Scattering of topological edge waves in Kekule structures

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GdR Complexe: Annual workshop

Collaboration with:

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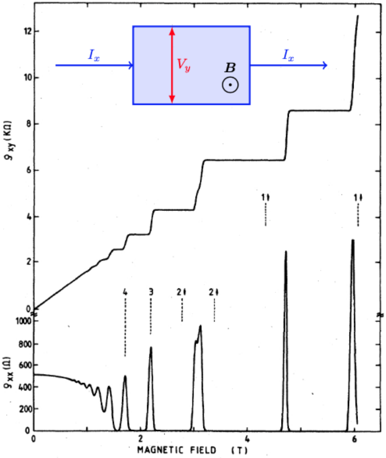
Laboratoire de Mécanique et d'Acoustique

Topological insulators: origins



First discovery:

- Quantum Hall effect (1986)

Later → broad field:

- In photonics (∼ 2008), in acoustics or elaticity (∼ 2015)

Interest:

Topological protection

- Properties **unchanged** through **continuous transformations**

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- 2D: Transport robust against defects or disorder
  $\rightarrow$ **Perfect transmission**

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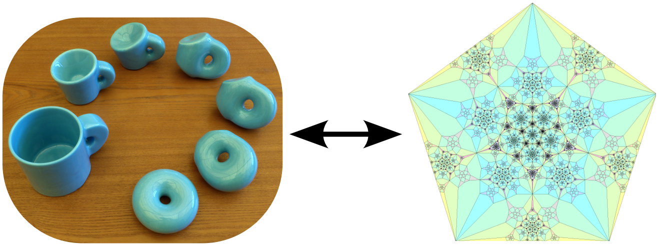

Topology   Symmetry

- **Careful:** topology is **not** enough
- **Need also symmetries** (broken or maintained)

Outline

Topological modes with classical waves:

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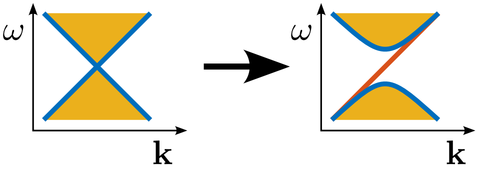
Quantum Hall effect:



Symmetry class: **Broken time reversal**

Topological invariant: Chern number

Topological modes with classical waves:

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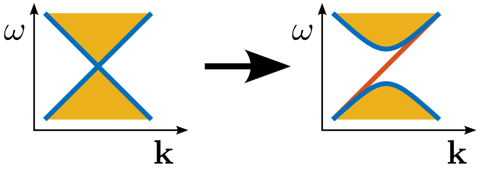

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  guaranteed by **unidirectional mode**

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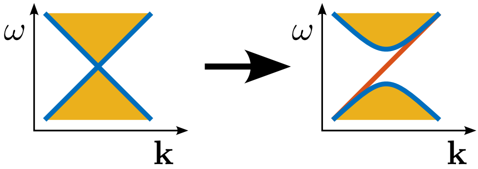


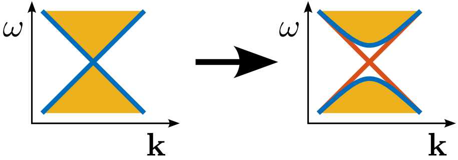
Symmetry class: **Broken time reversal**

Topological invariant: Chern number

- Backscattering immunity:
  guaranteed by **unidirectional mode**
- Drawbacks:
  - Active materials → energy to maintain TR breaking
  - Ex. in **acoustic**: Flow → dissipation/instabilities

Topological modes with classical waves:

Quantum Spin Hall effect:



Symmetry class: **Time reversal and spin**

Topological invariant: $\mathbb{Z}_2$ invariant

Topological modes with classical waves:

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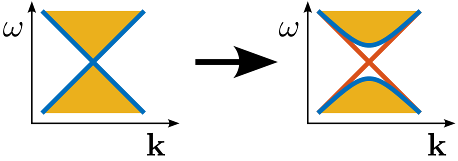

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$$\mathcal{T}^2 = -1 \qquad \text{(Time reversal operator)}$$

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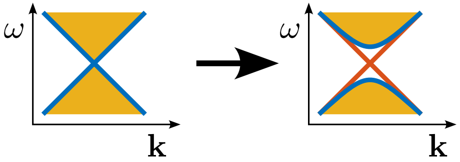

