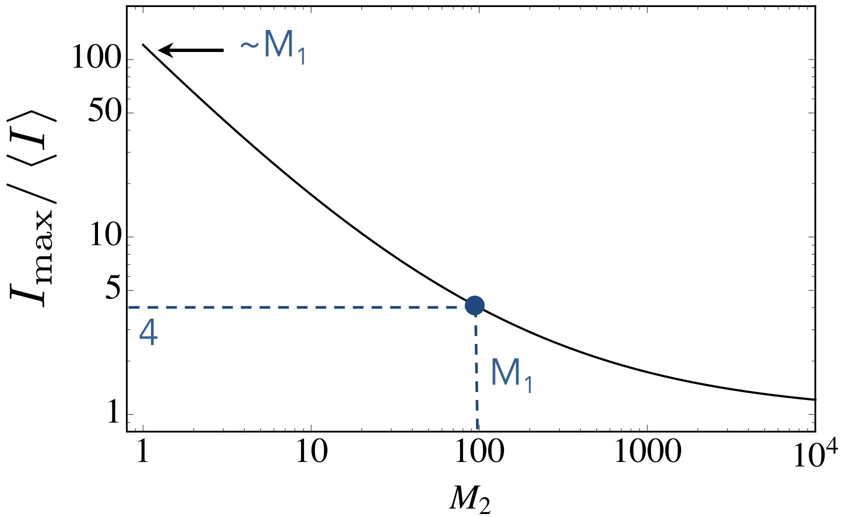


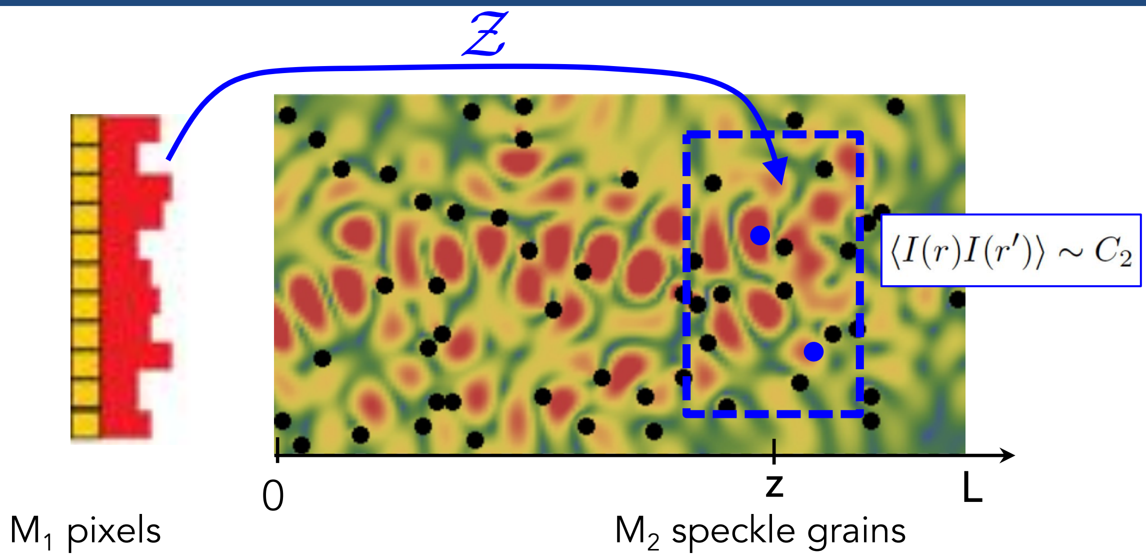
$$\frac{I_{\max}}{\langle I \rangle} = \left(1 + \sqrt{\frac{M_1}{M_2}}\right)^2 \simeq \frac{M_1}{M_2}$$

Total intensity indep of target size:

$$\langle I \rangle \simeq M_2 \langle I_{\text{speckle}} \rangle \implies I_{\max} \simeq M_1 \langle I_{\text{speckle}} \rangle$$