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- Drawbacks:
  - Classical waves **always** have $\mathcal{T}^2 = 1$ (Bosons)

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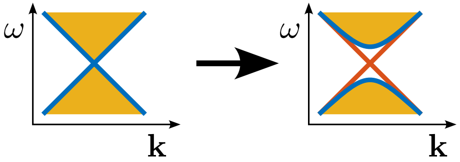

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Topological invariant: $\mathbb{Z}_2$ invariant

- Backscattering immunity: guaranteed by

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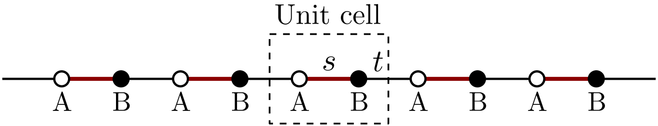
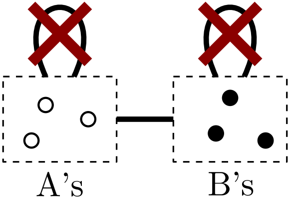
- Drawbacks:
  - Classical waves **always** have $\mathcal{T}^2 = 1$ (Bosons)
  - Can mimick a spin with $\mathcal{T}_{\text{eff}}^2 = -1$
    - Spatial symmetries (but broken by defects)
    - Valley degrees of freedom (but always approximate)

Outline

Chiral symmetry



Ex: SSH model has chiral symmetry

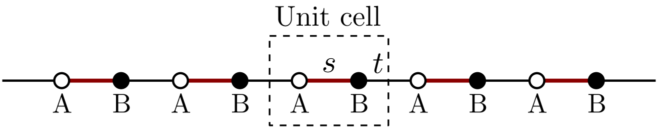


- Chiral ⇔ **sublattice** symmetry
- **Algebra:** unitary matrix Γ

$$\Gamma H + H\Gamma = 0$$

  with H Hamiltonian
- First consequence: spectrum symmetry $\varepsilon \to -\varepsilon$

Chiral symmetry



Ex: SSH model has chiral symmetry

- 1D chiral systems (class AIII)
- Topological invariant: winding number $\nu \in \mathbb{Z}$

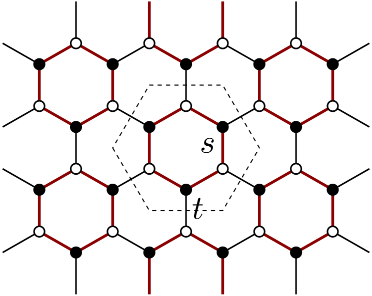
$$H(q) = \begin{pmatrix} 0 & Q \\ Q^\dagger & 0 \end{pmatrix} \quad \text{defines} \quad \nu = \frac{1}{2\pi} \int \mathrm{Tr}(Q^{-1} \partial_q Q)$$

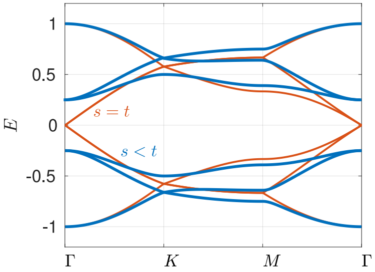
- $\nu \neq 0$: **Topologically protected edge mode at $\varepsilon = 0$**

In 2D, need something more:

Combination of symmetries

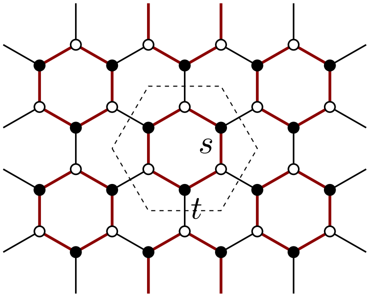
Kekule model

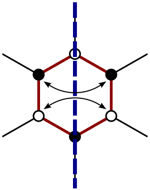




- Graphene with hopping modulation
- Intra-molecular hopping $s \neq$ extra-molecular hopping t
- Usually seen as a Quatum Spin Hall analogue

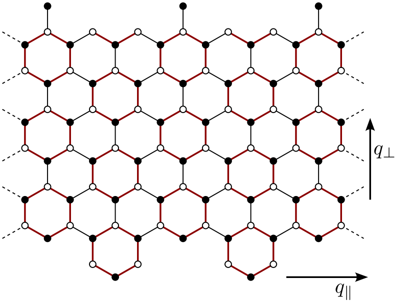
Kekule model

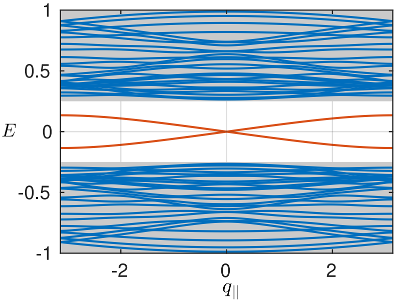




- Chiral symmetry
- Spatial symmetries: hexagonal group C_{6v}
- **In particular:** mirror symmetries along summits $M_{j=1..3}$

$$\Gamma M_j = M_j \Gamma$$



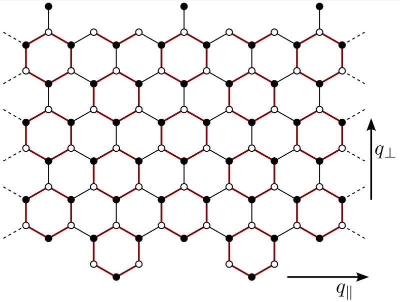


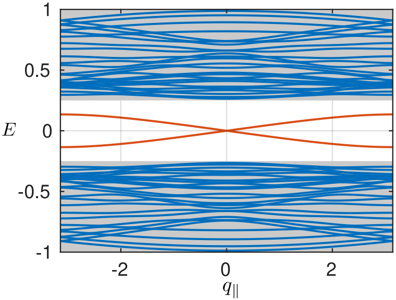
Idea of mirror winding number:

- $q = 0$ (high symmetry point):
  H splits in Symmetric and Anti-symmetric parts

$$H_S(q_\perp) = \begin{pmatrix} 0 & Q_S \\ Q_S^\dagger & 0 \end{pmatrix} \quad \text{and} \quad H_A(q_\perp) = \begin{pmatrix} 0 & Q_A \\ Q_A^\dagger & 0 \end{pmatrix}$$

- Each part is chiral $\rightarrow$ 1D winding number



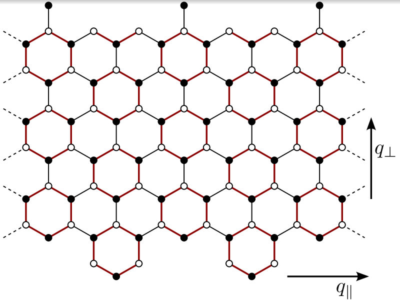


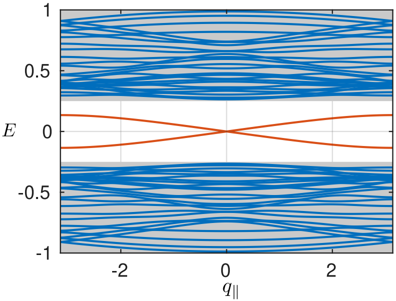
Idea of mirror winding number:

- We obtain a pair of mirror winding number:

$$\begin{aligned} \text{Topological:} \quad (n_S, n_A) &= (1, -1) \\ \text{Trivial:} \quad (n_S, n_A) &= (0, 0) \end{aligned}$$

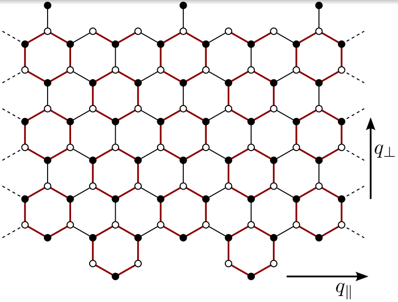
- 2 **zero energy modes** at $\varepsilon = 0$
  $\rightarrow$ **Edge waves with Dirac point**

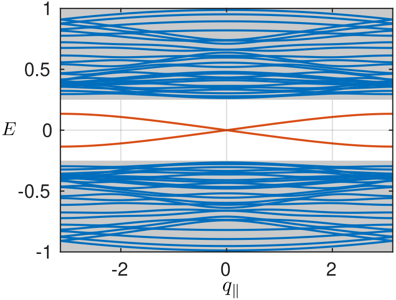




Careful:

- Choice of edge shape matters!
- Choose edge **mirror invariant**





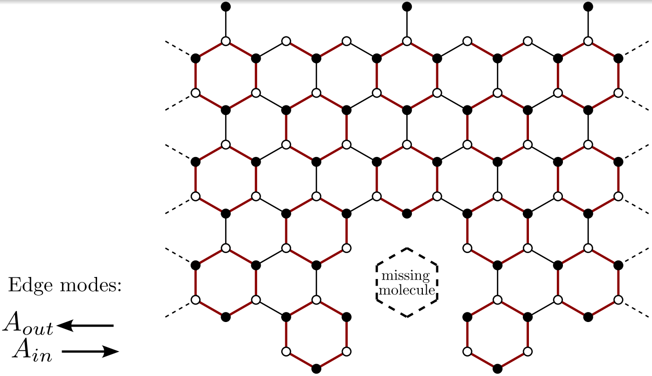
Careful:

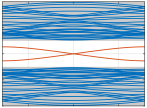
- Choice of edge shape matters!
- Choose edge **mirror invariant**
- Finite Ribbon:
  - Down: molecular zigzag → topological when $s < t$
  - Up: Partially bearded → topological when $s > t$

→ **exact crossing for finite width** (no mini-gap)

[AC, Zheng, Achilleos, Richoux, Theocharis, Pagneux *to appear*]

Outline





- Scattering problem:

$$\begin{pmatrix} A_{out} \\ B_{out} \end{pmatrix} = \begin{pmatrix} R(\varepsilon) & \tilde{T}(\varepsilon) \\ T(\varepsilon) & \tilde{R}(\varepsilon) \end{pmatrix} \begin{pmatrix} A_{in} \\ B_{in} \end{pmatrix}$$

- We consider **mirror symmetric** and **chiral** defects

[AC, Zheng, Achilleos, Richoux, Theocharis, Pagneux *to appear*]

Consequences of symmetries

- S-matrix:

$$S(\varepsilon) = \begin{pmatrix} R & \tilde{T} \\ T & \tilde{R} \end{pmatrix}$$

- **Unitarity** (energy conservation)

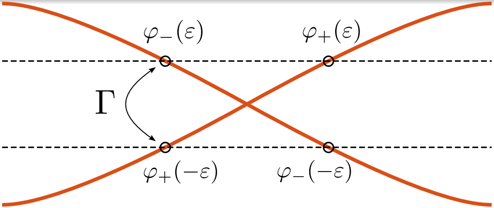
$$\begin{aligned} |R|^2 + |T|^2 &= 1, \\ R\tilde{T}^* + T\tilde{R}^* &= 0, \end{aligned}$$

- **Mirror symmetry**

$$R(\varepsilon) = \tilde{R}(\varepsilon) \quad \text{and} \quad T(\varepsilon) = \tilde{T}(\varepsilon)$$

- **Both:**

$$\mathrm{Re}(R(\varepsilon)T^*(\varepsilon)) = 0$$



Consequences of symmetries

- **Chiral symmetry:** relation between S and $S^{-1} = S^\dagger$
- Identity:

$$R(-\varepsilon) = R(\varepsilon)^* \quad \text{and} \quad T(-\varepsilon) = T(\varepsilon)^*$$

- At $\varepsilon = 0$: real-valued
- Combining all symmetries:

$$R(0)T(0) = 0$$

Conclusion:

- Combination of symmetries imposes:

$$R(0) = 0 \quad \text{or} \quad T(0) = 0$$

Perfect transmission ($R(0) = 0$) — Perfect reflection ($T(0) = 0$)

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- When is it **perfect transmission**?

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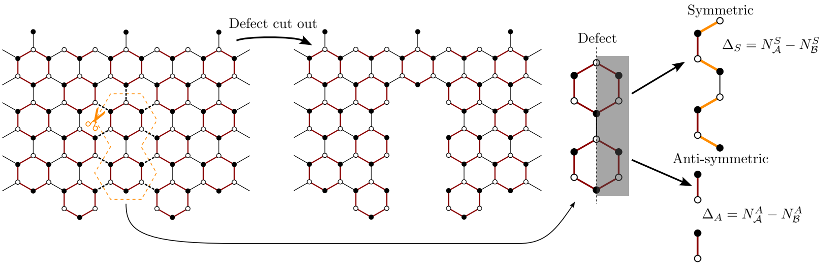
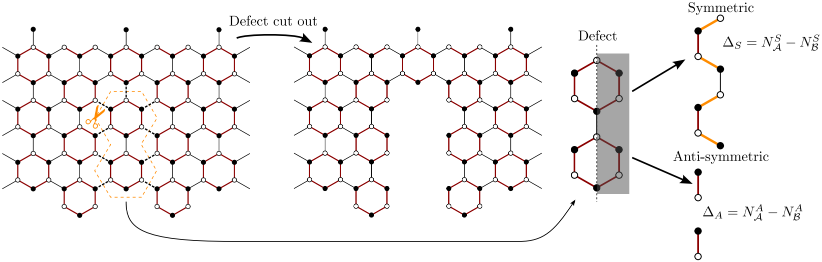
Perfect transmission ($R(0) = 0$) Perfect reflection ($T(0) = 0$)