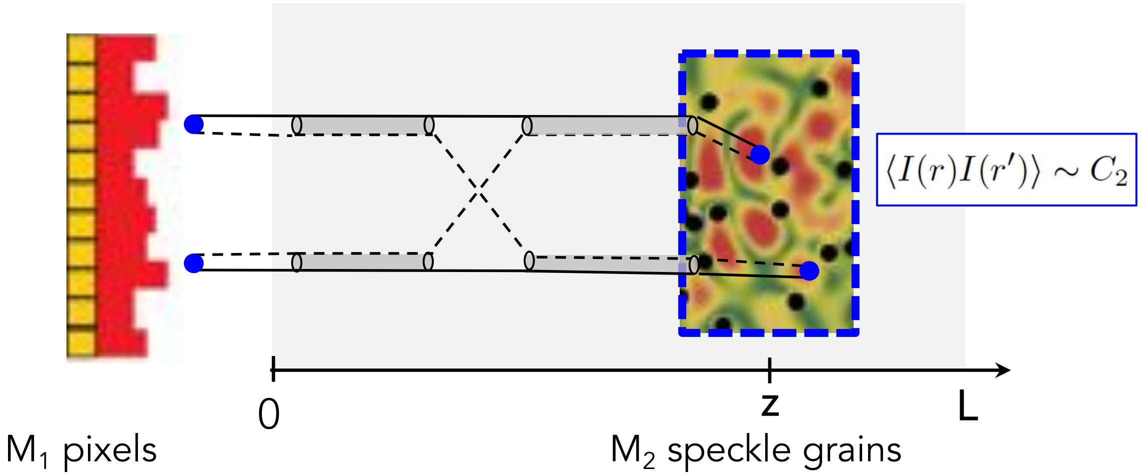
Delivering energy in deep targets

Maximum energy enhancement



Include long-range correlations $\mathcal{Z} \sim$ non Gaussian…

Maximum energy enhancement



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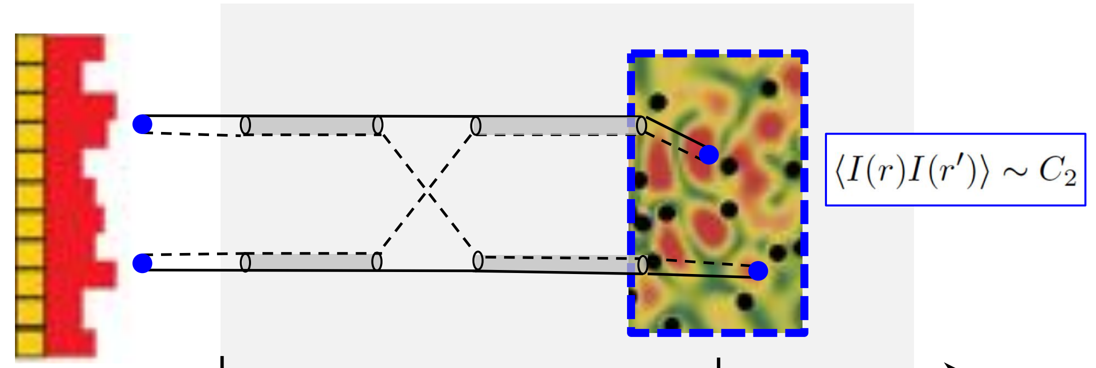


Include long-range correlations $\mathcal{Z} \sim$ non Gaussian…

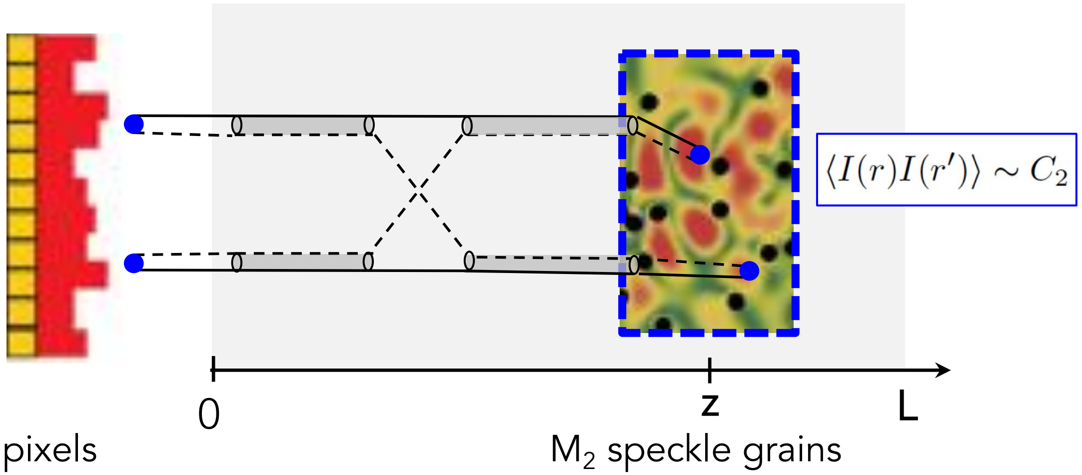
Example: phase conjugation

$$|\psi^{\rm in}\rangle = \mathcal{Z}^\dagger|\psi^{\rm target}\rangle \quad \Longrightarrow \quad |\psi^{\rm out}\rangle = \mathcal{Z}\mathcal{Z}^\dagger|\psi^{\rm target}\rangle$$

$$\Longrightarrow \quad I^{\rm out} = \langle\psi^{\rm out}|\psi^{\rm out}\rangle = \langle\psi^{\rm target}|\mathcal{Z}\mathcal{Z}^\dagger\mathcal{Z}\mathcal{Z}^\dagger|\psi^{\rm target}\rangle$$

4-field correlation

Maximum energy enhancement

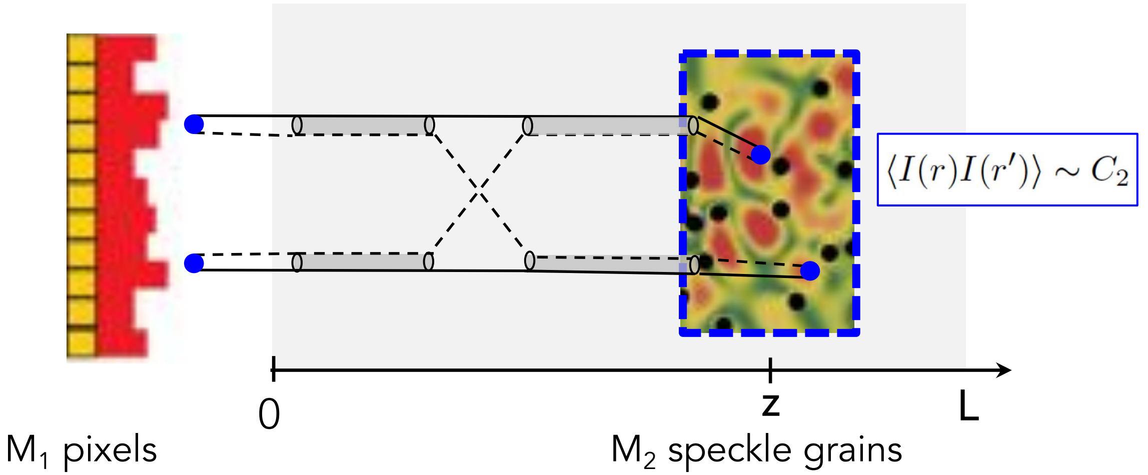


Include long-range correlations

$$\frac{I_{\max}}{\langle I\rangle} \simeq \frac{M_1}{M_2} + M_1 C_2$$

Extra contribution ☺

Maximum energy enhancement



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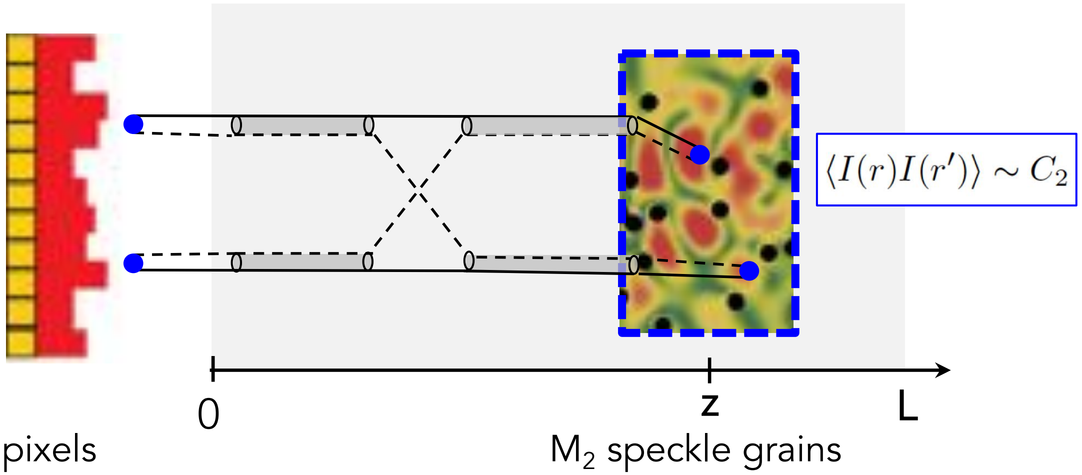
$$\equiv \frac{M_1}{M_2^{\mathrm{eff}}} \quad \text{with} \quad \frac{1}{M_2^{\mathrm{eff}}} = \frac{1}{M_2} + C_2 = \frac{1}{M_2} + \frac{1}{g}$$

Part I Delivering energy in deep targets

Maximum energy enhancement



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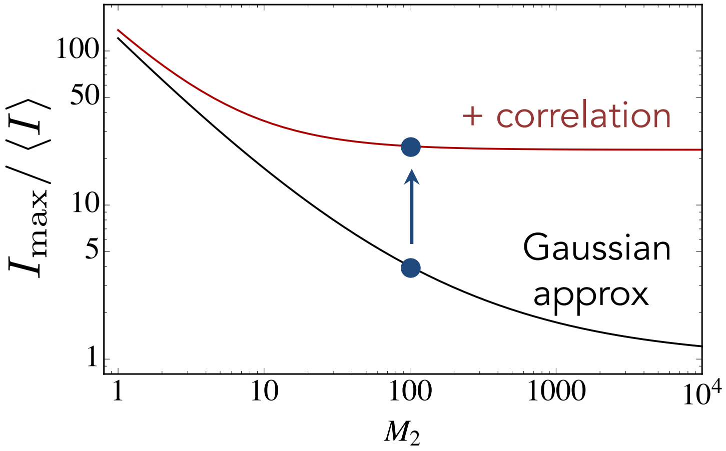
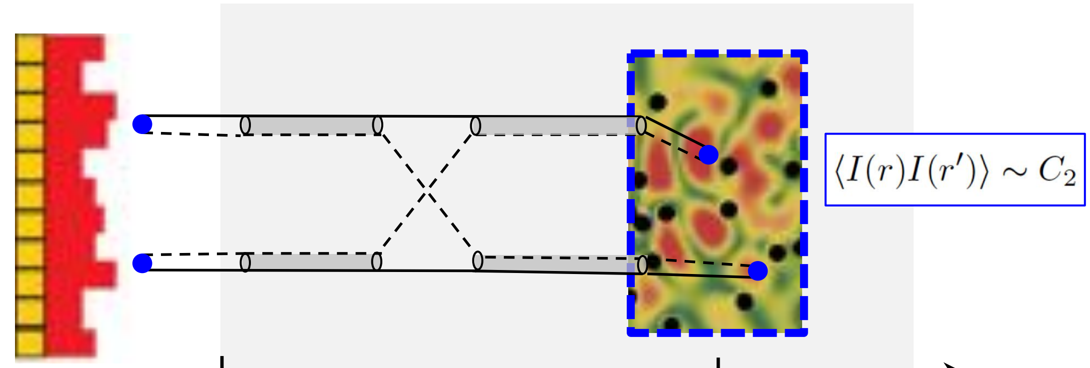


C_2 depends on:

- space dimension
- depth
- disorder strength
- illumination profile

Delivering energy in deep targets

Maximum energy enhancement



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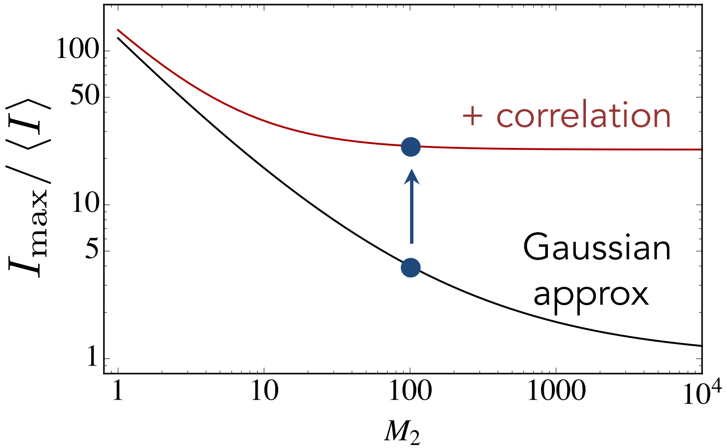
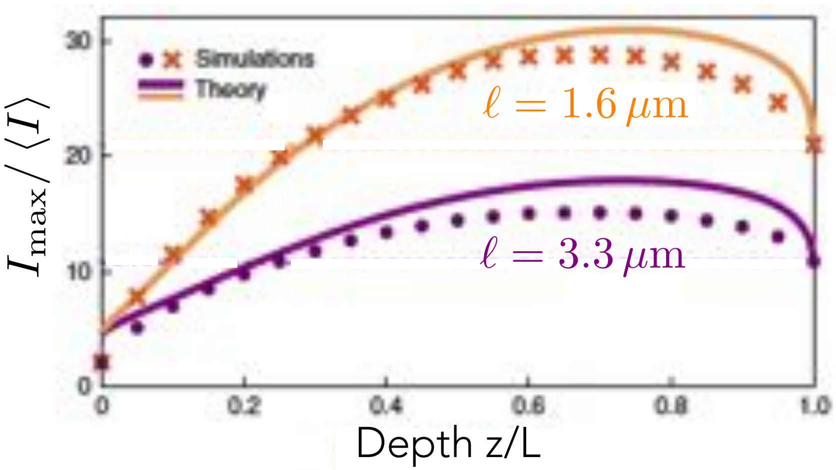
$$\frac{I_{\max}}{\langle I\rangle} \simeq \frac{M_1}{M_2} + M_1 C_2 \sim \frac{L}{\ell}$$

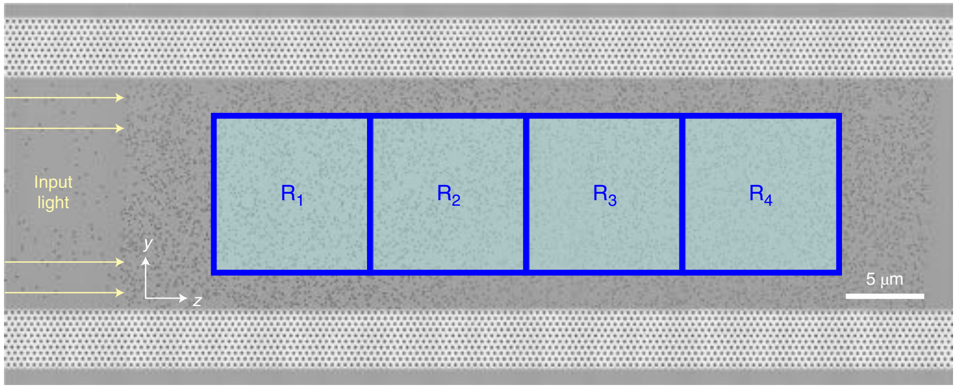
$$C_2(z) \simeq \frac{1}{M_1}\frac{L}{\ell}\left(\frac{3z}{L} - 2\frac{z^2}{L^2}\right)$$

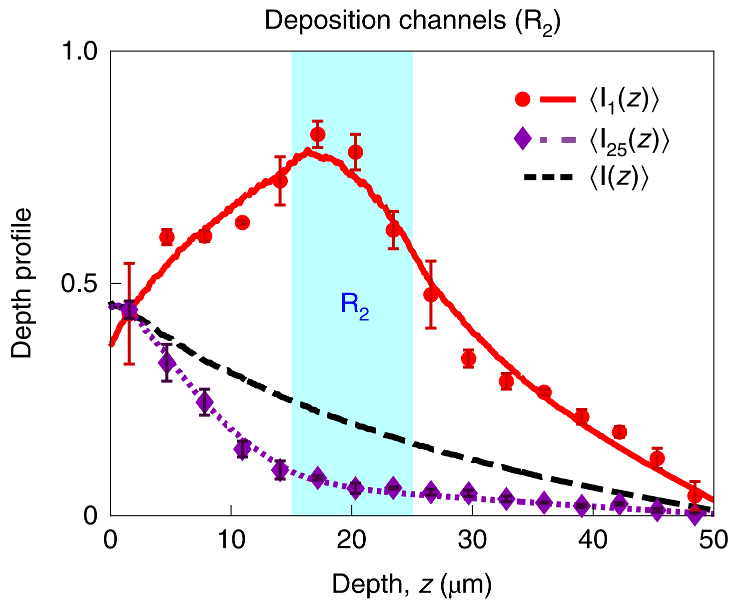


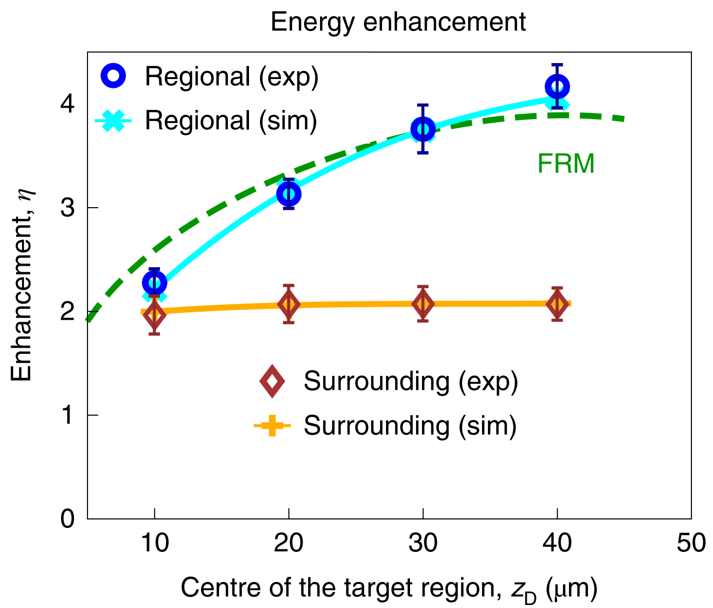


Part I Delivering energy in deep targets

In experimental conditions





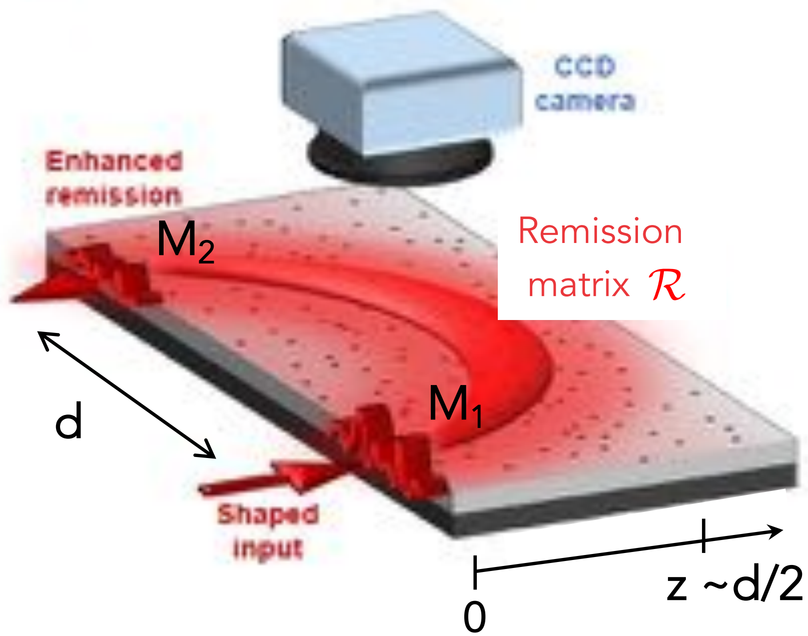


Part II Accessing in-depth information

Principle DOT config + WFS

DOT
- \+ Access deep $z \sim d/2$
- \- Weak signal $\langle \rho \rangle \sim M_2 \frac{k\ell}{(kd)^2} \ll 1$

WFS $\rho = \langle \psi^{\text{in}} | \mathcal{R}^\dagger \mathcal{R} | \psi^{\text{in}} \rangle$



Bender, Goetschy, Yilmaz, Hsu, Palacios, Yamilov, Cao, PNAS (2022)

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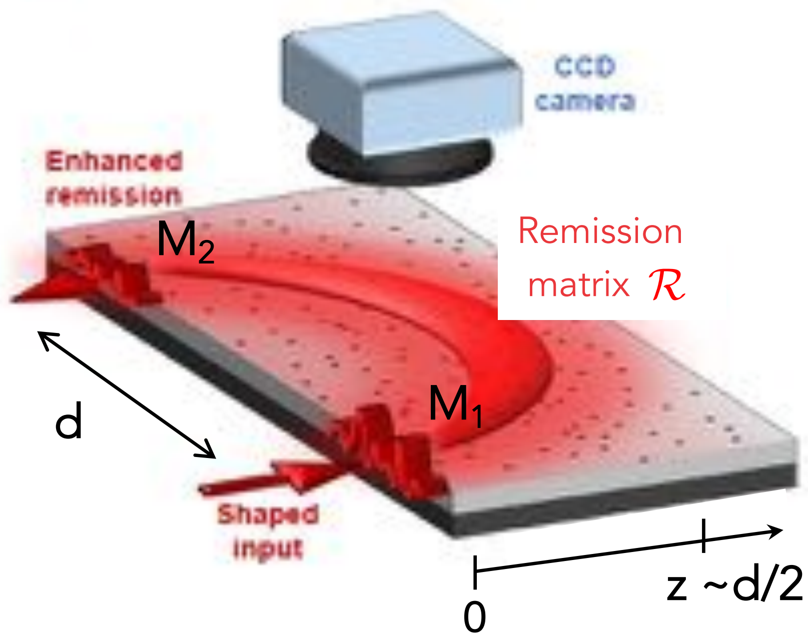
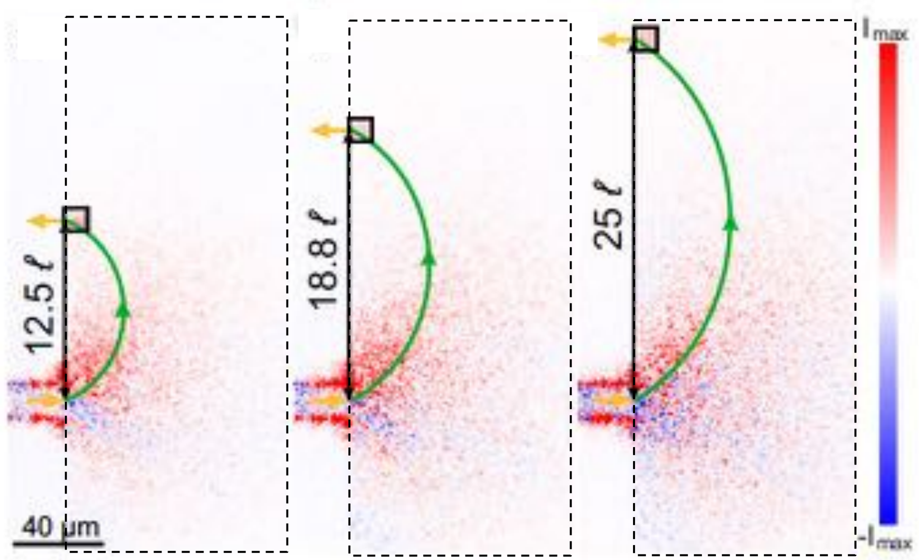
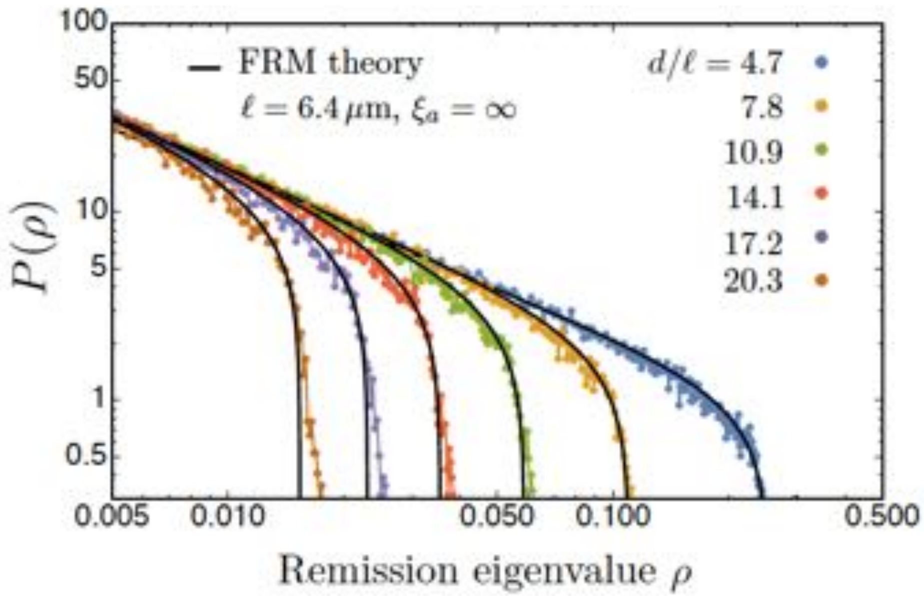




Eigenvalue distribution at different distances d

Example

$$\text{at } d \simeq 10\,\ell,\ \langle\rho\rangle = 1\% \ \text{ but } \ \rho_{\max} = 10\%$$



Accessing in-depth information

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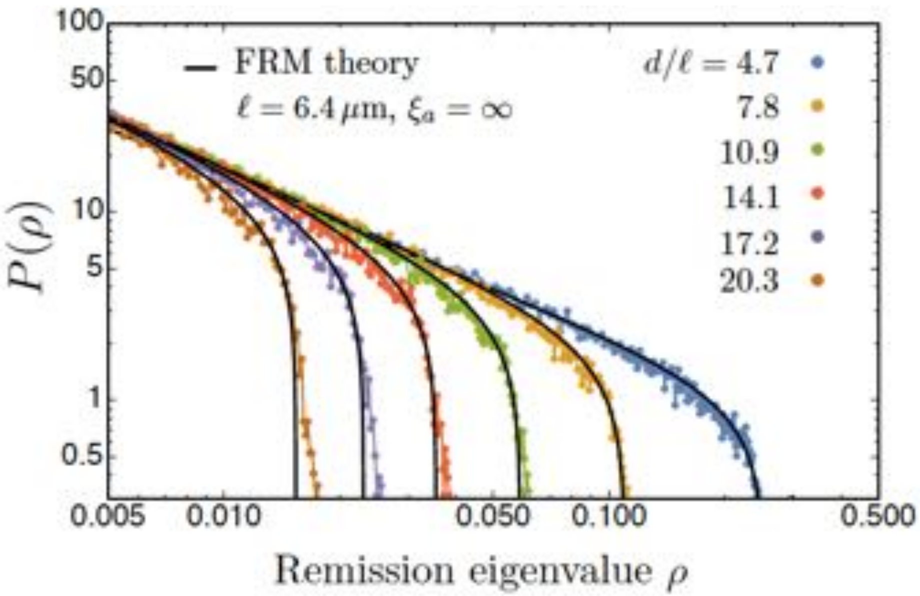
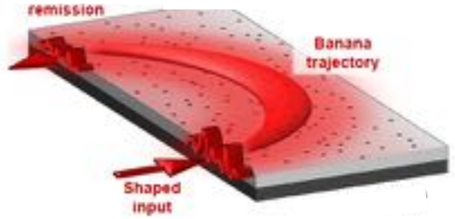


Example

at $d \simeq 10\,\ell,\ \langle\rho\rangle = 1\%$ but $\rho_{\max} = 10\%$

Maximum remission enhancement

$$\frac{\rho_{\max}}{\langle\rho\rangle} \simeq \frac{M_1}{M_2} + M_1 C_2$$



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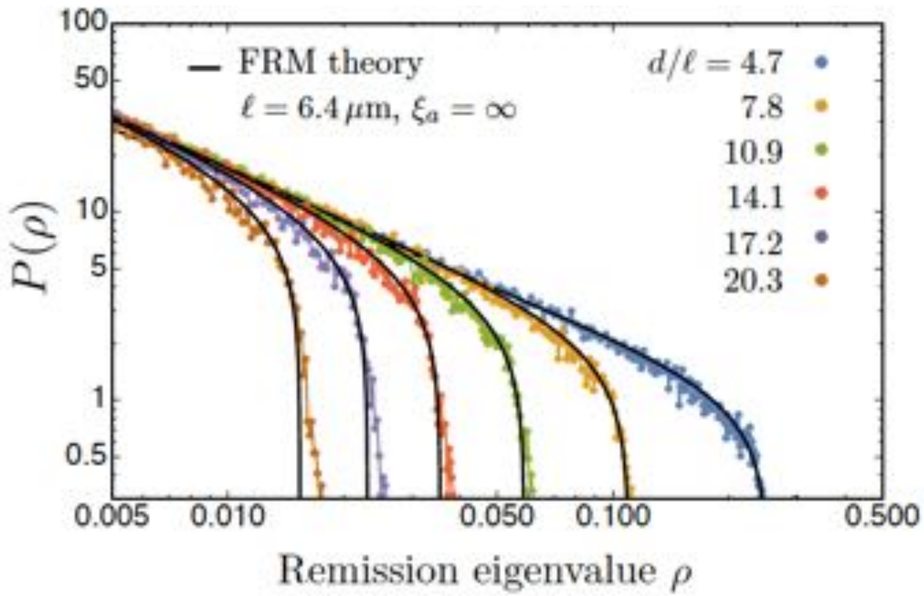

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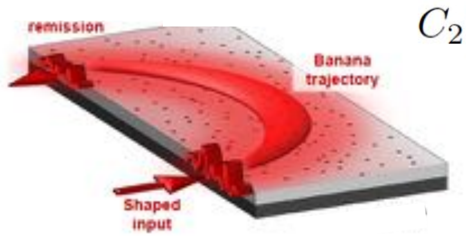
$$\frac{\rho_{\max}}{\langle\rho\rangle} \simeq \frac{M_1}{M_2} + M_1 C_2 \sim \frac{M_1}{k\ell}$$

$$C_2(d) \simeq \frac{\ln(kd/\pi M_1)}{k\ell}$$



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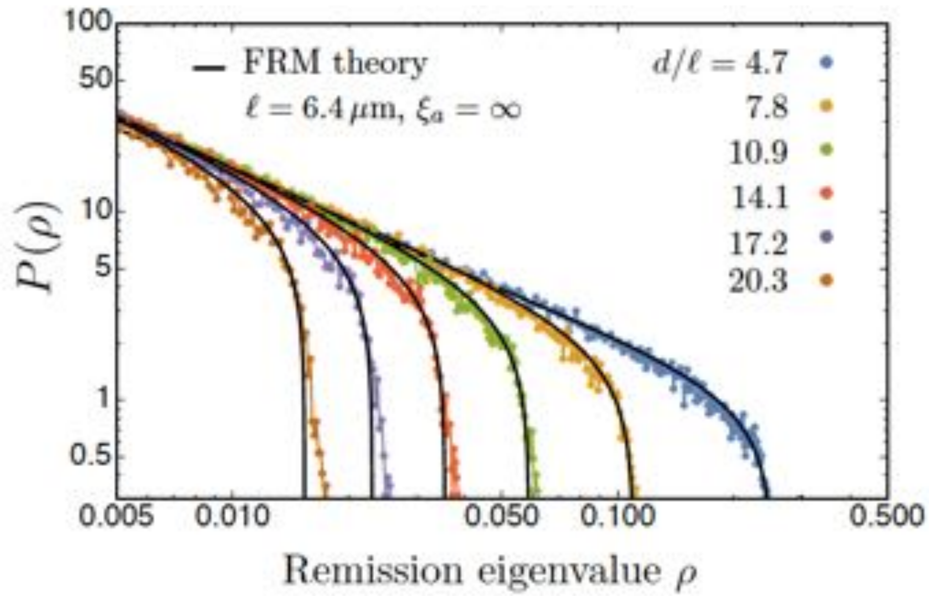

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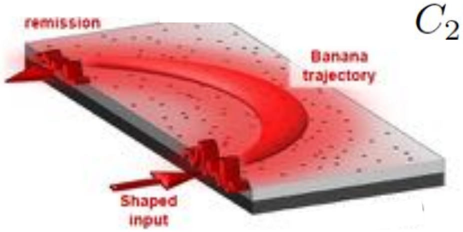
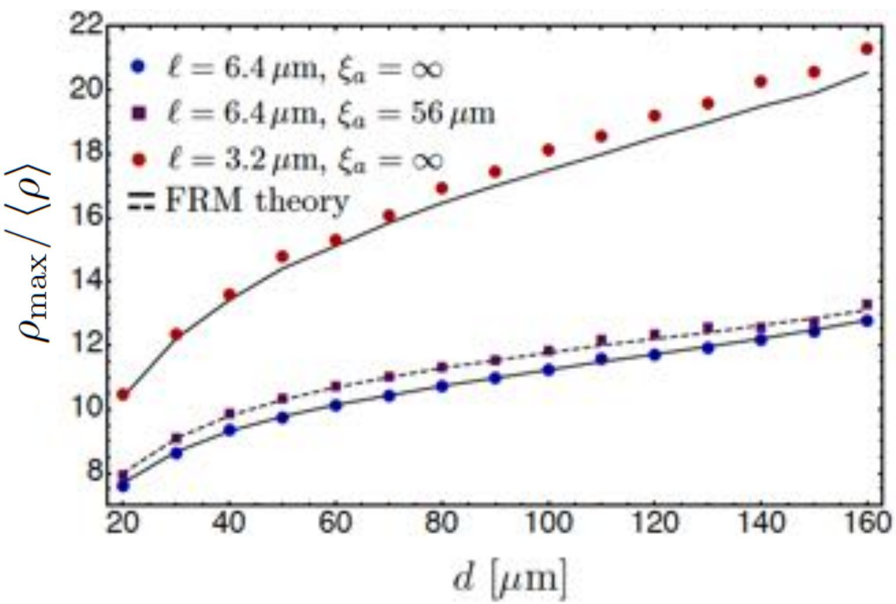




Part II

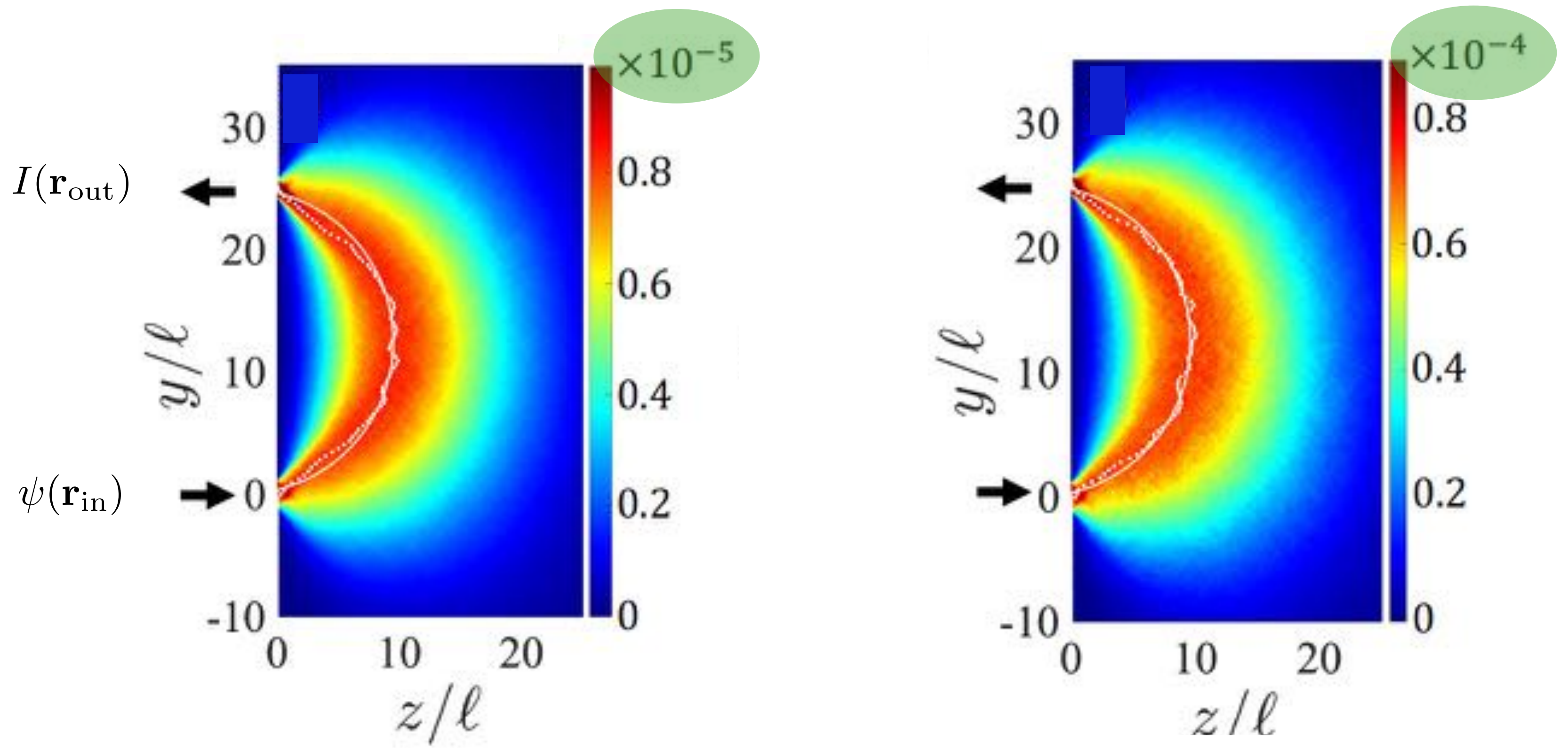
Consequence: higher sensitivity

Sensitivity map for the presence of a small absorber

$$S(\mathbf{r}) = \frac{dI(\mathbf{r}_{\text{out}})}{d\epsilon(\mathbf{r})} \sim \text{Re}\left[\psi(\mathbf{r}_{\text{out}})G_0(\mathbf{r}_{\text{out}},\mathbf{r})^*\psi(\mathbf{r})^*\right]$$

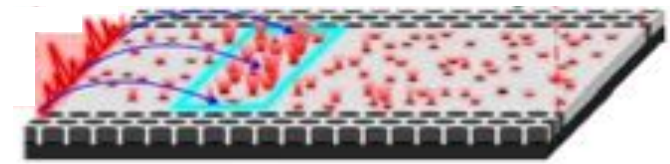
Standard DOT

Remission eigenchannel



Conclusions

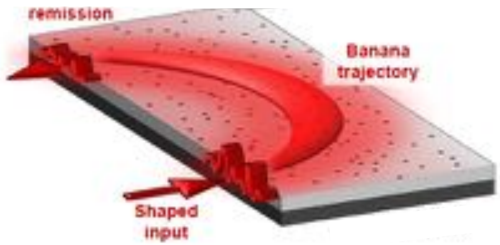
Deposition matrix $\mathcal{Z}$



- $I_{\max} \sim \frac{L}{\ell} \langle I \rangle \gg \langle I \rangle$
- Crucial role of long-range non-Gaussian correlations

Bender, Yamilov, Goetschy, Yilmaz, Hsu, Cao, Nature Phys. (2022)

Remission matrix $\mathcal{R}$



- Enhanced remission and sensitivity $\rho_{\max} \sim \frac{M_1}{k\ell} \langle \rho \rangle \gg \langle \rho \rangle$
- Applications: DOT, fNIRS

Bender, Goetschy, Yilmaz, Hsu, Palacios, Yamilov, Cao, PNAS (2022)

What next? Pulse excitation and time-gated matrix

Addendum

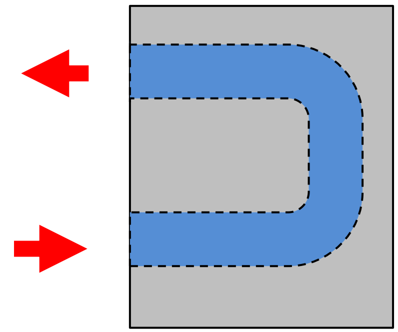
Long-range correlations and energy enhancement

	g (C_2=2/3g)	M_2 > g	$\frac{U_{\max}}{\langle U \rangle} \sim \frac{M_1}{g}$
Closed geometry (negligible transverse spreading)	$M_1 \frac{\ell}{L}$	$M_2 \gtrsim M_1 \frac{\ell}{L}$	$\frac{L}{\ell}$
Open geometry (transverse spreading) — 2D	$k\ell$	$M_2 \gtrsim k\ell$	$\frac{M_1}{k\ell}$
Open geometry (transverse spreading) — 3D	$\sqrt{M_1} k\ell$	$M_2 \gtrsim \sqrt{M_1} k\ell$	$\frac{\sqrt{M_1}}{k\ell}$

RMT model for the deposition matrix

Closed geometry

Eigenvalue distribution ~ bimodal



Open geometry

Incomplete channel control

↓

Filter the matrix with projectors: $\tilde{t} = P^{\text{out}}\, t\, P^{\text{in}} =$

$$\begin{pmatrix} t_{1,1} & t_{1,2} & t_{1,3} & t_{1,4} & t_{1,5} & t_{1,6} & t_{1,7} & t_{1,8} & t_{1,9} & t_{1,10} \\ t_{2,1} & t_{2,2} & t_{2,3} & t_{2,4} & t_{2,5} & t_{2,6} & t_{2,7} & t_{2,8} & t_{2,9} & t_{2,10} \\ t_{3,1} & t_{3,2} & t_{3,3} & t_{3,4} & t_{3,5} & t_{3,6} & t_{3,7} & t_{3,8} & t_{3,9} & t_{3,10} \\ t_{4,1} & t_{4,2} & t_{4,3} & t_{4,4} & t_{4,5} & t_{4,6} & t_{4,7} & t_{4,8} & t_{4,9} & t_{4,10} \\ t_{5,1} & t_{5,2} & t_{5,3} & t_{5,4} & t_{5,5} & t_{5,6} & t_{5,7} & t_{5,8} & t_{5,9} & t_{5,10} \\ t_{6,1} & t_{6,2} & t_{6,3} & t_{6,4} & t_{6,5} & t_{6,6} & t_{6,7} & t_{6,8} & t_{6,9} & t_{6,10} \\ t_{7,1} & t_{7,2} & t_{7,3} & t_{7,4} & t_{7,5} & t_{7,6} & t_{7,7} & t_{7,8} & t_{7,9} & t_{7,10} \\ t_{8,1} & t_{8,2} & t_{8,3} & t_{8,4} & t_{8,5} & t_{8,6} & t_{8,7} & t_{8,8} & t_{8,9} & t_{8,10} \\ t_{9,1} & t_{9,2} & t_{9,3} & t_{9,4} & t_{9,5} & t_{9,6} & t_{9,7} & t_{9,8} & t_{9,9} & t_{9,10} \\ t_{10,1} & t_{10,2} & t_{10,3} & t_{10,4} & t_{10,5} & t_{10,6} & t_{10,7} & t_{10,8} & t_{10,9} & t_{10,10} \end{pmatrix}$$

RMT + Feynman diagrams

Free probability →

$$p_{\tilde{t}^\dagger \tilde{t}}(\tilde{T}) = f\left(p_{t^\dagger t}(T), m^{\text{in}}, m^{\text{out}}\right)$$

fraction of controlled channels

Simple case ($m^{\text{out}} = 1$)

DOS ↓ Input beam ↓ Long-range correlations ↓

$$m^{\text{in}} = \frac{\text{Var}\tilde{T}}{\text{Var}T} = \frac{1}{1 - 3\bar{T}/2}\left[\int \mathrm{d}\mathbf{q}\, \rho(\mathbf{q}) \frac{I(\mathbf{q})I(-\mathbf{q})}{I(\mathbf{q}=0)} \frac{C_2(\mathbf{q})}{C_2(\mathbf{q}=0)} - \frac{3\bar{T}}{2}\right]$$