- When is it **perfect transmission**?
- Extra condition: **Defect must preserve topology**



Topological indices of defects

- For mirror symmetric and chiral defect
- Define **topological indices** (Δ_S, Δ_A)
- (Similar to mirror winding numbers)



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- Define **topological indices** (Δ_S, Δ_A)
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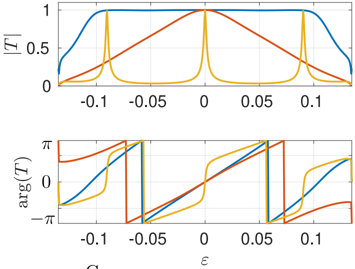
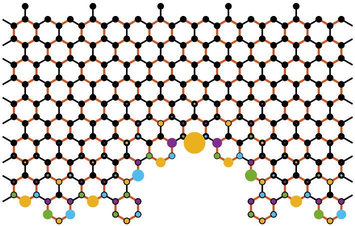
Nice result:

- If defect has $(0,0)$ (trivial) always have $T(0) = 1$
- It is true for **any** defect with missing whole molecules

Transmission across trivial defects:



Curve:



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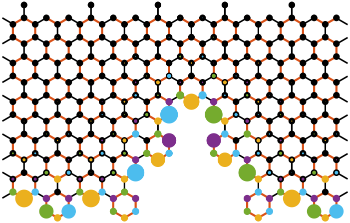

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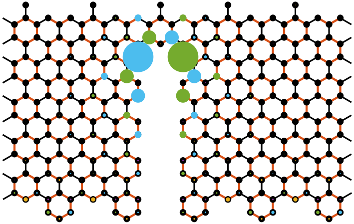
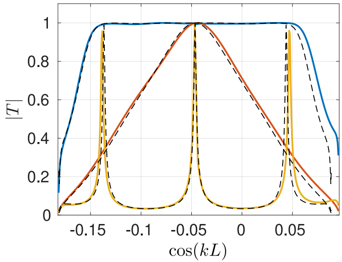
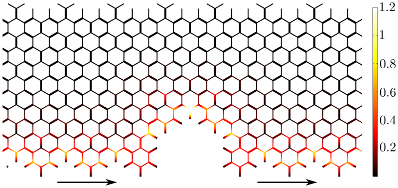
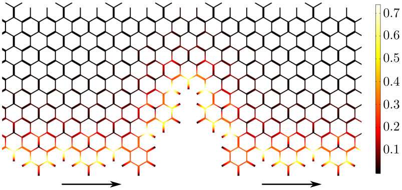


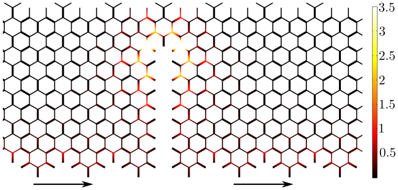
Acoustic realization: with networks of tubes

Dashed: lattice model
Color: 2D finite element simulations





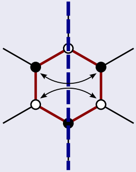




Conclusion

Topological edge modes in Kekule:

- Combination of symmetries
- Chiral and mirror
- Edge dependent



Kekule ribbon with defects:

- Symmetries guarantee

$$R(0) = 0 \quad \text{or} \quad T(0) = 0$$

- Topological index: trivial defect $\Rightarrow T(0) = 1$

[AC, Zheng, Achilleos, Richoux, Theocharis, Pagneux *to appear*]

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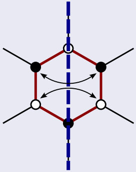

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Thank you.

Different types of edges: